

# Digital State Feedback Current Control using the Pole Placement Technique

<sup>1</sup>susanta Kumar Pradhan, <sup>2</sup>sourav Kumar Mishra,

Gandhi Institute of Excellent Technocrats, Bhubaneswar, India

Kruttika Institute of Technical Education, Khordha, Odisha, India

## ABSTRACT

A digital state feedback control method for the current mode control of DC-DC converters is proposed in this paper. This approach can precisely achieve interleaved current sharing among the converter modules. As the controller design and system analysis are performed in the time domain, the proposed method can easily satisfy the required converter specification by using the pole placement technique. The digital state feedback controller in the continuous and discrete time domain is derived for the robust tracking control. For the verification of the proposed control scheme, a parallel module bi-directional converter in a prototype 42V/14V hybrid automotive power system, which is a design example in the continuous time domain, and a parallel module buck converter, which is a design example in the discrete time domain, are implemented using a TMS320F2812 digital signal processor (DSP).

### Keywords:

DC converters, digital state feedback control, pole placement technique, parallel interleaved current

## INTRODUCTION

Ever increasing demands for the development of compact, lightweight power supplies with more power density, higher efficiency and fast dynamics often require power conversion through parallel connected converters. In order to achieve the current sharing among the modules, various analog current mode control methods such as peak current mode control, charge current mode control, average current mode control, etc., are used.

With recent advances in digital systems, digital control

has become increasingly visible even for high frequency, low-to-medium power switching converters. Digital control offers the potential advantages of immunity to analog component variations, programmability and possibilities to improve performance using more advanced and sophisticated control algorithms<sup>[1-5]</sup>.

However, a direct digital implementation of the analog current mode control is not easy. In analog current mode control, the switch current or the inductor current is sensed, and the switch duty cycles of each converter module are regenerated by comparing the sensed current to a reference. Because the switch or inductor current is a fast changing waveform with a high switching frequency, the need for a very fast analog to digital converter (ADC) to produce multiple samples of the sensed current per switching period, and the corresponding need for the large signal

processing capabilities, may require excessively complex hardware. Thus, a digital current mode control method that can match or exceed the performance of the standard analog current mode control has been of much interest. In [4, 5], current estimated algorithms are proposed. These algorithms focus on the current mode control to improve the dynamics. Even though the dynamics are improved, the current sharing among the parallel converter modules cannot be solved. Furthermore, a parallel interleaved converter control method using the sliding mode control was proposed<sup>[3]</sup>. However, this controller design technique is very complex and the expansion of the modules is not convenient.

experimental results from two 100W parallel module buck converters are presented. Conclusion of the paper are represented in the last section.

## **1. Robust Tracking Control using the Pole Placement Technique**

### **The state feedback controller structure in the continuous time domain**

Most switching converters operating in the continuous conduction mode (CCM) have two state equations within one switching period as follows;

In this paper, a digital state feedback control method for the current mode control is proposed in the continuous and discrete time domains. This approach can precisely

$$\dot{x}(t) = A_1 x(t) + b_{vg1} v_{g1}(t) + b_{io1} i_o(t),$$

$$\dot{x}(t) = A_2 x(t) + b_{vg2} v_{g2}(t) + b_{io2} i_o(t),$$

$$nT_s \leq t \leq (n+1)T_s$$

$$(n+1)T_s \leq t \leq (n+2)T_s$$

(1)

achieve the interleaved current sharing among the

where,

$x(t)$

is a state vector,

$v_g(t)$

is an input voltage,

converter modules.

Also, the required converter

$i_o(t)$

is an output current,

$d(t)$

is a duty ratio and  $T_s$  is

performance, such as the settling time and maximum overshoot of the step load response, can be easily satisfied because the controller design and system analysis using the pole placement technique are performed in the time domain<sup>[6]</sup>. For the verification of the proposed control

as a switching period. From the continuous differential equation (1), the average continuous time state equation can be derived for the continuous state feedback controller.

scheme, a parallel module bi-directional converter in a

$$\dot{x}(t) = A_C x(t) + B_C d(t) + B_{vg} v_{g1}(t) + B_{io} i_o(t),$$

$$y(t) \square C_C x(t)$$

prototype 42V/14V hybrid automotive power system, which is a design example in the continuous time domain, along with a parallel module buck converter, which is a  
where

$$A_C \square D A_1 \square D' A_2$$

$$B_C \square A_1 \square A_2 \square X \square B_{vg1} \square B_{vg2} V_g \square B_{io1} \square B_{io2} I_o$$

$$B_{vgc} \square D B_{vg1} \square D' B_{vg2}$$

(2)

design example in the discrete time domain, is implemented using a TMS320F2812 digital signal

$$B_{io1}$$

$$\square D B_{io1}$$

$$+ D' B_{io2}$$

processor (DSP).

where,

$$X, D, V_g$$

and

$I_o$  are the steady state values

This paper is organized as follows. In Section 2, the state feedback controller using the pole placement technique is derived for the robust tracking control. In Section 3, after reviewing the limitation of the bi-directional DC-DC converter using an analog control scheme, the proposed current mode control to overcome these problems is discussed for a controller design

of the state, duty ratio, input voltage and output current, respectively. The objective is to design an overall system such that the output  $y(t)$  will track, asymptotically, any step preference input,  $r(t) \square R$  (constant), even with the presence of an input disturbance and with plant parameter variations. Let an error state,  $e(t)$ , and a augmented state

example in the continuous time domain. For the

variables,

$$z(t),$$

$$u(t),$$

$w(t)$ , be defined, and assume that

verification of the theoretical analysis, a parallel module

the input voltage,

$$v_g$$
 and output current,

$$i_o$$
 are

bi-directional converter in a prototype 42V/14V hybrid automotive power system is implemented in Section 4. In sustained and slowly varying.

Section 5, for a design example in the discrete time

$$e(t) \square r(t) \square y(t),$$

$$z(t) \square \dot{x}(t),$$

$$u(t) \square \dot{d}(t)$$

domain, a parallel module current control scheme for the

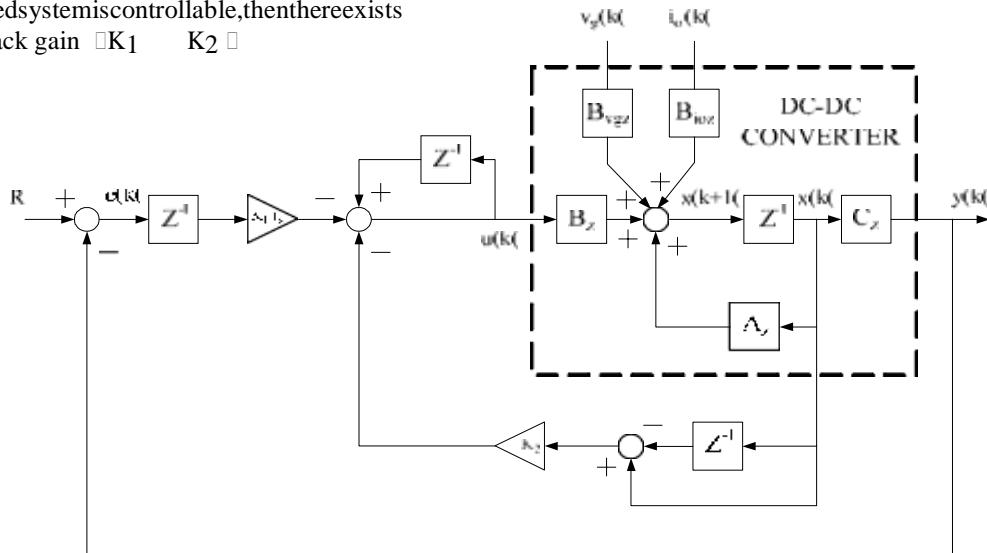
$$\dot{x}(t) \square \dot{d}(t) \square \dot{y}(t) \square C_C \dot{x}(t) \square C_C z(t)$$

(3)



Fig. 1 The continuous state feedback controller scheme for the robust tracking control

If the expanded system is controllable, then there exists a state feedback gain  $\begin{bmatrix} K_1 & K_2 \end{bmatrix}$



such that the expanded system is stable. That means the output of the converter tracks the reference value. Furthermore, using the state feedback gain, the system's eigenvalues can be placed to the desired poles, which determine the performance of the

Fig. 2 The discrete state feedback controller scheme for the robust tracking control

The objective is to design an overall system such that feedback control system. Fig. 1 shows the overall closed the output

$y(k)$

will track asymptotically any step

loop system using the continuous state feedback control. The duty ratio can be derived from equation (3) and (4) as reference input,  $r(k) = R$  (constant), even with the presence of an input disturbance and with plant parameter follows;

variations. Let an error state,

$e(k)$ , and an augmented state

variable,  $z(k)$ ,  $u(k)$ ,  $w(k)$ , be defined, and assume that the  $d(t) = u(t) - d(t) = K_1 e(t) - d(t) = K_2 x(t)$

(5)

input voltage,

$v_g$ , and output current,

$i_o$ , are sustained

Since the control input is the duty ratio, the magnitude of the control input must be checked because of its

and slowly varying.

$$e(k) \square r(k) \square y(k) \square R \square C(k), z(k) \square \dot{x}(k) \square \frac{x(k-1)}{z} \square x(k)$$

boundary constraint. This can be easily checked from the  $T_s$  allowed maximum duty ratio, the switching frequency, the  $u(k) \square \dot{d}(k) \square \frac{d(k-1)}{T_s} \square d(k)$ ,  $\dot{d}(k) \square C z(k)$

(8)

$z$  designed feedback gain and the required system  $s$  specification, as follows;

$w_1(k) \square \psi_g(k) \square 0$ ,  $w_2(k) \square \dot{t}_0(k) \square 0$  using Euler's method

$$| u(t) | \square \dot{d}(t) \square K_1 e(t)$$

$$\max + K_2 z(t)$$

From a similar procedure in the continuous time case,

$$\underline{D} \square D$$

(6)

the state equation of the overall closed loop system is

$$| K \dot{r} \square K \dot{x}(t) |$$

$$| \max \quad \min |$$

1            2            spec             $T$   
 $s$   
 given as follows

$$\begin{array}{l} e(k) = \\ z(k) = 0 \\ T_C z \end{array} \quad \begin{array}{l} e(k) = 0 \\ u(k) \\ A \quad z(k) \quad B \end{array}$$

components. Fig. 4 shows the typical analog control scheme of the 42V/14V bi-directional DC-DC converter,

$$\begin{array}{l} z \\ z \\ z \\ (9) \\ u(k) = K \\ e(k) \\ 2 z(k) \end{array} \quad ]_K$$

which is usually realized with general purpose PWMICs for switch mode power supplies. Since these PWMICs are

$$\begin{array}{l} \text{unable to change the role of the two switches in} \\ d(k) = d(k) = K_T e(k) = K_x x(k) = x(k) \\ (10) \quad 1_s \quad 2 \\ \text{synchronous buck/boost operation, it is general practice to} \end{array}$$

$$u(k) = K_e(k) \max(K_T z(k) \max(D))$$

$$(11) \quad \begin{array}{l} K_R = K_T \max(K_x) \\ \min \quad 42V \\ 1 \quad 2 \quad \text{spec} \\ s \end{array} \quad | \quad T \quad 14V \quad \text{Duty}$$

Fig. 2 shows the overall closed loop system using the discrete state feedback control.

## 2. The 42V/14V Bi-Directional DC-DC Converter Application for the Continuous Time Domain Design Example

Fig. 4 The typical analog control scheme of the 42V/14V bi-directional DC-DC converter

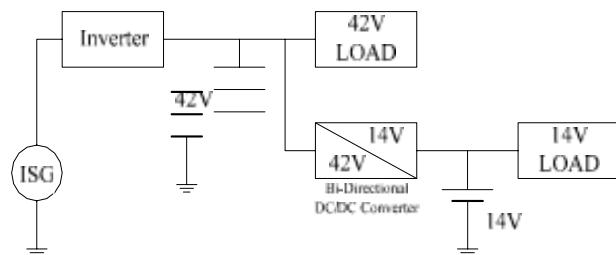


Fig. 3 The dual battery 42V/14V system

### **Problems of the conventional analog control scheme**

The 42V/14V dual voltage system have been adopted to provide backward compatibility with the existing components of the 14V system [7]. Fig. 3 shows one of the popular architectures for implementing the 42V/14V dual voltage system. Two batteries are connected to a 42V bus and a 14V bus, respectively and bi-directional DC-DC converter allows the power to be exchanged between the two buses.

When implementing a 42V/14V DC-DC converter, a simple non-isolated buck or boost converter is sufficient, since isolation between the two buses is not required in automobiles. Asynchronous buck/boost topology is considered to be more attractive since bi-directional operation is possible without the need for additional

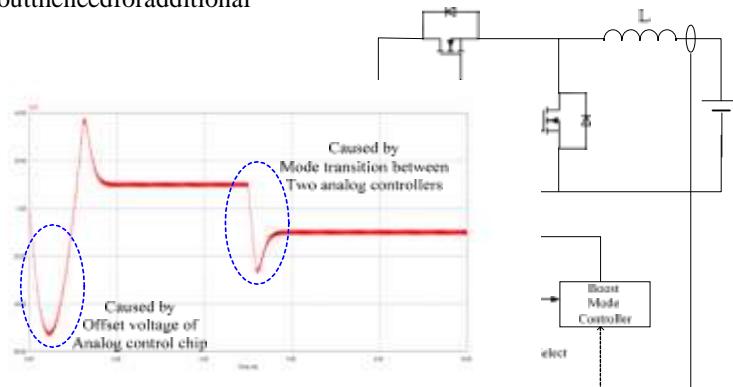


Fig. 5 The transient current simulation results of the analog control scheme

have a separate controller for each operation mode. Additionally, the PWMICs have a soft-start function and a voltage offset to increase the switching duty ratio slowly at the starting time and to allow for noise margin [10]. Due to these reasons, the transient current of the synchronous buck/boost converter may flow as shown in Fig. 5. This phenomenon, which is caused by the saturation of the error ramp output of the disabled controller, maybe observed during startup and in mode transitions as well [8,9]. However, the proposed digital control scheme does not have these transient problems. Thus, the system's stability and dynamics are additionally improved.

### **The proposed continuous state feedback control scheme**

Settling time  $\square$

4

$\square \square_n$

$\square 1\text{mS}$ ,

P.O.  $\square 100e^{\square \square \square} /$

$\square 1\%$

$\sqrt{1-\xi^2}$

(14)

As shown in Fig. 6, the bi-directional converter module has only one state variable, the inductor current. Using the un-terminated modeling method, the state equation is

$\square \square \frac{d}{dt} \square 2 \square \square$

$\square \square^2$ ,  
 $\square \square \underline{\text{sw}}(\text{Singular point})$

$n = 2$   
 derived as  
 SWON)i

$\square \frac{1}{v}$   
 $L = L^{42}$

$\square \frac{1}{v}$   
 $L^{14}$   
 ,SWOFF)i

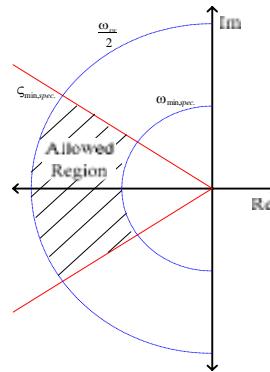
$\square \frac{1}{v}$   
 $L = L^{14}$

(12)

Fig. 7 shows the allowed region of the desired pole location. Using this pole placement technique, the continuous feedback controller for the system can be designed. To verify the theoretical analysis, the proposed digital

From the same procedure in Section 2.1, the state equation of the overall closed loop system is given as follows; state feedback control scheme has been tested by using the MATLAB Simulink software. The synchronous buck/boost converter parameters chosen for this simulation are:

$$\begin{aligned} & e(t) = 0 \\ & KV \\ & \underline{1} = |e(t)| \\ & KV \\ & z(t) = \end{aligned}$$



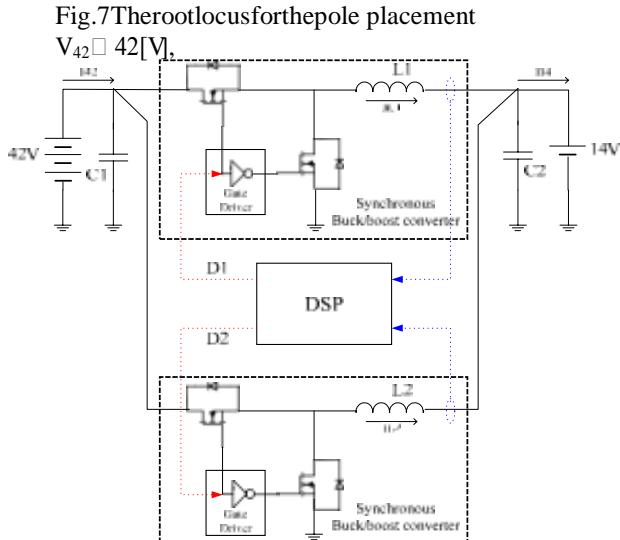
$$\begin{aligned} & \underline{1.42} \\ & \underline{2.42} z(t) \\ & \underline{L} \\ & L \end{aligned}$$

(13)

$$\frac{K_2 V_{42}}{L} s^2 + \frac{K_2 V_{42}}{L} = 0$$

The augmented system is a second order system. Therefore, the desired pole locations can be easily obtained from the given converter specification and the

Fig. 7 The root locus for the pole placement



$$V_{14} = 14[V]$$

$$L_1 = 11[\mu H]$$

$$rL_1 = 30[mH]$$

$$L_2 = 9[\mu H]$$

$$rL_2 = 50[mH]$$

, and  
Fsw = 100[kHz] where

$$rL_{1,2}$$

is the ESR of the inductor.

$$C_{1,2}$$

is neglected in this

simulation. From equation (14), the feedback gain is designed as

$$K_1 = 23.0895,$$

$$K_2 = 0.0047 \text{ (where)}$$

$$\alpha = 0.99,$$

$$r_n = 10k\Omega. \text{ When } r_n$$

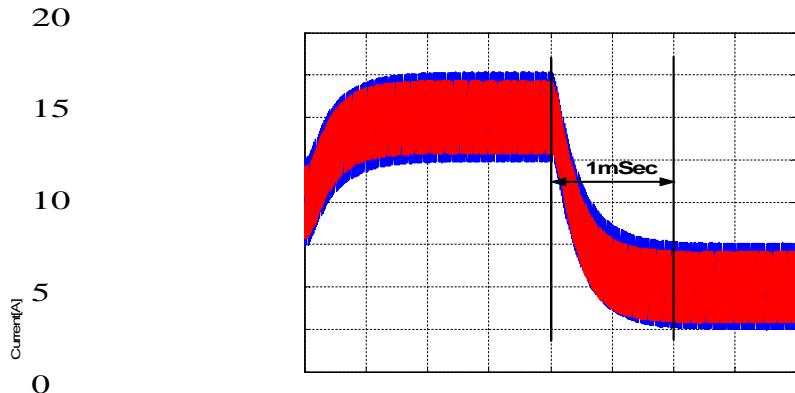
is selected, the digital sampling

Fig. 6 The parallel interleaved bi-directional converter control scheme effect should be considered because the controller is realized in a DSP.

Fig. 8 shows the current response during startup and transition from buck mode operation to boost mode operation. The proposed digital control scheme has a smooth transition between the two modes because the current controller can operate for the positive and negative general second order system's response. Let the given

converters specifications be defined as follows; settling time = 1mS, P.O. = 1%. From there required system dynamics, the characteristic equation of the desired poles is given by

reference, and can easily achieve the required dynamics for the given specification because time domain analysis is used. Also, it is observed that the interleaved current sharing among the converter modules is precisely achieved.



-5

-10

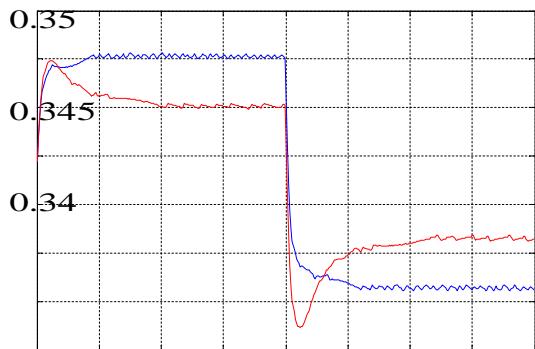
-15

-20

#### Inductor current of Each module

The proposed control scheme is realized using a TMS320F2812 32-bit fixed point DSP, which has a 12-bit ADC, 16-bit PWM, 150 MIPS performance, etc. Fig. 9 depicts the PWM duty ratio generation and the exact sampling of frequency generation using the synchronous PWM as well as the average inductor current sampling method without the use of a filter. Also, this PWM scheme can easily generate two phase shifted duty cycles by using the peak and valley of the PWM counter. Taking into account the A/D conversion time and the control

0      0.5      1      1.5      2      2.5      3      3.5      4



Time[Sec]

DutyRatioofEachmodule

$\times 10^{-3}$

algorithm calculation time, the duty cycle is updated at the next sampling time. This provision reduces control delays in the feedback loop.

The prototype hardware, which consists of two parallel buck/boost converter modules, is built as shown in Fig.6.

Dutyratio  
0.335

The converter parameters are:

$C_1 = C_2 = 300 \mu F$ ,

$L_1 = L_2$

0.33

$\square 10 \mu H$

and

$f_{sw} = 100 \text{ kHz}$ . A 36V battery made by GS

0.325

0.32

0.315

0 0.5 1 1.5 2 2.5 3 3.5 4

of Japan and a 12V battery made in Korea are used. Fig.10 shows the experimental result of the inductor current during startup transition. It is observed that there is no transient current flow, as shown in Fig.5. Fig.11 and 12

Time[Sec]

$\times 10^{-3}$

show the current response during the transition from buck

Fig. 8 The simulation results of the proposed digital state feedback current control

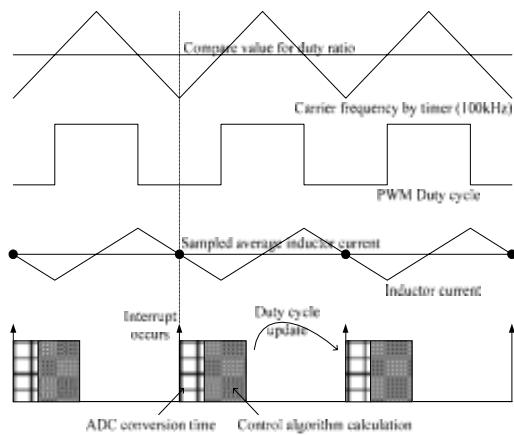


Fig.9 The DSP control strategy

### 3. Experimental Results of the Bi-Directional Converter

mode operation of 20[A], corresponding to 10[A] for each module, to boost mode operation of -20[A], and vice versa. The interleaved current sharing is possible during the transient period, as well as in steady state. The response time is in good agreement with the simulation results and thus the theoretical analysis is confirmed.

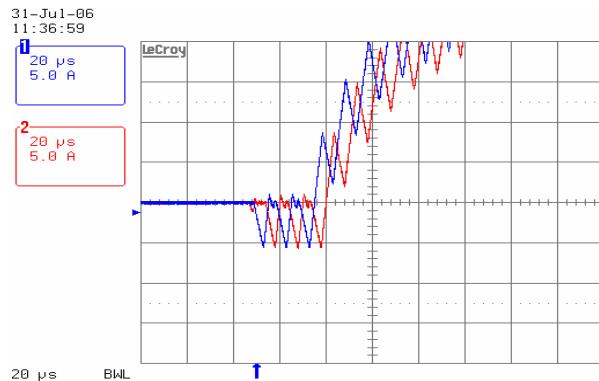
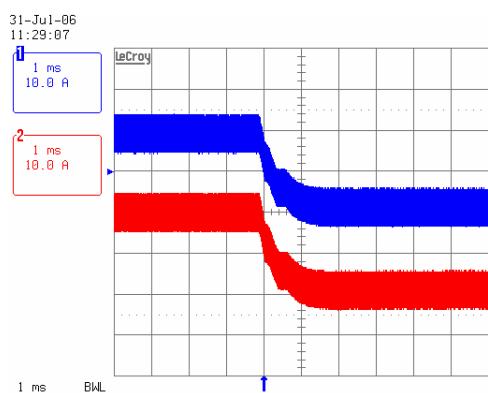


Fig.10 The startup transition to buck mode operation



The proposed discrete state feedback control scheme for the buck converter is illustrated in Fig. 13. Using the un-terminated modeling method, the state equation is derived as

$$\frac{v_1}{L_g} = \frac{v_o}{L_o} - \frac{v_{SWOFF}}{L_o}$$

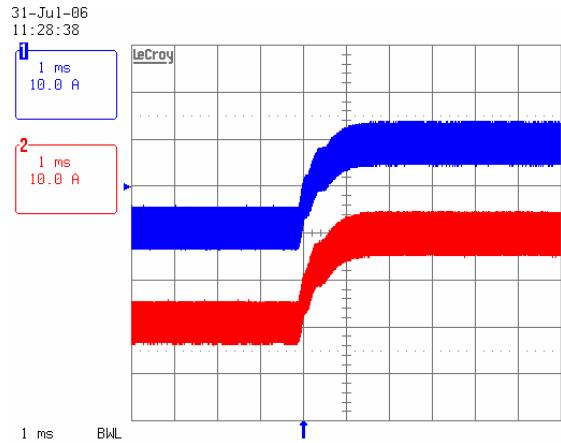
(15)

Fig.11 The mode transition from buck to boost mode operation

From same procedure in Section 2.2, the state equation of the overall closed loop system is given as follows;

$$\frac{e(k+1)}{z(k+1)} = \frac{1}{T_s} \left[ \frac{1}{K_1 V_g} + \frac{1}{K_2 V_g} \right]$$

(16)



$$T_s \frac{1}{L} z(k)$$

The augmented system is a second order system. Therefore, the desired pole location can be easily obtained from the given converter specification, as well as the general second order system's response using the continuous and discrete time domain relationship. Let the given converter specifications be defined as follows; Settling time  $\leq 100$  S, P.O.  $\leq 1\%$ . From the required system dynamics, the desired pole locations are given by

Fig.12 The mode transition from boost to buck mode operation

Settlingtime  $\frac{4T_s}{\ln(r)} \approx 100$  S,

$\ln(r)$

P.O.  $100e^{\frac{\ln(r)}{1}} \approx 1\%$

(17)

$$z_{desired} = r \cos(\theta) + j r \sin(\theta)$$

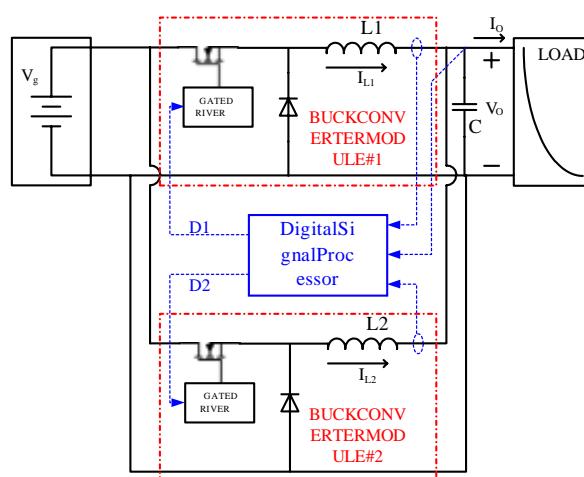
The control duty ratio is given from (10) and can be modified as follows;

$$d(k) = d(k-1) + K_1 T_s i_L, ref - K_2 i_L(k) - K_1 T_s K_2 i_L(k-1)$$

(18)

To verify the theoretical analysis, the proposed digital state feedback control scheme has been tested by using the MATLAB Simulink software. The buck converter parameters chosen for this simulation are:

$$\begin{aligned} V_g &= 52[V], V_o \\ &= 28[V] \end{aligned}$$



$$L_1 = 110[\mu H]$$

$$r_{L1}$$

$$= 30[mH]$$

$$L_2 = 110[\mu H]$$

$$r_{L2}$$

Fig.13 The parallel interleaved buck converter control scheme

$$= 30[mH]$$

$$F_{sw} = 100[kHz], \text{ where } C \text{ is neglected in}$$

this simulation. From equation (17), the feedback gain is designed as

$$K_1 T_s = 0.0304,$$

$$K_2 = 0.1363.$$

Fig.14 shows

#### 4. The Buck Converter Current Mode Control for Discrete Time Domain Design Example

the transient current response of the proposed control system. It is observed that the interleaved current sharing

among the converter modules is precisely achieved even though the parasitic resistance and the inductance are different. Also, the proposed discrete state feedback control scheme can meet the required dynamics for the given specifications.

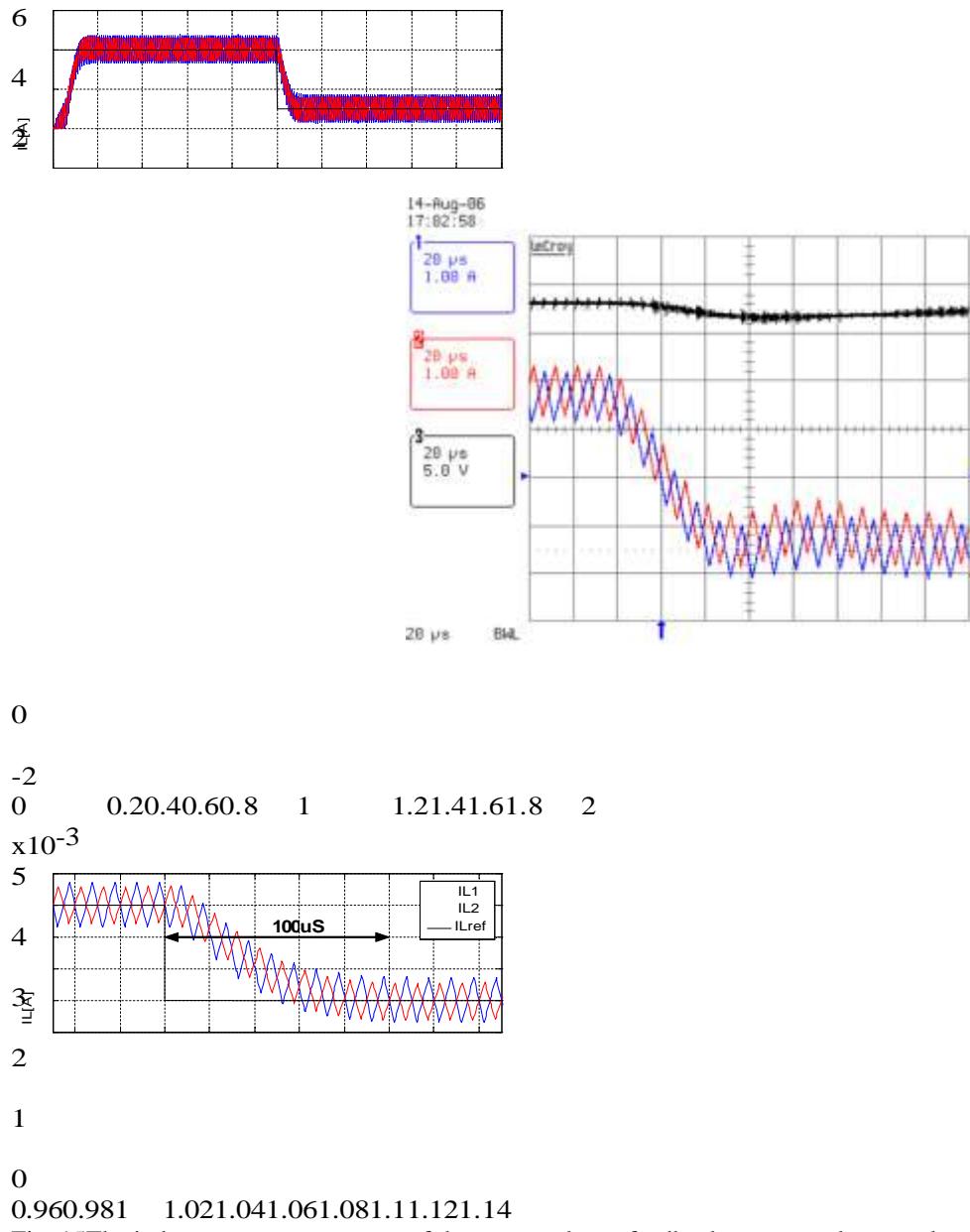


Fig. 15 The inductor current response of the proposed state feedback current mode control

The prototype hardware, which consists of two 100W parallel module buck converters, is built as shown in Fig.

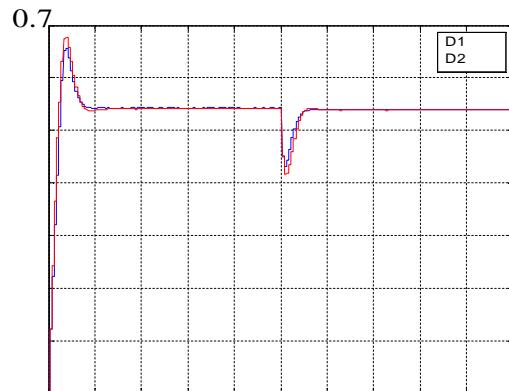
Time[sec]

$\times 10^{-3}$

13. The converter parameters are:

$V_g \square 52[V]$ ,

$V_o \square 28[V]$ ,



$L_1 \square L_2 \square 100[\square H]$ ,

$C \square 100[\square F]$ , and

$F_{sw} \square 100[kHz]$ . The

0.6

0.5

0.4

0.3

0.2

0.1

0

0

0.2 0.4 0.6 0.8 1 1.2 1.4 1.6

Time[sec]

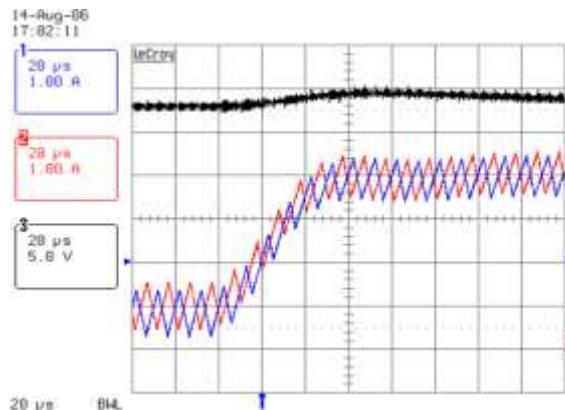
1.8 2

$\times 10^{-3}$

DSP implementation is the same as the bi-directional converter application as described in Section 4. Fig. 15 shows the inductor current of each module at the current control loop, which is designed and simulated in this section. In this

implementation, the output voltage is set to 28[V] using a constant voltage mode using electric load equipment. The response time and performance of the discrete state feedback controller are in good agreement with the simulation results and thus the theoretical analysis is confirmed.

Fig. 14 The simulation results of the proposed discrete state feedback current control



### CONCLUSIONS

A digital state feedback control approach using the pole placement technique is proposed in the continuous and discrete time domain. Since the analysis and design are performed in the time domain using the state equations, the controller can be systematically designed for the required system specifications. For the design example in the continuous time domain approach, a prototype 42V/14V bi-directional automobile system that consists of two synchronous buck/boost converter modules has been built and tested by using the digital state feedback current control, which can solve the problems of the analog control scheme. For the design example in the discrete time domain approach, a parallel module buck converter was built and tested by using the discrete state feedback current control. The proposed control system can achieve interleaved current sharing and can track the reference value.

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