ISSN (e): 2250 – 3005 || Volume, 08 || Issue, 9|| Sepetember – 2018 ||
International Journal of Computational Engineering Research (IJCER)

Properties Of Elements In Ternary-Semirings

G. Srinivasa Rao[1,2], D. Madhusudhana Rao[3], P. Sivaprasad[4], G. Srinivasa Rao[5]
[1]Research Scholar, Department of Mathematics, Acharya Nagarluna University, Nagarluna Nagar, Guntur, A.P. India.
[2]PGT Mathematics, APSWRS Lr College, Karampudi, Guntur(Dt), A.P. India.
[3] Associate Professor, Department of Mathematics, VSR & NVR College, Tenali, A.P. India.
[4]Department of BSH, VFSTR'S University,Vadlamudi, Guntur, A.P. India.
[5]Associate Professor, Department of Mathematics, Tirumala Engineering College, Narasarao Pet, A.P.
Corresponding Author: D. Madhusudhana Rao.

ABSTRACT *:In this paper we made a study on zeroed, regular & idempotent elements in ternary semirings and we proved some properties of such elements in ternary semirings.*
KEYWORDS: *Zeroed, Regular, Idempotent, Ordered ternary semiring, Simple.*
AMS Subject Classification: 16Y60, 03G25

Date of Submission: 03-11-2018 Date of acceptance: 17-11-2018

I. INTRODUCTION

In the year 2011 Madhusudhana Rao, Anjaneyulu and Gangadhara rao inverstigated and studied about gamma semigroups. In the year 2013, Y. Sarala, D. Madhusudhana Rao, A. Anjaneyulu made a study on ternary semigroups. Further in the year 2013 Subramaneswara rao seetamraj, D. Madhusudhana rao and A. Anjaneyulu introduce the some notions in partially ordered gamma semigroups. P. Siva Prasad, D. Madhusudhana Rao and G. Srinivasa rao developed some properties in partially ordered ternary semirings. In this paper we introduce the properties of ternary semirings.

1. Preliminaries:

Definition 1.1: A **ternary semi ring** is a nonempty set U on which operations of addition and ternary multiplication have been defined such that the following conditions are satisfied:
(1) (U ,+) is a commutative monoid with identity element 0;
(2) (U, [ ]) is a ternary monoid with identity element 1_T,
(3) Ternary Multiplication distributes over addition,
(4) 00r = 0r0 = r00 = 0 for all r ∈ U. For more preliminaries refer the references.

2. Zeroed Elements in ternary semi rings:

Def 2.1: U is said to be **multiplicatively left cancellative** (MLC) if pqd = pqf implies that d = f for all p, q, d, f ∈ U.

Def 2.2: An element f ∈ U is known as **left (right) zeroed element** of U if ∃ h, g, l ∈ U ∋ fgh + l = l(l + ghf = l). An element f ∈ U is known as **zeroed element** of U if ∃ l ∈ U ∋ l + ghf = l or fgh + l = l. Every idempotent element of U is a zeroed element of U. The zeroed of U is the set of all f ∈ U ∋ fgh + l = l or l + ghf = l for some l ∈ U and it is denoted by Z.

Note 2.3: Some of the authors were defined the zeroed elements as left zeroed f + u = u as well as right zeroed as u + f = u, but both the conditions are equivalent, because if f is fgh = ghf = f. Since f is zero of U.

Theorem 2.4: Let U be ternary semi ring with unity element, $l + l^3 = l$ ∀ l ∈ U and l + 1 = l for all l ∈ U. If ternary semi ring U is left cancellative then every element of U is a zeroed.
Proof: Let l ∈ U ⇒ $\mathbf{l + l^3 = l}$. Since l.1.1 = l.
$\mathbf{l + l^3 = l}$ ⇒ $l^2v + l + l^3 = l^2v + l$ for v ∈ U
⇒ $l^2v + l.1.1 + l^3 = l^2v + l.1.1$
⇒ $l(lv + 1) + l^3 = l(lv \underline{+ 1)}$
⇒$\underline{llv}$ + l^3 = llv ⇒ ll(v + l) = llv ⇒ v + l = v.
Therefore, every element of U is a zeroed.

Def 2.5 [ ]: A nonempty subset V of a ternary semi ring U is known as **k-ideal** of U if $l \in U$, $l + v \in V$, $v \in V \Rightarrow l \in V$.

Theorem 2.6: Let U be the additively commutative ternary semi ring. The zeroed Z is a k-ideal of U.

Proof: Let $l, v \in Z$, $\exists$ f, g, p, $q \in U \ni lfg + p = p$, $p + fgl = p$ & $vfg + q = q$, $q + fgl = q$.
Suppose, $lfg + p = p$ & $vfg + q = q$, then

$$(l + v)fg + p + q = lfg + vfg + p + q = lfg + vfg + q + p$$
$$= lfg + q + p = lfg + p + q = p + q \Rightarrow l + v \in Z$$

lly, we can prove in each case $l + v \in Z$.
Now, l, v, $w \in Z$, then $lfg + p = p$, $p + fgl = p$, $vfg + q = q$, $q + fgv = q$ & $wfg + r = r$, $r + fgw = r \Rightarrow lvwfg + vwp = vwp$, $lvwfg + lwq = lwq$ & $lvwfg + lvr = lvr \Rightarrow lvw \in Z$. Therefore Z is a ternary subsemiring of U.
To show that Z is a k-ideal of U, let $t \in U$ and $l \in Z$ such that $l + t \in Z$, therefore there exist x, y, $z \in U$ such that$(l + t)yz + x = x$. But $lqr + p = p \ \forall$ p, q, $r \in U$.
Then we have $x + p = (l + t)yz + x + lqr + p = tyz + (lyz + p + lqr + x)$

$$= tyz + (lyz + p + x) = tyz + (p + x) = tyz + (x + p).$$

Therefore $t \in Z$ and hence Z of T is k-ternary ideal of U.

Corollary 2.7: Let U be a additive commutative simple ternary semiring and $Z \neq \{0\}$. Then every element of U is zeroed.

Theorem 2.8: Let U be a ternary semiring and every element of U is zeroed. Suppose ternary semiring is a singular then for each $l \in U$, $\exists$ $v \in U \ni l + lvvwx = v$ or $lvvwx + l = v$.

Proof: Let $l \in U \Rightarrow \exists$ w, x, $v \in U \ni lwx + v = v$ or $v + wxl = v$. Suppose, $v + wxl = v$, since U is singular w. r. t ternary multiplication i.e. $vvl = l$, $lvv = l$. Therefore, $v + wxl = v \Rightarrow v + wxvvl = v$. Suppose, $lwx + v = v \Rightarrow lvvwx + v = v$.

Theorem 2.9: Let U be a ternary semiring and (U, +) is right cancellative. If l is right zeroed of U iff $l^3 + l^5 = l^3$.

Proof: Let l is a right zeroed of U, then $v \in U \ni v + lll = v$ or $lll + v = v$.

$$\Rightarrow ll(lll + v) = llv \Rightarrow lllll + llv = llv \Rightarrow lll + lllll + llv = lll + llv$$
$$\Rightarrow lll + lllll = lll \Rightarrow l^3 + l^5 = l^3.$$

Conversely suppose that $l \in U \Rightarrow l^3 + l^5 = l^3$ & $l.1.1 = l \Rightarrow l^3 + l^5 = l^3 \Rightarrow l^3(1 + l^2) = l^3 \Rightarrow 1 + ll1 = 1 \Rightarrow l$ is a zeroed of U and hence every element of U is zeroed.

Corollary 2.10: Let U be a ternary semiring and (U, +) is left cancellative. If l is left zeroed of U iff $l^5 + l^3 = l^3$.

Theorem 2.11:In a ternary semiring U with unity element and unit is zeroed then every element of U is zeroed.

Proof: Suppose U is a ternary semiring with unity element and $l \in U$ so $\exists$ $v \in U \ni 1 + v = v = v + 1$ & $l.1.1 = l = 1.1.l$. Let $v + 1 = v \Rightarrow l.1(v + 1) = l.1.v \Rightarrow l.1.v + l.1.1 = l.1.v \Rightarrow l1v + l = l1v$.
Let $1 + v = v \Rightarrow l.1.(1 + v) = l.1.v \Rightarrow l.1.1 + l.1.v = l.1.v \Rightarrow l + l1v = l1v$. Hence every element of U is zeroed.

Def 2.12: A ternary semiring U is said to be **semi-subtractive** if for any elements a; $b \in T$; there is always some $x \in T$or some $y \in U \ni a + y = b$ or $b + x = a$. Every ring is a semi-subtractive ternary semiring.

Theorem 2.13: Let the zeroed of a semi subtractive ternary semiring U is empty, then (U, +) is cancellative.

Proof: Suppose $v + l = w + l \Rightarrow v \neq w$ for l, v, $w \in U$. then $\exists$ some $a \in U \ni v = a + w$ or $w = a + v$. Suppose $v = a + w \Rightarrow v + l = a + w + l \Rightarrow v + l = a + v + l \Rightarrow v + l = awx + (v + l)$, since $awx = a$ and hence a is zeroed element of U, it is a contradiction and hence (U, +) is a right cancellative. Therefore (U, +) is cancellative.

Def 2.14: A ternary semiring U is known as **zero cube ternary semiring** if $l^3 = 0 \ \forall \ l \in U$.

Theorem 2.15: Suppose U is a zero cube ternary semiring and l is a zeroed of U then $\exists$ v, w, $x \in U$ $\ni$ $lvwxx = 0$ or $xxvwl = 0$ or $xvwlx = 0$.

Proof: Suppose $l \in U$, $\exists$ v, w, $x \in U \ni uvw + x = x$ or $x + vwl = 0$. Let $lvw + x = x$

$\Rightarrow$ (lvw + x)xx = xxx $\Rightarrow$ lvwxx + x^3 = x^3 $\Rightarrow$ lvwxx + 0 = 0 $\Rightarrow$ lvwxx = 0. Similarly we can prove the other parts.

3. Idempotent and Regular Elements in Ternary Semirings:

Def 3.1: An element l $\in$ U is known as **+ve idempotent** if l + l = l. The set of all additive idempotent of a ternary semiring U is denoted by $E^+(U)$.

Example 3.2: R$\cup\{-\infty\}$ is a commutative, additive idempotent ternary Γ-semiring with the addition and multiplication operations where Γ = R defined as: $l \oplus v$ = max(l,v) and $l \otimes v \otimes w$ = l + v + w where + is the ordinary addition as ternary semiring multiplication. Clearly, $-\infty$ is the zero element, and 0 is the unity.

Def 3.3: An element l $\in$ U is known as an **idempotent** if $u^3 = u$. The set of all idempotent elements in a ternary semiring U is denoted by E(U). Every identity, zero elements are idempotent elements.

Def 3.4: An element l $\in$ U is known as a **proper dempotent** element provided l is an idempotent which is not the identity of U if identity exists. A ternary semiring U is said to be an **idempotent ternary semiring** provided every element of U is an idempotent.

Def 3.5: An element l $\in$ U is known as **ternary multiplicatively sub-idempotent** provided l + l^3 = l. A ternary semiring U is said to be a **sub-idempotent ternary semiring** provided each of its element is sub-idempotent.

Def 3.6: A ternary semiring U is said to be **Viterbi ternary semiring** if U is +vely idempotent and multiplicatively sub-idempotent.

Theorem 3.7: Let U be a ternary semiring satisfying the identity l + lvl + l = l $\forall$ l, v$\in$ U. If U contains the ternary multiplicative identity which is also an additive identity, then U is a multiplicatively sub-idempotent ternary semiring.

Proof: Since u + lvl + l = l $\forall$ l, v$\in$ U. Let e be the multiplicative identity in e is also an additive identity. i.e., lee = ele = eel = l and l + e = e + l = l. Given l + lvl + l = l $\forall$ l, v$\in$ U. Taking l = v. Then l + l^3 + l = l $\Rightarrow$ l+ l(l^2 + e^2) = l $\Rightarrow$ l + l^3 = l + 0 $\Rightarrow$ l + l^3 = l $\forall$ l $\in$ U. Therefore, U is multiplicatively sub-idempotent ternary semiring.

Theorem 3.8: Let U be a multiplicatively idempotent ternary semiring. If U satisfying the identity l + lvl + l = l $\forall$ l, v$\in$ U. Then U is additively idempotent.

Proof: Let l $\in$ U. Consider l + l = $(l + l)^3$ $\forall$ l $\in$ U. l + l = $(l + l)^3$ = $l^3 + l^3 + l^3 + l^3 + l^3 + l^3 + l^3 + l^3$ = (l + lll + l) + (l + lll + l) + l + l = l + l + l + l = l + lll + lll + l = l + (l + l)l + l = l. Therefore, U is additively idempotent.

Theorem 3.9: Every ternary semiring U in which U is left singular semigroup w.r.t addition and left singular ternary semigroup w.r.t ternary multiplication, then U is a Viterbi ternary semiring.

Proof: By the hypothesis U is left singular semigroup w. r. t addition and left singular ternary semigroup w. r. t ternary multiplication. i.e., l + v = l and lvl = l $\forall$ l, v $\in$ U. Consider l + v = l. Taking l = v $\Rightarrow$ l + l = l$\longrightarrow$ (1) & lvl = l. Taking l = v $\Rightarrow$ l^3 = l $\Rightarrow$ l + l^3 = l + l = l $\Rightarrow$ l + l^3 = l$\longrightarrow$ (2). Therefore, from (1) and (2), U is a Viterbi ternary semiring.

Note 3.10: The converse of the theorem 3.9, is not necessarily true. This is evident from the following example.

Example 3.11:

+	l	v
l	l	l
v	l	v

[ ]	l	v
l	l	l
v	v	v

Def 3.12: An element l of a ternary semiring U is known as **+vely regular** if there exist v $\in$U $\ni$ l + v + l = l. An element l $\in$ U is known as **ternary multiplicatively regular** if $\exists$ v, w $\in$ U $\ni$ lvlwl = l. U is known as **regular ternary semiring** if every element is regular.

Example 3.13: Let U = {0, u, v} be any nonempty set. If we define a binary operation + and ternary multiplication on U as the following Cayley table, then U is a regular ternary semiring.

[]	0	u	v
0	0	0	0
u	0	u	u
v	0	v	v

+	0	u	v
0	0	u	v
u	u	v	u
v	v	u	u

Lemma 3.14: Every idempotent element is regular.

Def 3.15[ ]: A ternary semiring U is known as an **Ordered Ternary Semiring** if U is partially ordered set such that $l \leq u$ then (1) $l + v \leq u + v$ & $v + l \leq v + u$, (2) $lvw \leq uvw$, $vlw \leq vuw$ & $vul \leq vwu$ $\forall$ l, u, v, w $\in$ U.

Th 3.16: Let U be a ordered ternary semiring in which (U, +) is +vely ordered ternary semiring. If u $\in$ U & v be a idempotent of (U, +), uzx $\leq$ v implies u is zeroed forz, x $\in$ U.

Proof: Let u, z, x $\in$ U & v be a idempotent, $uzx \leq v \Rightarrow uzx + v \leq v + v \Rightarrow uzx + v \leq v$, but $uzx + v \geq v$ and therefore, $uzx + v = v$. Thus u $\in$ Z, hence u is zeroed of U.

Corollary 3. 17: Let U be a ordered ternary semiring in which (U, +) is -vely ordered ternary semiring. If u $\in$ U & v be a idempotent of (U, +), uzx $\leq$ v implies v is zeroed forz, x $\in$ U.

Theorem 3.18: Let U be a ternary semiring, every element U is zeroed. If ternary semigroup U is band then for each u $\in$ U, $\exists$ v, z, x $\in$ U$\ni$ uzx + uzxvv + v = v.

Proof : Let u $\in$ U, $\exists$ v $\in$ U $\ni$ $v = uzx + v$ & $v^3 = v$. $v = uzx + v \Rightarrow v = uzx + v^3$
$\Rightarrow v = uzx + (uzx + v)v^2 \Rightarrow v = uzx + uzxvv + v^3$
$\Rightarrow v = uzx + uzxvv + v$.

Theorem 3.19: Let U be a ternary semiring with identity u + uuv + v = v for each u, v, z, x $\in$ U, every element U is zeroed & (U, +) is left cancellative. Then ternary semigroup U is a band.

Proof: Let u $\in$ U, $\exists$ v $\in$ U $\ni$ $v = u + v$ or $v + u = v$ we have $u + v + uuv = v$
$\Rightarrow u + v + uu(v + u) = v + u \Rightarrow v + u^3 = v + u \Rightarrow u^3 = u$.
Therefore U is a band.

Theorem 3.20: Let U be an ordered ternary semiring in which ternary semigroup is –vely ordered. If u is a regular ternary semiring iff u is an idempotent of U.

Proof: Suppose u is regular in U, then $\exists$ x, y $\in$ U $\ni$ $u = uxuyu \leq u^3 \leq u \Rightarrow u^3 = u \Rightarrow$ u is idempotent of U. Conversely, suppose u is a idempotent of U, then we have $u = u^3 \Rightarrow u = uuuuu$. Thus u is regular.

Theorem 3.21: Let U be an ordered ternary semiring as well as u $\in$ U in which ternary semigroup is –vely ordered. Then u = uxu = xuy for x, y $\in$ U.

Proof: Let u $\in$ U, $\exists$ x, y $\in$ U $\ni$ $u = uxuyu \leq uxu \leq u$ and $u = uxuyu \leq xuy \leq u$. Therefore $u = uxu = xuy$.

Def 3.22: An ordered ternary semiring is known as **sum ordered ternary semiring** if $u \leq v$ there exist w $\in$ U $\ni$ $u + w = v$.

Theorem 3.23: Let U be an +vely idempotent ordered ternary semiring. Then
(i) 0 is the least element of U.
(ii) $l \leq l + v$ $\forall$ l, v $\in$ U.
(iii) $l \leq v$ iff $l + v = v$
(iv) U is sum ordered ternary semiring.

Proof: Let U be an +vely idempotent ordered ternary semiring.
(i) $\Rightarrow$ (ii): Suppose 0 is the least element of U & l, v $\in$ U. $0 \leq l \Rightarrow 0 + v \leq l + v \Rightarrow v \leq l + v$. Similarly, $l \leq l + v$.
(ii) $\Rightarrow$ (iii): Suppose assume (ii) & $l \leq v \Rightarrow l + v \leq v + v \Rightarrow l + v \leq v \leq l + v \Rightarrow l + v = v$.
(iii) $\Rightarrow$ (iv): obvious.
(iv) $\Rightarrow$ (i): Suppose U is a sum ordered ternary semiring. Then $\exists$ v $\in$ U $\ni$ $0 + v = l \Rightarrow l = v$.
Now, $0 + l = l$ $\forall$ l $\in$ U $\Rightarrow 0 \leq l$. Therefore 0 is the least element of U.

Theorem 3.24: Let U be an ordered ternary semiring with unity 1. If 1 + 1 = 1 holds implies that (U, +) is a idempotent semigroup of U.

Proof: Let $l \in U \Rightarrow l11 = l$ & $l + l = l11 + l11 = l1(1 + 1) = l11 = l$. Therefore, (U, +) is a idempotent semigroup of U. Here, the set of all additive idempotents is denoted $E^+(U) = \{ l \in U / l + l = l\}$.

Theorem 3.25: Let U be an ordered ternary semiring in which (U, +) cancellative, +ve ordered commutative semigroup. If $E^+(U)$ is nonempty as well as ternary semigroup U is cancellative implies that $E^+(U)$ is k-prime ideal of an ordered ternary semiring U.

Proof: Let $l \in E^+(U) \Rightarrow l + l = l$ & let $v, w \in U$

$\Rightarrow lvw = (l + l)vw = lvw + lvw \Rightarrow lvw \in E^+(U)$.

Similarly, $vwl, wlv \in E^+(U)$. Suppose $l, v \in E^+(U) \Rightarrow l + l = l$ & $v + v = v$ then

$(l + v) + (l + v) = (l + l) + (v + v) = l + v \Rightarrow l + v \in E^+(U)$.

Suppose, $l \leq v$, $v \in E^+(U)$ then $l \leq v \Rightarrow l + v \leq v + v \Rightarrow l + v \leq v$, but $v \leq l + v$.

Therefore, $v = l + v \Rightarrow l + v = l + l + v \Rightarrow l = l + l \Rightarrow l \in E^+(U)$. Hence $E^+(U)$ is an ideal of ternary semiring U.

Suppose $l, l + v \in E^+(U) \Rightarrow l + l = l$ & $l + v + l + v = l + v$.

Now $l + v + l + v = l + v \Rightarrow (l + l) + (v + v) = l + v$

$\Rightarrow l + (v + v) = l + v \Rightarrow v + v = v \Rightarrow v \in E^+(U) \Rightarrow E^+(U)$ is a k-ideal of U.

Suppose, $lvw \in E^+(U)$ & $l, v, w \notin E^+(U) \Rightarrow lvw + lvw = lvw \Rightarrow lv(w + w) = lvw \Rightarrow w + w = w$ it is a contradiction and hence one of the $l, v, w \in E^+(U)$. Therefore $E^+(U)$ is k-prime ideal of U.

Corollary 3.26: Suppose U is a simple ternary semiring in which (U, +) cancellative, +ve ordered commutative semigroup. If $E^+(U)$ is nonempty as well as ternary semigroup U. Then every element of U is additive idempotent.

Proof: By the theorem 3.24, $E^+(U)$ is an ideal of ordered ternary semiring and since U is simple, therefore $U = E^+(U)$. Hence every element of U is additive idempotent.

Corollary 3.27: Let U be a ordered simple ternary semiring and (U, +) cancellative, +ve ordered commutative ternary semigroup. If $E^+(U) = 1$, then $|U| = 1$

Theorem 3.28: Let U be the ordered regular ternary semiring in which ternary semigroup U is negatively ordered with unity. If $l, v \in U$ & $l \leq v \ni l = lll = vll = lvv$.

Proof: Let $l, v \in U$ **&** $l \leq v$, since U is regular $\Rightarrow l = lxlyl$ for $x, y \in U$.

We have $xly \leq l \Rightarrow lxlyl \leq lll \Rightarrow l \leq lll \leq l \Rightarrow l = lll$.

Now $l \leq v \Rightarrow lll \leq vll \Rightarrow l \leq vll \leq l \Rightarrow l = vll$.

Again, $l \leq v \Rightarrow lll \leq llv \Rightarrow l \leq llv \leq l \Rightarrow l = llv$. Therefore, $l = lll = vll = lvv$.

II. CONCLUSION:

Here, The notion of zeroed in ternary semiring, regular & idempotent elements in ordered ternary semiring are introduced. We investigated the properties of zeroid, regular & idempotent elements and relations between them.

REFERENCES

[1]. **Francisco E. Alarcon & Daniel D. Anderson,** Commutat ive semirings and their lattices of ideals , Houston L . Math. 20 (1994), 571 -590.

[2]. **Gudikondulu. Srinivas, Madhusudhana Rao.D., Srinivasa Rao. G** -Ternary Hemirings and Ternary ssemirings-Defs & Examples- Asian Lournal of Science & Technology, Vol. 8, Issue -12, Decmber-2017, pp: 7153-7156.

[3]. **Laved Ahsan**, Fully idempotent semirings, Proc. Lapan Acad. 69, Ser . A (1993), 185 - 188.

[4]. **Madhusudhana Rao. D.**,Theory ol Γ-ideal inΓ-semigroups – Thesis, ANU (2011).

[5]. **Madhusudhana Rao. D, Anlaneyulu. A and Gangadhara Rao. A.**, Prime Γ- Radicalsin Γ-Semigroups, International e-lournal of Mathematics and Engineering, 138 (2011), 1250-1259.

[6]. **Madhusudhana Rao. D, Anlaneyulu. A and Gangadhara Rao. A.**, Prime Γ- Ideals in Duo Γ-Semigroups, International Lournal of Mathematics and Engineering 174 (2012), 1642-1653.

[7]. **Madhusudhana Rao. D. and Srinivasa Rao. G.,** Prime Radicals and Completely Prime Radicals in Ternary Semirings, International Lournal of Basic Science and Applied Computing (ILBSAC) Volume-1, Issue 4, February 2015.

[8]. **Madhusudhana Rao. D. and Srinivasa Rao. G.,**Structure ol P-Prime and P-Semiprime Ideals in Ternary Semirings- International Lournal of Innovation in Science and Mathematics(ILISM) Vol 3, Issue 2, February 2015, pp 24-31.

[9]. **Madhusudhana Rao. D. and Srinivasa Rao. G.**, Special Elements ol a Ternary Semirings, International Lournal of Engineering Research and Applications, Vol. 4, Issue 11(Version-5), November 2014, pp. 123-130.

[10]. **Muralikrishna Rao. M, Venkateswarlu. B, Ravikumar. B** – zeroid, regular& idempotent elements in gamma semirings –Journal of International Mathematical Virtual Institute, Vol 9(2019), 37-52.

[11]. **SalaniLavanya. M., Madhusudhana Rao. D., and V. Syam Lulius Ralendra,**On Quasi-Ternary **Γ**-Ideals and Bi-Ternary**Γ**-Ideals in Ternary **Γ**-Semirings-International Lournal of Mathematics and Statistics Invention, Volume 6, Issue 3, (September-2015), PP 05-14.

[12]. **SalaniLavanya. M., Madhusudhana Rao. D., and V. Syam Lulius Ralendra,**On Lateral Ternary Γ-Ideals ol Ternary Γ-Semirings-American International Lournal of Research in Science, Technology, Engineering &Mathematics (AILRSTEM), 12(1), September-November, 2015, pp: 11-14.

[13]. **SalaniLavanya. M., Madhusudhana Rao. D., and V. Syam Lulius Ralendra,**Prime Bi-ternary**Γ**-Ideals in Ternary **Γ**-Semirings-British Lournal of Research, Volume 2, Issue 6, November-December, 2015, pp: 156-166.

[14]. **SalaniLavanya. M., Madhusudhana Rao. D., and V. Syam Lulius Ralendra,** A Study on Lacobson Radical ol a Ternary Γ-Semieing-International Lournal of Mathematics and Computer Applications Research, Volume 6, Issue 1, (Feb-2016), PP 17-30.

[15]. **SalaniLavanya. M., Madhusudhana Rao. D., and VB. Subrahmanyeswara Rao Seetamralu,** On Right Ternary Γ-ideals ol Ternary Γ-Semiring, IMPACT-International Lournal of Applied, Natural and Social Sciences, Vol. 4, Issue 5, May 2016, 107-114.

[16**[** **SalaniLavanya. M., Madhusudhana Rao. D., and Vasantha. M,** A Study on Inverses Strongly Ternary Gamma Semiring-International Research Journal of Mathematics, Engineering and IT, Vol 3, Issue 5, May 2016, pp:1-10.

[17[**SalaniLavanya. M., Madhusudhana Rao. D., and VB. Subrahmanyeswara Rao Seetamralu,** Properties ol Right Strongly Prime Ternary Gamma Semiring- Accepted for publication in International Organization of Scientific Research Lournal of Mathematics.

[18] **Syamluliusralendra. V, Dr. Madhusudhana rao. D, Salanilavanya. M**-A Study on Completely Regular po-Ternary **Γ**-Semiring, Asian Academic Research Lournal of Multidisciplinary, Volume 2, Issue 4, September 2015, PP: 240-246.

[19] **Syam Lulius Ralendra. V, Dr.MadhusudhanaRao. D**and **SalaniLavanya. M**-On Pure PO-Ternary Γ-Ideals in Ordered Ternary Γ-Semirings, IOSR Lournal of Mathematics (IOSR-LM), Volume 11, Issue 5 Ver. IV (Sep. - Oct. 2015), PP 05-13.

D. Madhusudhana Rao "Properties Of Elements In Ternary-Semirings "International Journal of Computational Engineering Research (IJCER), vol. 08, no. 09, 2018, pp 56-61