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Oscillatory MHD Flow and Heat Transfer through a Porous Medium with Constant Suction Bounded By Two Infinite Vertical Plates in Presence of a Heat Source

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ABSTRACT

The study of two dimensional oscillatory flow of an electrically conducting incompressible viscous fluid through a porous medium with constant suction bounded by two infinite vertical porous plates in presence of a heat source have been presented. A uniform magnetic field is applied transversely to the direction of the flow. The governing equations are solved by regular perturbation technique. The magnetic Reynold's number is assumed to be very small so that the induced magnetic field can be neglected. The discussion is confined to small Eckert Number E. The expressions for velocity field, temperature field, non-dimensional skin friction from the plate into the fluid and the co-efficient of rate of heat transfer from the plates to the fluid in terms of Nusselt number are obtained in non-dimensional forms. The amplitudes and the phases of the fluctuating parts of the skin friction and co-efficient of rate of heat transfer are demonstrated graphically for different values of the parameters involved and results obtained are discussed.

Keywords: MHD, electrically conducting, viscous incompressible, oscillatory flow, suction, skin friction, Nusselt Number.

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I. INTRODUCTION

The study of MHD flows have got considerable importance due to its applications in the fields of Geophysics, Astrophysics, Cosmic fluid dynamics, Meteorology, Solar physics, and in the motion of earth's core. In broader sense the MHD has its applications in various areas like astrophysical, geophysical and engineering problems. In recent years, the flow of fluids through porous media has become an important topic because of the recovery of crude oil from the pores of the rocks. Meanwhile, there has been a renewed interest in studying MHD flow and heat transfer in porous media because of the effect of magnetic fields on the performance of many systems. MHD has reached the present form due to contributions of notable authors like Alfven (1942), Cowling (1957), Cramer and Pai (1973), etc. Meanwhile, there has been a considerable interest in studying MHD flow and heat transfer in porous medium due to the effect of magnetic fields on performance of many systems. In view of such applications, a series of investigations have been made by Nigam and Singh (1996), Soundalgekar and Bhat (1971), Vajravelu (1988) and Attia and Kotb (1996). Bodosa and Borthakur (2003) have studied the MHD flow and heat transfer of a viscid elastic fluid past between two horizontal plates with heat sources or sinks. The study of a two dimensional free convection with mass transfer in case of viscous incompressible fluid was presented by Acharya et.al. (2000). Makinde and Mhone (2005) have considered MHD flow in a channel fluid with porous medium. Chaudhary et.at. (2006) have studied the combined effect s of a magnetic field and Ohmic dissipation on unsteady flow with saturated porous medium. Ahmed and Barman (2010) investigated effect of porosity on a MHD oscillatory flow bounded by two horizontal plates in presence of a heat source.

The present paper is an extension work done by Ahmed and Barman (2010). Here our aim is to investigate the combined effect of transverse magnetic field and heat transfer on a flow of a conducting fluid through a porous medium with constant suction bounded by two infinite vertical plates.

In the following sections, the problem is formulated, solved and the relevant results are discussed.

II. MATHEMATICAL FORMULATION

The equations governing the motion of an incompressible viscous electrically conducting fluid through a porous medium in presence of a magnetic field are:

The equation of continuity:
div $\bar{q}$ =0 (1)

The Gauss's Law of magnetism:
div $\bar{B}$ =0 (2)

The momentum equation:
$$\rho\left[\frac{\partial \bar{q}}{\partial \bar{t}} + (\bar{q}.\bar{\nabla})\bar{q}\right] = -\bar{\nabla}p + \bar{J} \times \bar{B} + \mu\nabla^2\bar{q} - \frac{\mu\bar{q}}{\bar{K}} + \rho\bar{g} \qquad (3)$$

The energy equation:
$$\rho C_p\left[\frac{\partial \bar{T}}{\partial \bar{t}} + (\bar{q}.\bar{\nabla})\bar{T}\right] = \kappa\nabla^2\bar{T} + \varphi + \frac{\bar{J}^2}{\sigma} + Q(\bar{T} - \bar{T}_0) \qquad (4)$$

The Ohms Law:
$$\bar{J} = \sigma[\bar{E} + \bar{q} \times \bar{B}] \qquad (5)$$

Where
$\bar{B}$ The magnetic induction vector
C_p Specific Heat at constant pressure
$\bar{E}$ The Electric Field
$\bar{g}$ The Acceleration due to gravity
$\bar{J}$ The electric current density
$\bar{J} \times \bar{B}$ The Lorentz force per unit volume
$\bar{K}$ The permeability parameter
κ The thermal conductivity
p The Pressure
Q The first order heat source
$\bar{t}$ The time
$\bar{T}$ The temperature
$\bar{T}_0$ The temperature of the stationary plate
$\bar{q}$ The velocity vector
μ The coefficient of viscosity
σ The electrical conductivity
ρ The density of the fluid

We consider the oscillatory flow of an incompressible viscous electrically conducting fluid through a porous medium bounded by two infinite horizontal porous plates separated by a distance h in presence of a transverse magnetic field by making the following assumptions:

(I) All the fluid properties except the density in the buoyancy force term are constant.
(II) The Eckert Number E is small
(III) The magnetic Reynolds number is so small that the induced magnetic field can be neglected
(IV) The magnetic dissipation term in the energy equation is negligible

We introduce a co-ordinate system $(\bar{x}, \bar{y}, \bar{z})$ with x-axis vertically upwards along a plate, the y-axis perpendicular to it and directed into the fluid region and z-axis perpendicular to the xy-plane. Let $\vec{q} = \hat{i}\,\bar{u} + \hat{j}\,\bar{v}$ be the fluid velocity at the point $(\bar{x}, \bar{y}, \bar{z})$ and $\vec{B} = B_0\hat{j}$ be the applied magnetic field, $\hat{i}$ and $\hat{j}$ being the unit vectors along x-axis and y-axis respectively. The distance between the plates is assumed to be h. Since the plates are infinite in length, therefore all the quantities except possibly the pressure are assumed to be independent of $\bar{x}$. Under above assumptions, the equations governing the flow and heat transfer reduce to the following set of equations:

The equations governing the fluid motion are

$$\frac{\partial \bar{u}}{\partial \bar{t}} - v_0 \frac{\partial \bar{u}}{\partial \bar{y}} = g\beta(\bar{T} - \bar{T}_\infty) + \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\nu \bar{u}}{\bar{K}_0} - \frac{\sigma B_0^2 \bar{u}}{\rho} \quad (6)$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} - v_0 \frac{\partial \bar{T}}{\partial \bar{y}} = \alpha \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\nu}{C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \frac{\sigma B_0^2 \bar{u}}{\rho C_p} + \bar{Q}(\bar{T}_\infty - \bar{T}) \quad (7)$$

Under boundary conditions:

,

$$\left.\begin{array}{l} \text{At } y = 0: \ \bar{u} = 0, \bar{T} = \bar{T}_w + \varepsilon(\bar{T}_w - \bar{T}_\infty)e^{i\bar{\varpi}t} \\ \text{At } y \to \infty: \ \bar{u} \to 0, \bar{T} \to 0, \end{array}\right\} \quad (8)$$

We introduce the following non-dimensional quantities:

$$y = \frac{v_0}{\nu}\bar{y}, t = \frac{v_0^2}{\nu}\bar{t}, \ w = \frac{4\nu\bar{w}}{v_0^2}, u = \frac{\bar{u}}{v_0}, \ T = \frac{(\bar{T} - \bar{T}_\infty)}{(\bar{T}_w - \bar{T}_\infty)},$$

$$G = \frac{\nu\beta(\bar{T}_w - \bar{T}_\infty)}{v_0^3}, \ P = \frac{\mu C_p}{\lambda}\left(= \frac{\nu}{\alpha}\right) \ \text{[Prandtle Number]}$$

$$Q = \frac{\bar{Q}\nu}{v_0^2}, \ M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, E = \frac{v_0^2}{C_p(\bar{T}_w - \bar{T}_\infty)} \ \text{[Eckert No.]}, \ K = \frac{\bar{K} v_0^2}{\nu^2}$$

The non-dimensional forms of equations (1) and (2) are:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = GT + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right)u \quad (9)$$

$$\frac{\partial T}{\partial t} - \frac{\partial T}{\partial y} = \frac{1}{P}\frac{\partial^2 T}{\partial y^2} + E\left(\frac{\partial u}{\partial y}\right)^2 + MEu^2 + QT \quad (10)$$

And the relevant boundary conditions are:

$$\left.\begin{array}{l} \text{At } y = 0: \ u = 0, T = (1 + \varepsilon e^{int}) \\ \text{At } y = \infty: \ u \to 0, T \to 0 \end{array}\right\} \quad (11)$$

III. METHOD OF SOLUTION:

In order to solve the equations (4) and (5) under the boundary conditions (6), we put

$$\left.\begin{array}{l} u(y,t) = u_0(y) + \varepsilon u_1(y)e^{int} \\ T(y,t) = T_0(y) + \varepsilon T_1(y)e^{int} \\ C(y,t) = C_0(y) + \varepsilon C_1(y)e^{int} \end{array}\right\} \quad (12)$$

Substituting relations from (7) in equations (4) and (5) and equating the harmonic terms and neglecting $\varepsilon^{2.}$, we get following differential equations:

$$u_0'' + u_0' - \left(M + \frac{1}{K}\right)u_0 = -GT_0 \quad (13)$$

$$u_1'' + u_1' - \left(M + \frac{1}{K}\right)u_1 = -GT_1 \quad (14)$$

$$T_0'' + P\,T_0' + PQ\,T_0 = -EPu_0'^2 - MEPu_0^2 \quad (15)$$

$$T_1'' + P\,T_1' + PQT_1 = -2EPu_0'u_1' - 2MEPu_0u_1 \quad (16)$$

Equations (8) to (11) are to be solved subject to the boundary conditions:

$$\left.\begin{array}{l} \text{At } y = 0: \ u_0 = u_1 = 0, \ T_0 = T_1 = 1, \\ \text{when } y \to \infty: \ u_0 \to 0, \ u_1 = 0; T_0 = T_1 \to 0, \end{array}\right\} \quad (17)$$

To solve the equations from (8) to (11), we assume the following for E<1

$$\left.\begin{array}{l} u_0(y) = u_{00}(y) + Eu_{01}(y) \\ u_1(y) = u_{10}(y) + Eu_{11}(y) \\ T_0(y) = T_{00}(y) + ET_{01}(y) \\ T_1(y) = T_{10}(y) + ET_{11}(y) \end{array}\right\} \quad (18)$$

Substituting (13) in equations from (8) to (11) and equating the coefficients of E^0 and E^1 (neglecting E^2), we get the following equations:

$$u_{00}'' + u_{00}' - a_0 u_{00} = -GT_{00} \quad (19)$$
$$u_{01}'' + u_{01}' - a_0 u_{01} = -GT_{01} \quad (20)$$
$$u_{10}'' + u_{10}' - a_0 u_{10} = -GT_{10} \quad (21)$$
$$u_{11}'' + u_{11}' - a_0 u_{11} = -GT_{11} \quad (22)$$
$$T_{00}'' + PT_{00}' + PQT_{00} = 0 \quad (23)$$
$$T_{01}'' + PT_{01}' + PQ\,T_{01} = -Pu_{00}'^2 - MPu_{00}^2 \quad (24)$$
$$T_{10}'' + PT_{10}' + PQ\,T_{10} = 0 \quad (25)$$
$$T_{11}'' + PT_{11}' + PQ\,T_{11} = -2Pu_{00}'\,u_{01}' - 2MPu_{00}u_{10} \quad (26)$$

Here $a_0 = \left(M + \frac{1}{K}\right)$

And the boundary conditions (12) become

$$\left.\begin{array}{ll} \text{At } y = 0: & u_{00} = u_{01} = 0;\ u_{10} = u_{11} = 0,\ T_{00} = T_{10} = 1, \\ & T_{01} = T_{11} = 0;\ C_{00} = C_{10} = 1;\ C_{01} = C_{11} = 0 \\ \text{when } y \to \infty: & u_{00} = u_{01} \to 0;\ u_{10} = u_{11} \to 0,\ T_{00} = T_{10} \to 1, \\ & T_{01} = T_{11} \to 0;\ C_{00} = C_{10} \to 1;\ C_{01} = C_{11} \to 0 \end{array}\right\} \quad (27)$$

Equations from (14) to (21) have been solved, but the solutions are not presented here for the sake of brevity. Now with the convection that the real parts of complex quantities have physical significance in the problem, the velocity and temperature fields can be expressed as follows:

$$u(y,t) = u_0(y) + \varepsilon(u_r \cos\varpi t - u_i \sin\varpi t) \quad (28)$$
$$T(y,t) = T_0(y) + \varepsilon(T_r \cos\varpi t - T_i \sin\varpi t) \quad (29)$$

Where: $u_r = \mathrm{Re}\,(u_1)$, $u_i = \mathrm{Im}\,(u_1)$, : $T_r = \mathrm{Re}\,(T_1)$, $T_i = \mathrm{Im}\,(T_1)$,

The expressions for u_r, u_i, T_r and T_i are obtained but not presented here for the brevity.

Skin Friction and Rate of Heat transfer:

The Skin friction coefficient (τ_0) at the plate in terms of amplitude and phase is given by

$$\tau_0 = \left(\frac{\partial u}{\partial y}\right)_{y=0} = u_0'(0) + \varepsilon e^{i\omega t} u_1'(0)$$

Splitting this equation into real and imaginary parts and taking the real parts only we get:

$$\tau_0 = \tau_0^0 + \varepsilon|B|cos(\omega t + \theta)$$

Where

$$\left.\begin{array}{r} B = B_r + iB_i = u_1'(0) \\ \tau_0^0 = u_0'(0) \\ |B| = \sqrt{B_i^2 + B_i^2} \\ \tan\theta = \frac{B_i}{B_r} \end{array}\right\} \quad (30)$$

Also the Heat Transfer coefficient (Nu_0) at the plate in terms of the amplitude and phase is given by

$$Nu_0 = \left(\frac{\partial T}{\partial y}\right)_{y=0} = T_0'(0) + \varepsilon e^{i\omega t} T_1'(0)$$

Splitting this equation into real and imaginary parts and taking the real parts only we have-

$$Nu_0 = Nu_0^0 + \varepsilon|H|cos(\omega t + \varphi)$$

Where

$$\left.\begin{array}{r} H = H_r + iH_i = T_1'(0) \\ Nu_0^0 = T_0'(0) \\ |H| = \sqrt{H_i^2 + H_i^2} \\ \tan\varphi = \frac{H_i}{H_r} \end{array}\right\} \quad (31)$$

The expressions for $\tau_0^0, Nu_0^0, B_r, B_i, H_r$ *and* H_i are obtained but not presented here for the sake of brevity.

IV. DISCUSSION OF THE RESULTS

The graphs of the amplitude $|B|$, the phase θ of the fluctuating part of the non-dimensional skin friction τ_0 and the amplitude $|H|$, the phase ϕ of the fluctuating part of the Nusselt number Nu_0 against the permeability parameter K have been displayed in the figures 1 to 19. The Eckert number (E) is taken to be 0.04 and the discussion is confined for S=0.1 and the others parameters involved are chosen arbitrarily

Figures 1 to 3 demonstrate the behaviour of $|H|$ against K for variations of P and M. From figure 1, we observe that in absence of magnetic field, $|H|$ slowly and steadily increases as K, whereas in presence of magnetic field porosity almost ceases to act on $|H|$. The same figures also indicate that an increase in Prandtle Number results a growth in $|H|$ whether there is a magnetic field or not. It is inferred from figure 3 that there is decay in $|H|$ due to the application of the magnetic field for large K. For small values of K, the effect of the magnetic field is negligible on $|H|$.

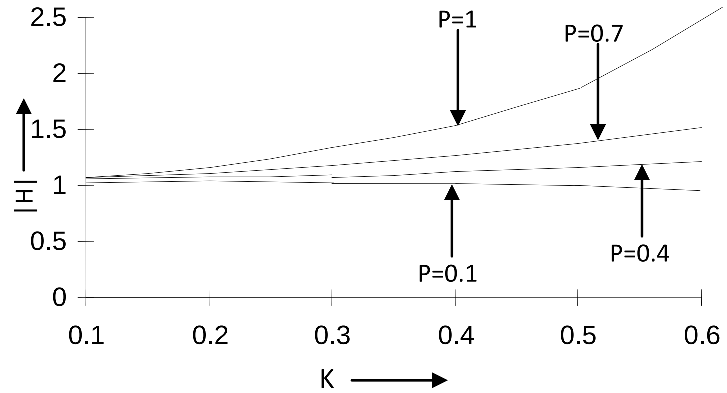

Fig. 1: $|H|$ against K for variation of P (S=0.1, E=0.04, G=5, M=0)

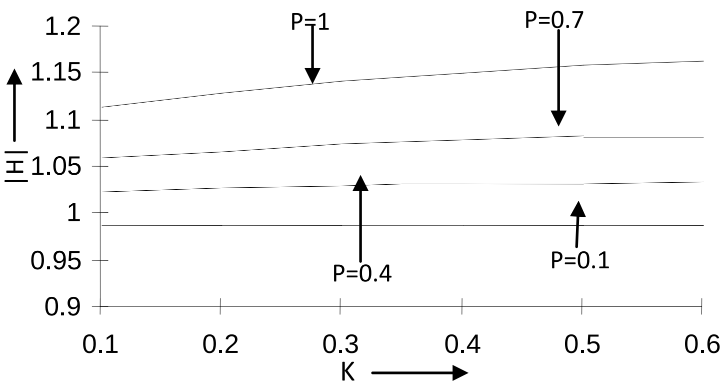

Fig. 2: $|H|$ against K for variation of P (S=0.1, E=0.04, G=5, M=2)

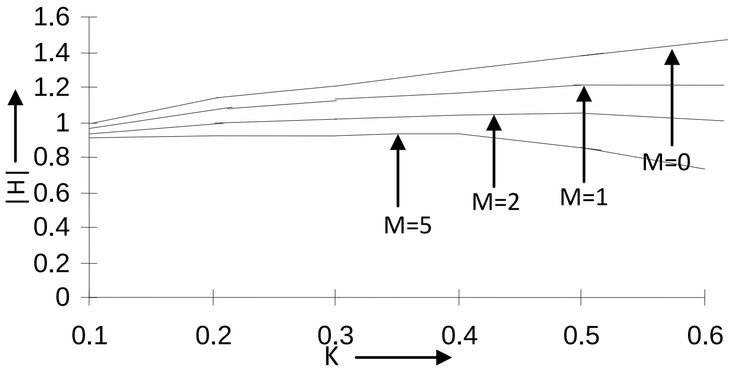

Fig. 3: $|H|$ against K for variation of M (S=0.1, E=0.04, G=5, P=0.7)

Figures 4, 5 and 6 represent the variation of the phase $\tan\phi$ of the fluctuating part of the Nusselt number Nu_0 against the permeability parameter K for different values of P and M. In figures 4 and 5, it is observed that for M=0 $\tan\phi$ increases gradually with K with increase of P, for $M \neq 0$ $\tan\phi$ is not significantly affected by K.

Figure 6 describes the behaviour of $\tan\phi$ with K for variation of M. It has been seen that when there is no magnetic field $\tan\phi$ increases with K but when M increases the effect of K on $\tan\phi$ gradually decreases and finally for higher values of M effect of K on $\tan\phi$ becomes nil.

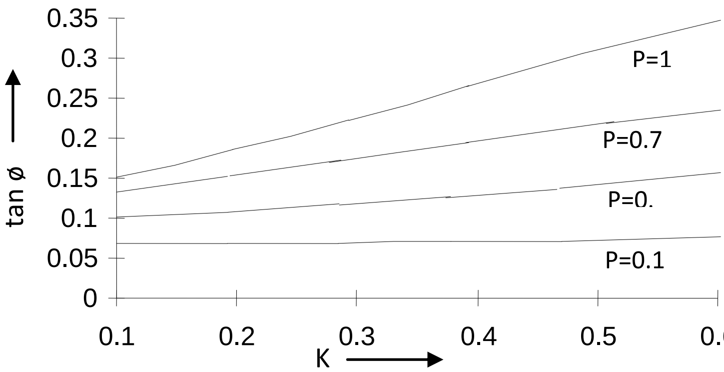

Fig. 4: $\tan\phi$ against K for variation of P (S=0.1, E=0.04, G=5, M=0, U=0)

Figures 7 and 8 describe nature of the amplitude $|B|$ of the fluctuating part of the non-dimensional skin friction τ_0 for variation of P and M.

Figure 7 shows that $|B|$ increase very slowly with the increase of K, but an increase of P results a decrease of $|B|$

From figure 8 we observe that $|B|$ increases gradually with K, but when M increases the value of $|B|$ gradually decreases.

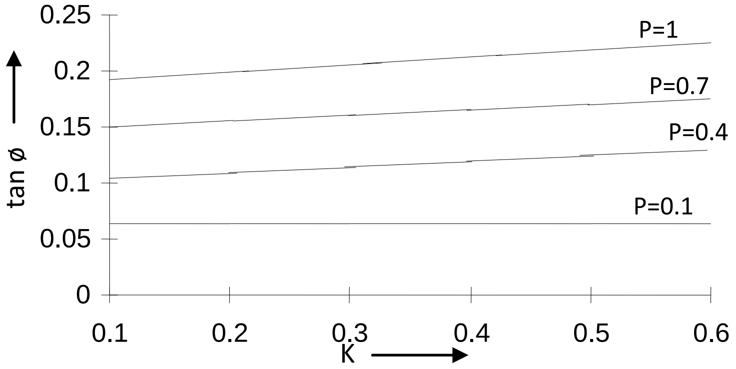

Fig. 5: $\tan\phi$ **against K for variation of P (S=0.1, E=0.04, G=5, M=5)**

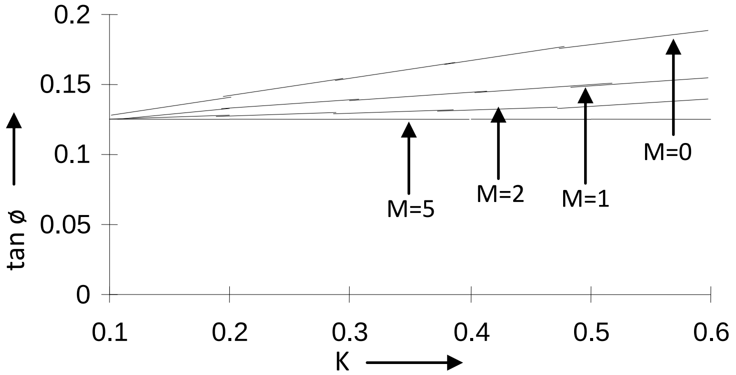

Fig. 6: $\tan\phi$ **against K for variation of M (S=0.1, E=0.04, G=5, U=0, P=0.7)**

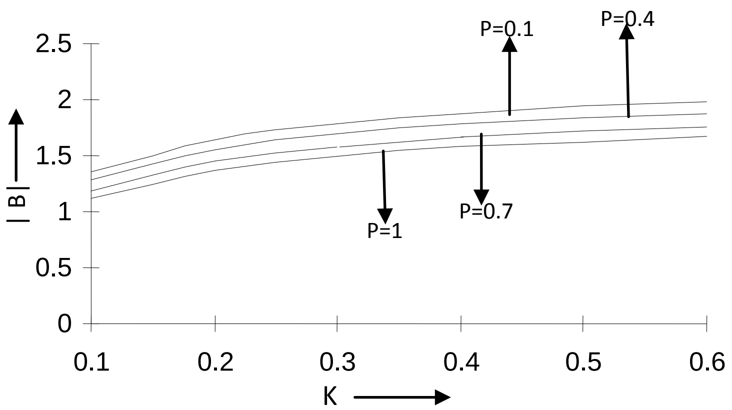

Fig. 7: $|B|$ **against K for variation of P (S=0.1, E=0.04, G=5, U=0, M=5)**

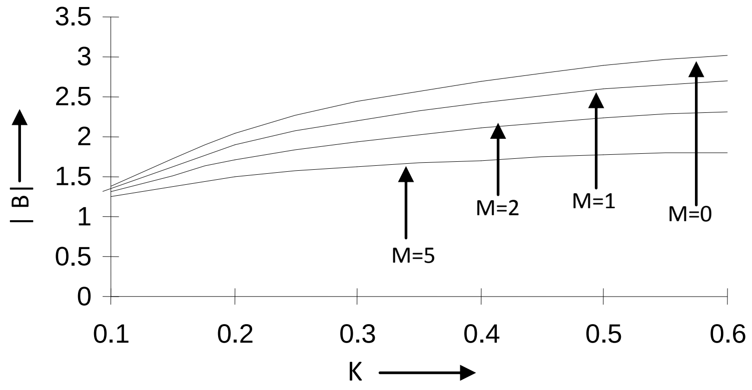

Fig. 8: $|B|$ **against K for variation of M (S=0.1, E=0.04, G=5, P=0.7)**

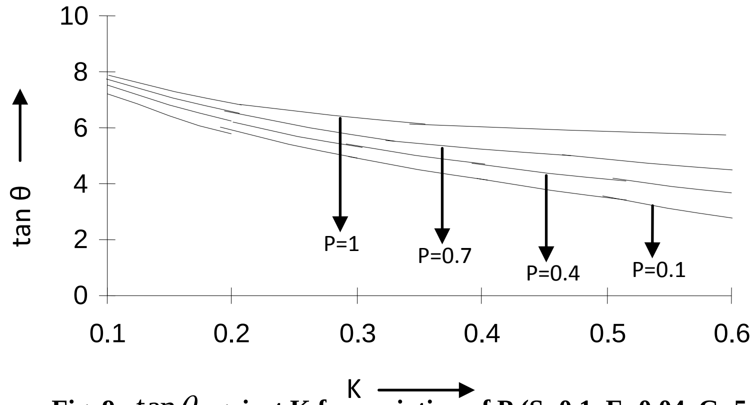

Fig. 9: $\tan\theta$ **against K for variation of P (S=0.1, E=0.04, G=5, M=5)**

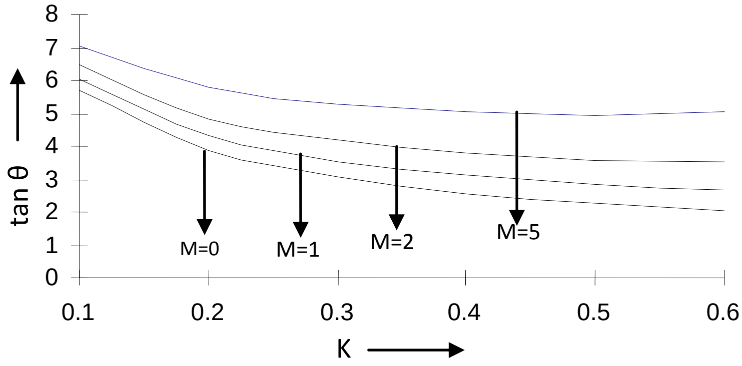

Fig. 10: $\tan\theta$ **against K for variation of M (S=0.1, E=0.04, G=5, P=0.1)**

Figures 9 and 10 demonstrate the change of the trend of the phase $\tan\theta$, the fluctuating part of the non-dimensional skin friction τ_0 with variation of the porosity parameter and the magnetic field M..

In figure 9, we see that $\tan\theta$ gradually decreases with the increase of K but it increases slowly and steadily with the increase of P.

Figure 10 shows that $\tan\theta$ increases with the increase of M but decreases slowly with variation of K.

5. CONCLUSION:

From the above discussions we conclude that:

a) The amplitude $|H|$, of the fluctuating part of the Nusselt Number Nu_0 increases with the Prandtl Number (P) but it decreases with the increase of the Magnetic field (M). When there is no magnetic field, $|H|$ increases very slowly and steadily with the porosity of the medium (K), but the porosity has almost no effect on it in presence of a magnetic field.

b) The phase $\tan\phi$ of the fluctuating part of the Nusselt number Nu_0 increases gradually with the porosity of the medium (K) in absence of a magnetic field, but when there is a magnetic field it is not significantly affected by K. It also increases with the Prandtl Number (P).

c) The amplitude $|B|$, of the fluctuating part of the non-dimensional skin friction τ_0 increases with the Porosity parameter, but it decreases with the increase of the Prandtl Number (P) and the Magnetic field (M).

d) The phase $\tan\theta$, the fluctuating part of the non-dimensional skin friction τ_0 gradually decreases with the increase of the Porosity of the medium, but it increases slowly and steadily with the increase of M

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