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Edge Domination in Picture Fuzzy Graphs

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ABSTRACT: In this paper, the concepts of edge domination in picture fuzzy graphs are introduced. The edge domination number $\gamma_e(G)$ for some standard picture fuzzy graphs was determined. The edge independent set of a picture fuzzy graphs are defined. Theorems related to edge domination and edge independent set in picture fuzzy graphs are stated and proved.

KEYWORDS: Picture fuzzy graph, Edge dominating set, Edge domination number, Edge independent set, Edge independent number of picture fuzzy graph.
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I. INTRODUCTION

L A Zadeh[11] introduced the concept of fuzzy sets in the year 1965. Rosenfeld[9] gave the notion of fuzzy graph and several fuzzy concepts such as paths, cycles and connectedness in 1975. Somasundaram[10] investigated the concept of domination number in fuzzy graphs in the year 1998. In 1986 Atanassov introduced intuitionistic fuzzy sets. Parvathi and Tamizhendhi[8] introduced domination number in intuitionistic fuzzy graph. Ismayil and Ali[6] discussed about strong interval valued intuitionistic fuzzy graph in the year 2014. Cuong[4] introduced the picture fuzzy sets(PFS) which is a direct extention of fuzzy set and intuitionistic fuzzy set. The picture fuzzy set was constructed by adding a neutral membership degree of an element to the existing intuitionistic fuzzy set. In a picture fuzzy set every element contains positive, negative and neutral membership degree. Cuong and Kreinovich[4] studied some compositions of picture fuzzy relations. Al-Hawary T etal.,[1] provided the concept of picture fuzzy graph(PFG). They also discussed bridge and cut vertex in PFG. A Mohamed Ismayil and Asha Bosley[5] defined order and size of a PFG, also introduced domination in PFG.
In this paper, the concept of edge dominating set in a picture fuzzy graph, edge domination number$\gamma_e(G)$, edge independent set, maximal edge independent number $\beta_e(G)$and edge cover numbers $\alpha_e(G)$are defined with suitable example. The bounds on edge dominating set for various standard PFG are obtained and also some theorems related to this concepts are stated and proved.

II. PRELIMINARIES

In this section, the definition of fuzzy graph, intuitionistic fuzzy graph and picture fuzzy graphs are given. The arc, effective edges, strong and complete picture graphs are also given which are useful to understandedge domination in picture fuzzy graphs.

Definition 2.1[8]:A fuzzy graph $G = (\mu, \rho)$defined on $G^* = (V, E)$is a set with two functions $\mu: V \to [0,1]$ and $\rho: E \to [0,1]$ such that$\rho(\{x, y\}) \leq \mu(x) \wedge \mu(y) \forall x, y \in V$. Hereafter we write $\rho(xy)$ for $\rho(\{x, y\})$.

Definition 2.2[7]: A pair $G = (\mu, \rho)$is known as intuitionistic fuzzy graph(IFG) if

(i) $V = \{v_1, v_2, \dots, v_n\}$ suchthat $\mu_1: V \to [0,1]$, $\rho_1: V \to [0,1]$ denote the degree of membership and non-membership $\forall v_i \in V$, respectively and $0 \leq \mu_1(v_i) + \rho_1(v_i) \leq 1$, for every $v_i \in V$, $i = 1,2, \dots, n$.

(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$ and $\rho_2: V \times V \to [0,1]$ are such that $\mu_2(v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$, $\rho_2(v_i, v_j) \geq \rho_1(v_i) \vee \rho_1(v_j)$ and $0 \leq \mu_2(v_i, v_j) + \rho_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$ $(i, j = 1,2, \dots, n)$. Here the triple $(v_i, \mu_{1i}, \rho_{1i})$ denotes the degree of membership and degree of non-membership of the vertex v_i. The triple $(e_{ij}, \mu_{2ij}, \rho_{2ij})$ denote the degree of membership and degree of non-membership of the edge relation $e_{ij} = (v_i, v_j)$ on V.

In an intuitionistic fuzzy graphG, when $\mu_{2ij} = 0$ and $\rho_{2ij} = 1$for some i and j, then there is no edge between v_i and v_j. Otherwise there exists an edge between v_i and v_j.

Definition 2.3[3]: A pair $G = (\widetilde{V}, \widetilde{E})$ defined on $G^* = (V, E)$is known as Picture Fuzzy Graph(PFG) if

(i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1: V \to [0,1]$, $\sigma_1: V \to [0,1]$, $\rho_1: V \to [0,1]$ denote the degree of truth membership, abstinence membership and false membership of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \sigma_1(v_i) + \rho_1(v_i) \le 1$, for every $v_i \in V$, $i = 1,2,\dots,n$.
(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$, $\sigma_2: V \times V \to [0,1]$ and $\rho_2: V \times V \to [0,1]$ are such that $\mu_2(v_i, v_j) \le \mu_1(v_i) \wedge \mu_1(v_j)$, $\sigma_2(v_i, v_j) \le \sigma_1(v_i) \wedge \sigma_1(v_j)$ and $\rho_2(v_i, v_j) \ge \rho_1(v_i) \vee \rho_1(v_j)$ where $0 \le \mu_2(v_i, v_j) + \sigma_2(v_i, v_j) + \rho_2(v_i, v_j) \le 1$ for every $(v_i, v_j) \in E$ $(i, j = 1,2,\dots,n)$.
Here the 4-tuple $(v_i, \mu_{1i}, \sigma_{1i}, \rho_{1i})$ denotes the degree of truth membership, abstinence membership and false membership of the vertex v_i. The 4-tuple $(e_{ij}, \mu_{2ij}, \sigma_{2ij}, \rho_{2ij})$ denotes the degree of truth membership, abstinence membership and false membership of the edge relation $e_{ij} = (v_i, v_j)$ on V.

Remark 2.4: In a picture fuzzy graph G, if $\mu_{2ij} = \sigma_{2ij} = 0$ and $\rho_{2ij} = 1$ for some i and j then there is no edge between v_i and v_j. Otherwise there exists an edge between v_i and v_j.

Definition 2.5: The weight of the vertex v_i in a PFG $G = (\tilde{V}, \tilde{E})$ is defined by $\frac{1+\mu_1(v_i)+\sigma_1(v_i)-\rho_1(v_i)}{2}$ and is denoted by $w(v_i)$. Similarly the weight of the edge e_j is defined by $\frac{1+\mu_2(e_j)+\sigma_2(e_j)-\rho_2(e_j)}{2}$ and is denoted by $w(e_j)$.

Remark 2.6: If $w(e_{ij}) = 0$ then there exists no edge between v_i and v_j.

Definition 2.7: Let $G = (\tilde{V}, \tilde{E})$ be a picture fuzzy graph. Then the vertex cardinality of G is defined by $|V| = \sum_{v_i \in V} \frac{1+\mu_1(v_i)+\sigma_1(v_i)-\rho_1(v_i)}{2}$ and $|V|$ is called the order of a PFG G and is denoted by p. Similarly the edge cardinality of G is defined by $|E| = \sum_{(v_i,v_j) \in E} \frac{1+\mu_2(v_i,v_j)+\sigma_2(v_i,v_j)-\rho_2(v_i,v_j)}{2} \forall v_i, v_j \in E$ and $|E|$ is called the size of a PFG $G = (V, E)$ and is denoted by q.

Definition 2.8: An arc (u, v) in a PFG, $G = (\tilde{V}, \tilde{E})$ is said to be strong arc, if $\mu_2(u,v) \ge \mu_2^{\infty}(u,v)$, $\sigma_2(u,v) \ge \sigma_2^{\infty}(u,v)$ and $\rho_2(u,v) \le \rho_2^{\infty}(u,v)$.

Definition 2.9: An edge (u, v) is said to be effective edge in a PFG if $\mu_2(u,v) = \mu_1(u) \wedge \mu_1(v)$, $\sigma_2(u,v) = \sigma_1(u) \wedge \sigma_1(v)$ and $\rho_2(u,v) = \rho_2(u) \vee \rho_2(v)$. Otherwise the edge (u, v) is called non effective edge.

Definition 2.10: A PFG $G = (\tilde{V}, \tilde{E})$ is said to be strong PFG if $\mu_{2ij} = \mu_1(v_i) \wedge \mu_1(v_j)$, $\sigma_{2ij} = \sigma_1(v_i) \wedge \sigma_1(v_j)$ and $\rho_{2ij} = \rho_1(v_i) \vee \rho_1(v_j)$ for every $(v_i, v_j) \in E$.

Definition 2.11: A PFG $G = (\tilde{V}, \tilde{E})$ is said to be complete PFG if $\mu_{2ij} = \mu_1(v_i) \wedge \mu_1(v_j)$, $\sigma_{2ij} = \sigma_1(v_i) \wedge \sigma_1(v_j)$ and $\rho_{2ij} = \rho_1(v_i) \vee \rho_1(v_j)$ for every $v_i, v_j \in V$.

Definition 2.12: Two vertices u and v are said to be neighbors in PFG $G = (\tilde{V}, \tilde{E})$ if either one of the following conditions hold
(i) $\mu_2(u,v) > 0$, $\sigma_2(u,v) > 0$, $\rho_2(u,v) > 0$
(ii) $\mu_2(u,v) = 0$, $\sigma_2(u,v) \ge 0$, $\rho_2(u,v) > 0$
(iii) $\mu_2(u,v) > 0$, $\sigma_2(u,v) = 0$, $\rho_2(u,v) \ge 0$
(iv) $\mu_2(u,v) \ge 0$, $\sigma_2(u,v) > 0$, $\rho_2(u,v) = 0$, $u, v \in V$.

Definition 2.13: Let u be a vertex in a PFG $G = (\tilde{V}, \tilde{E})$ then $N_E(u) = \{v \in V: (u,v)$ is an effective edge$\}$ is called open effective neighborhood of u. $N_E[u] = N_E(u) \cup \{u\}$ is called the closed effective neighborhood of u.

Definition 2.14: If (u, v) is an effective edge then u dominates v and v dominates u.

Definition 2.15[4]: A subset D of V is called a dominating set in a PFG G if for every vertex $v \in V - D$, there exists a vertex $u \in D$ which dominates v. A dominating set D of a PFG $G = (V, E)$ is said to be minimal dominating set if for each vertex $v \in D$, $D - \{v\}$ is not a dominating set of G. The minimum cardinality among all minimal dominating sets in G is called the domination number of G and is denoted by $\gamma(G)$ or simply γ. The maximum cardinality among all minimal dominating sets in G is called the upper domination number of G and is denoted by $\Gamma(G)$ or simply Γ.

Definition 2.16: Two vertices $u, v \in V$ in a PFG, $G = (\tilde{V}, \tilde{E})$ are said to be independent if $\mu_2(u,v) < \mu_1(u) \wedge \mu_1(v)$ or $\sigma_2(u,v) < \sigma_1(u) \wedge \sigma_1(v)$ or $\rho_2(u,v) > \rho_1(u) \vee \rho_1(v)$.

Definition 2.17: A subset I of V in a PFG, $G^* = (V, E)$ is said to be independent set of G if $\mu_2(u,v) < \mu_1(u) \wedge \mu_1(v)$ or $\sigma_2(u,v) < \sigma_1(u) \wedge \sigma_1(v)$ or $\rho_2(u,v) > \rho_1(u) \vee \rho_1(v) \forall u, v \in I$.

Remark 2.18: The set I is an independent set if and only if no two vertices of I are adjacent.

Definition 2.19: An independent set I of a PFG $G = (\tilde{V}, \tilde{E})$ is said to be maximal independent set, if for every $v \in V - I$, the set $I \cup \{v\}$ is not independent.

Definition 2.20: The maximum cardinality among all maximal independent set in G is called the independence number of G and is denoted by $\beta(G)$. The minimum cardinality among all maximal independent set in G is called the lower independence number of G and is denoted by $i(G)$.

Definition 2.21: Let $e = (u, v)$ be an effective edge in a PFG $G = (\tilde{V}, \tilde{E})$ then u, v and e are incident on each other.

Definition 2.22: A vertex v and an effective edge (u, v) in a PFG $G = (\tilde{V}, \tilde{E})$are said to be cover each other if they are incident.
Definition 2.23: A subset S of V in a PFG $G = (\tilde{V}, \tilde{E})$which covers all edges in G is called a vertex covering set of G. The minimum cardinality among all vertex covering set in G is called the vertex covering number of G and is denoted by $\alpha(G)$.
Definition 2.24: The complement of PFG$G = (\tilde{V}, \tilde{E})$ is PFG$\bar{G} = (\bar{V}, \bar{E})$where
(i) $\bar{V} = V$.
(ii)$\overline{\mu_{1i}} = \mu_{1i}$; $\overline{\sigma_{1i}} = \sigma_{1i}$; $\overline{\rho_{1i}} = \rho_{1i} \forall i = 1,2, \dots, n$.
$\overline{\mu_{2j}} = \mu_{1i} \wedge \mu_{1j} - \mu_{2ij}$, $\overline{\sigma_{2j}} = \sigma_{1i} \wedge \sigma_{1j} - \sigma_{2ij}$, $\overline{\rho_{2j}} = \rho_{1i} \vee \rho_{1j} - \rho_{2ij}$ for all $i, j = 1,2, \dots, n$.

III. EDGE DOMINATION IN PICTURE FUZZY GRAPHS

In this section, picture fuzzy graphs without isolated vertices are considered. The bounds on picture fuzzy graphs are obtained and theorems based on picture fuzzy graphs are stated and proved.
Definition 3.1: Let e_i and e_j are adjacent if they incident with a common vertex.
Definition 3.2: Let e_i, e_j aretwo edges in a PFG $G = (\tilde{V}, \tilde{E})$.The set$N_E(e_i) = \{e_j \in E: e_i$ is adjacent to e_j and e_j is an effective edge} is called open effective neighborhood of e_i. $N_E[e_i] = N_E(e_i) \cup \{e_i\}$ is called the closed effective neighborhood of e_i.
Definition 3.3: An edge $e$in a PFG $G = (\tilde{V}, \tilde{E})$is an isolated edge if $N_E(e) = \emptyset$.
Definition 3.4: Let e_i, e_j are two edges in a PFG $G = (\tilde{V}, \tilde{E})$. If e_i is an effective edge and adjacent to e_j then $e_i$dominatese_j.
Remark 3.5: Let e_i ande_j are two adjacent edges in a PFG$G = (\tilde{V}, \tilde{E})$.
(i)If $e_i and e_j$ are two effective edges then e_i dominatese_j and also e_j dominatese_i.
(ii)If $e_i$is an effective edge and e_j is a non effective edge then e_i dominatese_j but e_j does not dominatee_i.
(iii)If e_i, e_j are two non effective edges, then $e_i and e_j$ does not dominate each other.
Example 3.6: Consider the PFG given in figure 1,e_1 dominatese_2, but $e_2$does not dominate e_1.
Definition 3.7: Let $G = (\tilde{V}, \tilde{E})$be a picture fuzzy graph. A subset S of E is called an edge dominating set in a PFG G if for every edge$e_j \in E - S$, then there exists an edge $e_i \in S$ such that $e_i$dominatese_j.
Definition 3.8: An edge dominating set S in a PFG $G = (\tilde{V}, \tilde{E})$is said to be minimal edge dominating set if for each edge $e_i \in S$, $S - \{e_i\}$ is not an edge dominating set of$G$.The minimum cardinality among all the minimal edge dominating sets in G is called the edge domination number of G and is denoted by $\gamma_e(G)$ or simplyγ_e.The maximum cardinality among all the minimal edge dominating set in G is called the upper domination number of G and is denoted by $\Gamma_e(G)$ or simplyΓ_e.
Remark 3.9:
1. For any effective edge$e_i \in E$, $N_E(e_i)$ is precisely the set of all edges in E which are dominated bye_i.
2. If $\mu_2(u, v) < \mu_1(u) \wedge \mu_1(v)$, $\sigma_2(u, v) < \sigma_1(u) \wedge \sigma_1(v)$ and $\rho_2(u, v) > \rho_1(u) \vee \rho_1(v) \forall u, v \in E$ then the only edge dominating set of $G$is$E$.
3. Let e_i be an edge in a PFG$G = (V, E)$. If$N_E(e_i) = \emptyset$, then every edge dominating set of Gcontainse_i.
4. Non adjacent non effective edges with effective edges are members of an edge dominating set.
Example 3.10: Consider the picture fuzzy graph.

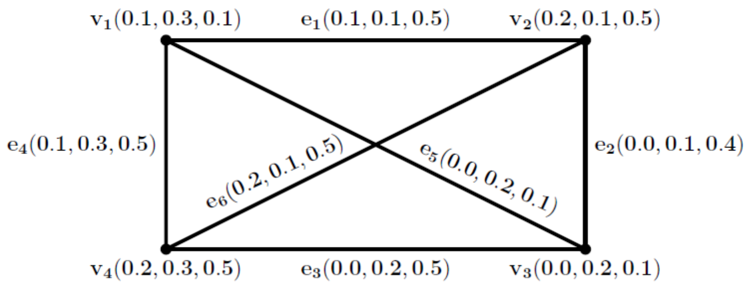

Figure: 1

The edge dominating sets are $D_1 = \{e_5, e_6\}$, $D_2 = \{e_1, e_3\}$ and $D_3 = \{e_4\}$.
$|D_1| = |\{e_5, e_6\}| = 0.35 + 0.3 = 0.65$.
$|D_2| = |\{e_1, e_3\}| = 0.25 + 0.15 = 0.4$.
$|D_3| = |\{e_4\}| = 0.15$.
Therefore $\gamma_e(G) = 0.15$ and $\Gamma_e(G) = 0.65$.
Theorem 3.11: If K_σ is a complete PFG of order p and n vertices then$\gamma_e(K_\sigma) \leq \frac{n}{2}$.

Proof : In a complete PFG, Choose an edge $e_1$it is adjacent to $2(n-2)$ edges. Delete all these $2(n-2)$ edges we get the complete graph with $n-2$ vertices. Similarly choose an edge e_2 from new PFG it is adjacent to $2(n-4)$ edges. Delete all these $2(n-4)$ edges we get the complete graph with $n-4$ vertices. Continuing this process we get the edge dominating set with the vertices$\begin{cases} \frac{n}{2} & , ifniseven \\ \frac{n-1}{2}, & ifnisodd \end{cases}$. Hence $\gamma_e(K_\sigma) \leq \frac{p}{2}$.

Preposition 3.12: For any picture fuzzy path of order p, $\gamma_e(G) \leq p$.

Preposition 3.13: In a strong picture fuzzy cycle of order pand$n$ vertices, $\gamma_e(C_\sigma) \leq \frac{p}{3}$.

Preposition 3.14: In a strong picture fuzzy path of order p and$n \geq 3$ vertices, $\gamma_e(P_\sigma) \leq \frac{p+3}{3}$.

Theorem 3.15: An edge dominating set S of a PFG, $G=(\tilde{V},\tilde{E})$is a minimal edge dominating set if and only if for each $e_i \in S$ one of the following conditions hold:

(i) $N_E(e_i) \cap S = \phi$

(ii) There is an edge $e_j \in E-S$ such that $N_E(e_j) \cap S = \{e_i\}$.

Proof: Let S be a minimal edge dominating set of G. Then for every $e_i \in S,\ S-\{e_i\}$ is not an edge dominating set of G and hence there exists an edge $e_j \in E-(S-\{e_i\})$ which is not adjacent with any edge in $S-\{e_i\}$.

Case(i): If $e_i = e_j$ then $N_E(e_j) \cap S = \phi$

Case(ii): If $e_i \neq e_j,\ e_j$ is not dominated by any edge in $S-\{e_i\}$, but is dominated by S, then the edge e_j effective neighbor with only to e_i in S. Hence $N_E(e_j) \cap S = \{e_i\}$. Conversely, assume that S is an edge dominating set and for each edge $e_i \in S$ one of two conditions hold. Suppose S is not a minimal edge dominating set, then there exists an edge $e_i \in S,\ S-\{e_i\}$ is an edge dominating set. Hence e_i is a effective neighbor to atleast one edge in$S-\{e_i\}$, then condition(i) does not hold. If $S-\{e_i\}$ is a dominating set then every edge in $E-S$ is a effective neighbor to atleast one edge in $S-\{e_i\}$ then second condition does not hold which contradicts to our assumption that atleast one of this conditions hold. Hence S is a minimal edge dominating set.

Observation 3.16: Let $G=(\tilde{V},\tilde{E})$be a PFG without isolated edges. If S is a minimal edge dominating set of G. Then $E-S$not an edge dominating set of$G$.

Example 3.17: Consider the PFG $G=(\tilde{V},\tilde{E})$where $\tilde{V}=\{v_1|(0.1,0.3,0.1), v_2|(0.2,0.1,0.5), v_3|(0.0,0.2,0.1)\}and\tilde{E}=\{(v_1,v_2)|(0.1,0.1,0.5),(v_2,v_3)|(0.0,0.1,0.7)\}$. Here$(v_1,v_2)$ is an effective edge but (v_2,v_3) is not an effective edge. Therefore (v_1,v_2) is an edge dominating set but (v_2,v_3) is not an edge dominating set.

Theorem 3.18: Let $G=(\tilde{V},\tilde{E})$be a strong PFG and without isolated edges. If S is a minimal edge dominating set of G. Then $E-S$ is an edge dominating set of$G$.

Proof: Let e_i be any effective edge in$S$. Since G had no isolated edges, then there is an edge$e_j \in N_E(e_i)$. It follows from the theorem 3.15, that$e_j \in E-S$. Since G is a strong PFG then e_j is not isolated. Thus every element of S is dominated by some edge in $E-S$ and hence $E-S$ is an edge dominating set of$G$.

Definition 3.19: Two edges $e_i, e_j \in E$ in a PFG $G=(\tilde{V},\tilde{E})$are said to be independent if $e_i \notin N_E(e_j)$ or$e_j \notin N_E(e_i)$.

Definition 3.20: The set I is an edge independent set if and only if no two edges of $I$are adjacent.

Definition 3.21: An edge independent set I of a PFG $G=(\tilde{V},\tilde{E})$is said to be maximal edge independent set, if for every$e_i \in E-I$, the set $I \cup \{e_i\}$ is not independent. The maximum cardinality among all the maximal edge independent set in G iscalled the independence number of G and is denoted by$\beta_e(G)$. The minimum cardinality among all the maximal edge independent set in G is called the lower independence number of G and is denoted by$i_e(G)$.

Theorem 3.22: An edge independent set I of a PFG $G=(\tilde{V},\tilde{E})$is a maximal edge independent set of G if and only if it is edge independent and edge dominating set.

Proof: Let I be a maximal edge independent set in PFG $G=(\tilde{V},\tilde{E})$and hence for every edge $e_j \in E-I$, the set $I \cup \{e_j\}$ is not edge independent. For every edge$e_j \in E-I$, there is an edge $e_i \in I$ such that e_i and e_j are effective edges. Thus I is an edge dominating set. Hence I is both edge dominating and edge independent set.

Conversely, assume I is both edge independent and edge dominating set. Suppose I is not maximal edge independent, then there exists an edge$e_j \in E-I$, the set $I \cup \{e_j\}$ is edge independent. If $I \cup \{e_j\}$ is edge independent then no edge in I is effective neighbor toe_j. Hence I is not an edge dominating set, which is a contradiction to our assumption. Hence I is maximal edge independent set.

Definition 3.23: Let $e=(u,v)$ be an edge in a PFG $G=(\tilde{V},\tilde{E})$then e covers u and$v$. The set $I \subseteq E$ covers all the vertices of V is called edge cover set of$G$. An edge independent set I is said to be minimal edge cover set, if for every$e_i \in E-I$, the set $I-\{e_i\}$ is not an edge cover set. The minimum cardinality among all the minimal edge cover set in G is called the edge cover number of G and is denoted by$\alpha_e(G)$. The maximum cardinality among all the minimal edge cover set in G is called the upper edge cover number of G and is denoted by$\beta_e(G)$.

Example 3.24: In figure-1 $\{e_1, e_3\}, \{e_5, e_6\}$ are edge cover set of$G$.

Theorem 3.25: In a PFG$G = (\tilde{V}, \tilde{E})$, if$I$ be the maximal edge independent set then $E - I$ is an edge cover set.

Proof: Let I be the maximal edge independent set. From theorem 3.22,I is also a dominating set. Therefore for every$v_i \in I$ there exists$v_j \in E - I$such that $v_i$dominatesv_j. By the definition of edge cover every edge $e_i \in E - I$ is incident with atleast one vertex$v_i \in I$. Therefore $E - I$ is the edge cover of the maximal independent set$I$.

Theorem 3.26: Every maximal edge independent set I in a PFG $G = (\tilde{V}, \tilde{E})$is a minimal edge dominating set.

Proof: Let I be a maximal edge independent set in a PFG$G = (\tilde{V}, \tilde{E})$. By theorem 3.22, I is an edge dominating set. Suppose I is not a minimal edge dominating setand then there exists atleast one edge $e_j \in I$ for which $I - \{e_j\}$ is an edge dominating set. Then atleast one edge in $I - \{e_j\}$ must be an effective neighbor to e_j. This contradicts to the fact that I is an edge independent set of$G$. Therefore I must be minimal edge dominating set.

Theorem 3.27: For any PFG $G = (\tilde{V}, \tilde{E})$without isolated edges$\alpha_e(G) + \beta_e(G) \leq q$.

Proof: Let I be an maximum edge independent set of G such that $|I| = \beta_e(G)$ and$K$be a minimum edge cover of Gsuch that$|K| = \alpha_e(G)$. If I isa maximum edge independent set, then $E - I$ is an edge covering set of$G$. Hence$|K| \leq |E - I|$. This implies that$\alpha_e(G) \leq q - \beta_e(G)$ and hence$\alpha_e(G) + \beta_e(G) \leq q$.

Theorem 3.28: For any PFG$G = (\tilde{V}, \tilde{E})$, $\gamma_e(G) + \gamma_e(\bar{G}) \leq 2q$ and equality holds if and only if $0 < \mu_2(u, v) < \mu_1(u) \wedge \mu_1(v)$, $0 < \sigma_2(u, v) < \sigma_1(u) \wedge \sigma_1(v)$ and $0 < \rho_2(u, v) > \rho_1(u) \vee \rho_1(v) \forall u, v \in V$.

Proof: The inequality is trivial. Further $\gamma_e = q$ if and only if $\mu_2(u, v) < \mu_1(u) \wedge \mu_1(v)$, $\sigma_2(u, v) < \sigma_1(u) \wedge \sigma_1(v)$ and $\rho_2(u, v) > \rho_1(u) \vee \rho_1(v) \forall u, v \in V$, $\gamma_e(\bar{G}) = q$ if and only if $\overline{\mu_2(u, v)} = \mu_1(u) \wedge \mu_1(v) - \mu_2(u, v)$, $\overline{\sigma_2(u, v)} = \sigma_1(u) \wedge \sigma_1(v) - \sigma_2(u, v)$ and $\overline{\rho_2(u, v)} = \rho_1(u) \vee \rho_1(v) - \rho_2(u, v) \forall u, v \in V$ which is equivalent to $\overline{\mu_2(u, v)} < \mu_1(u) \wedge \mu_1(v)$ since $\mu(u, v) > 0$; $\overline{\sigma_2(u, v)} < \sigma_1(u) \wedge \sigma_1(v)$ since $\sigma(u, v) > 0$ and $\overline{\rho_2(u, v)} > \rho_1(u) \vee \rho_1(v)$ since $\rho(u, v) > 0$. Hence $\gamma_e(G) + \gamma_e(\bar{G}) = 2p$ if and only if $0 < \mu_2(u, v) < \mu_1(u) \wedge \mu_1(v)$, $0 < \sigma_2(u, v) < \sigma_1(u) \wedge \sigma_1(v)$ and $0 < \rho_2(u, v) > \rho_1(u) \vee \rho_1(v) \forall u, v \in V$.

Theorem 3.29: Let $G = (\tilde{V}, \tilde{E})$be a PFG without isolated edges. Let p be the order and q be the size of G then $\frac{q}{\Delta_E(G)+1} \geq \gamma_e(G)$.

Proof: Let S be an edge dominating set of G.

Since $|S|\Delta_E(G) \leq \sum_{e \in S} d_E(e) = \sum_{e \in S} |N_E(e)| \leq |\cup_{e \in S} N_E(e)| \leq |E - S| \leq q - |S|$

$\Rightarrow |S|\Delta_E(G) + |S| \leq q$. Hence $\frac{q}{\Delta_E(G)+1} \geq \gamma_e(G)$.

Theorem 3.30: If S is a minimal edge dominating set in a complete PFG $G = (\tilde{V}, \tilde{E})$, then the edges of one of the edge dominating set S incident with the vertices have maximum degree.

Proof: Let S be an edge dominating set in$G$. Assume that the edges in an edge dominating set S are not incident with the vertices having maximum degree. Then edges of edge dominating set S are effective, which are incident with the vertices containing minimum degree. By the definition of edge dominating set, for each $e_j \in E - S$ there exists $e_i \in S$ such that $e_i$dominatese_j. Hence edge dominating set S must contain more number of effective edges. This implies S is not minimum, then it lead to contradiction. Hence edges of egde dominating set S should be incident with the vertices containing maximum degree.

Definition 3.31: Let $G = (\tilde{V}, \tilde{E})$ be a PFG and Sbe an edge dominating set in$G$.

(i) if the induced subgraph $E - S$ is disconnected then S is called split-edge dominating set of$G$.

(ii) if the induced subgraph $E - S$ is connected then S is called a non-split edge dominating set of$G$.

(iii) if the induced subgraph $E - S$ is a path then S is called a path non-split edge dominating set of$G$.

(iv) if the induced subgraph $E - S$ is a cycle then S is called a cycle non-split edge dominating set of$G$.

Example 3.32: (i) Consider the figure-2, Let $S = \{e_8, e_2, e_4\}$ is an edge dominating set and $E - S = \{e_1, e_3, e_5, e_6, e_7\}$ is disconnected.

(ii) Consider the figure-3, Let $S = \{e_2, e_6\}$ is an edge dominating set and $E - S = \{e_1, e_3, e_4, e_5\}$ is connected.

(iii) Consider the figure-4, Let $S = \{e_6, e_5\}$ is an edge dominating set and $E - S = \{e_1, e_2, e_3, e_4\}$ is a path.

(iv) Consider the figure-5, Let $S = \{e_7, e_8\}$ is an edge dominating set and $E - S = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ is a cycle.

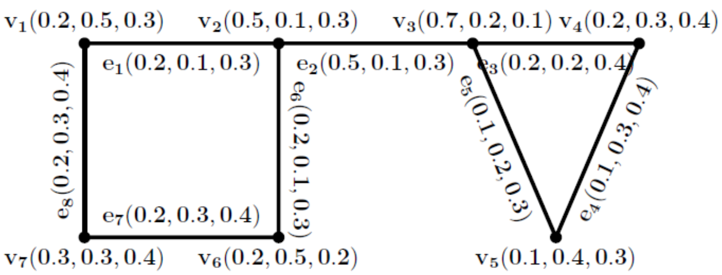

Figure-2

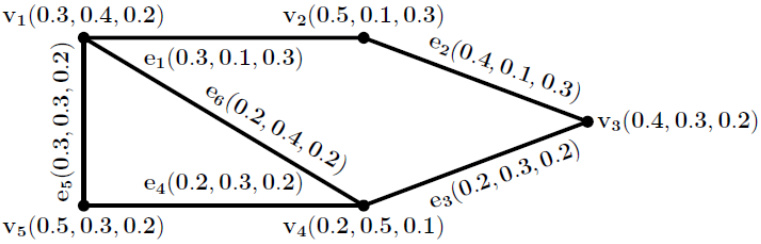

Figure-3

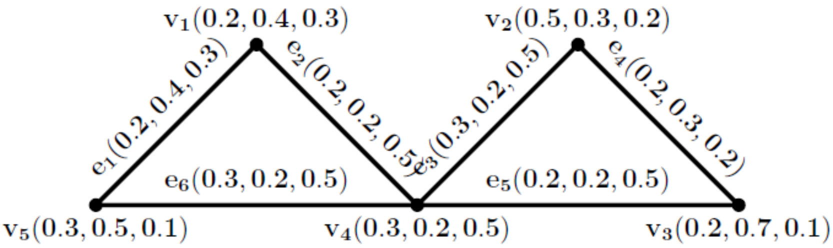

Figure-4

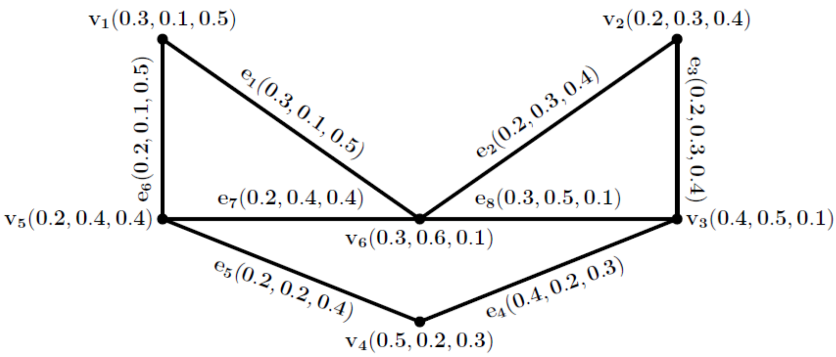

Figure-5

Theorem 3.33: Let S be a cycle non-split edge dominating set in a PFG $G = (\tilde{V}, \tilde{E})$, if $E - S$ vertex cover includes all the vertices of$G$.

Proof: Let$S$ is a cycle non-split edge dominating set of$G$. Assume that vertex cover of $E - S$ does not include all the vertices of$G$ then $E - S$ must be connected. If $E - S$ is connected, then any two edges of edge dominating set S contains no common vertex and is disconnected. This implies that S is a split edge dominating set that contradicts the assumption that S is a cycle non-split dominating set. Hence $E - S$ contains all the vertices.

Theorem 3.34: If S is an edge dominating set in a PFG $G = (\tilde{V}, \tilde{E})$then$\delta(G) \leq \gamma_e(G)$.

Proof: Let S be an edge dominating set of PFG G. By the definition of edge dominating set, S must be minimum and the edges should be effective. Minimum degree of G is nothing but minimum of degrees of all the vertices in G. Since degree of a vertex v is the sum of the weights of all effective edges incident in v, implies cardinality of an edge dominating set S should be maximum. Hence the minimum degree of G is less than the cardinality of an edge dominating set$S$. Therefore$\delta(G) \leq \gamma_e(G)$.

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