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A Literature Survey on Mathematical Modeling and its Perspectives

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ABSTRACT:
Mathematical modelling has a long tradition in mathematics education and has been gaining international attention, not only in research and practice, but also in official perspectives reflected explicitly in programs of studies around the world. Despite extensive publications on diverse aspects of mathematical modelling, systematic literature surveys on this topic are scarce. We highlight some qualitative results from a systematic survey of 452 publications related to different perspectives on mathematical modelling, extending and complementing previous reviews of the state of the art. In particular, we elaborate on the notion of 'authenticity' and the purposes of mathematical modelling in education. Finally, we identify two trends in Latin Americancountries.

NEED FOR A SURVEY

The roots of mathematical modelling, as per Niss, Blum and Galbright (2007), date from the late 1950s when mathematical modelling advocates attempted to restore focus on the utility and applications of mathematics in schools and universities. By the 1970s, several countries incorporated mathematical modelling their curriculum. A key moment for the international move toward mathematical modelling in education was the inauguration of the biennial Conference on the Teaching of Mathematical Modelling and Applications in 1983, organized by the International Community of Teachers of Mathematical Modelling and Applications (ICTMA). Another key event was the publication of the 14thInternational Commission on Mathematical Instruction (ICMI) study (Blum, Galbraith, Henn, &Niss, 2007). Since then, international research has increased significantly, and research methods and focuses have extended beyond traditional approaches (Stillman, Blum, & Kaiser, 2017). Current international focuses on science, technology, engineering and mathematics (STEM) education have also stressed the importance of mathematical modeling (English, 2015). Despite the large number of publications on mathematical modelling in education, systematic reviews in the literature arescarce.

Different perspectives have influenced the integration of mathematical modelling in educational contexts. In particular, Kaiser &Sriraman (2006) proposed a classification

consisting of six perspectives, as described in this paragraph. The realistic perspective aims to solve real-life problems beyond mathematics: authentic problems from industry and science are particularly relevant here. In contrast, the epistemological perspective focuses on the development of mathematical theories, and includes intra-mathematical models that are used to advance theory in mathematics. The educational perspective, based on an integrative approach (Blum &Niss, 1991), considers different aims for modelling that serve scientific, mathematical and pragmatic purposes harmoniously. The contextual perspective, also called the model-eliciting approach, focuses on problem solving activities constructed using specific instructional design principles. According to Kaiser and Sriraman, with this approach, "students make sense of meaningful situations, and invent, extend, and refine their own mathematical constructs" (p. 306). The socio-critical perspective emphasises the need to develop a critical stance towards the role and nature of mathe- matical models, as well as their impact on social issues. The cognitive perspective on modelling is transversal to the previous five and focuses on cognitive aspects of the mathematical modellingprocess.

Due to this diversity of perspectives, it is difficult to provide a single definition for mathematical modelling. In this paper, we highlight two common elements of a mathematical model consistent across diverse perspectives, namely: a situation or phenomena of interest, commonly but not exclusively, from the world beyond mathe- matics; and a collection of mathematical entities and relationships that correspond to certain aspects of the situation or phenomena of interest. The collection of entities and relationships is often represented visually and can be manipulated and studied with mathematical tools to make predictions or inferences about the situation or phenomena of interest. Modelling can therefore be understood as the creation or the application of a model to

solve a problem, make predictions or estimations, study certain phenomena, inform decisions, or even create policy (Blum &Niss,1991).

Because there are diverse perspectives and purposes related to mathematicalmodelling, it is important for researchers, teachers, administrators and policy makers to understand and be explicit about the differences among such perspectives. In this paper, we synthetize some results from a literature survey that includes key publicca- tions in journals and books. We focus on aspects that complement some of the key reviews of the state of the art in this field, including Latin Americantrends.

SURVEYING THE LITERATURE

This survey is the result of a seminar consisting of graduate and undergraduate students and educators from the National Pedagogical University and the Mathematics Education Department of the Center for Research and Advanced Studies in Mexico, and the University of Calgary in Canada. We identified multiple perspectives on modelling and found the literature on this topic to be vast. We also noticed that systematic reviews, such as the one conducted by Frejd (2013), were scarce; this influencedourdecisiontoconductourownreview.Thispaperpresentsinsightsfrom

an exploration in the literature focused on the different perspectives on mathematical modeling. Kaiser and Sriraman's (2006) widely cited classification served as a point of contrast to identify salient themes in this exploration.

We initiated our survey by searching peer-reviewed articles with 'modeling' or 'modelling' in the title through the SpringerLink database. Then, we refined the search using 'Education' as discipline and 'Mathematics Education' as subdiscipline for each of the two words. Book reviews and other articles that did not relate to mathematical modelling were excluded, resulting in a list with 73 articles. This list can be considered as representative of the literature because: (a) Springer publishes many of the most influential journals in mathematics education identified by Toerner and Arzarello (2012), and (b) searching the key words in the titles suggests that mathematical modelling is a main focus for the selected articles. The list served as a starting point for the survey, and the initial analysis not only helped to clarify and refine the categories that guided the review, but also allowed for the identification of key publications in books and articles in special journal issues.

In a second stage of the survey, we included: (a) articles from the special issues on mathematical modelling, and articles published in 2017 not included previously; (b) articles from the Journal for Research in Mathematics Education (JRME); (c) five books related to ICTMA and the 14th ICMI Study (Blum, Galbraith, Henn, &Niss, 2007); (d) articles from journals on mathematics education published in Spanish; and

(e) a recent Latin-American book addressing research on mathematical modelling (Arrieta Vera &Díaz Moreno, 2016).

We chose JRME because it is at the top of the list of journals identified by Toerner and Arzarello (2012). The same title criterion as in the first stage was followed to search articles in this journal. The five books related to ICTMA, extracted from its biannual conference, correspond to the series International Perspectives on the Teaching and Learning of Mathematical Modellingpublished by Springer. We included the Spanish journals and the Latin-American book to extend the scope of the review beyond publications in English. The selected journals were RevistaEducaciónMatemáticaand RevistaLatinoamericana de Investigación en MatemáticaEducativa, because they are specialized in mathematics education and are the most relevant among the Spanish journals. Similar to our search of the journals in English, we searched for articles with words in the title related to modelling.

A total of 452 documents were included for this paper: 111 journal articles, and 341 book chapters. Here, we report results from a thematic analysis on the perspectives on mathematical modelling as presented in these documents.

EMERGENT THEMES AND PERSPECTIVES ON MODELLING

We identified two themes with strong connection to the perspectives on mathematical modelling: authenticity and purpose. We also identified two common trends in mathematical modelling from Latin American countries from the Spanish literature.Within the recent literature, there is clearly a debate on the notions of 'authenticity' and 'real world,' commonly invoked by several authors from different perspectives on modelling. In one sense or another, most (and perhaps all) perspectives allude to something 'authentic.' Regarding the realistic perspective, Kaiser and Sriraman (2006) claimed that "modelling processes are carried out as a whole and not as partial processes, like applied mathematicians would do in practice" (p. 305). Something similar could be said for mathematicians and scientists who have developed mathe- matical theories based on phenomena from other fields, such as financing, chemistry, astronomy or biology. Such theories are often extended to models that are applied to subjects beyond mathematics. In this sense, the process of theory generation can be considered as authentic to the work of mathematical modelling.

With respect to this debate, Jablonka (2007) suggested that authentic mathematical modelling in the classroom

can take place "when students and teachers are bona fide engaging in a modelling or application activity about an issue relevant to them or to their community" (p. 196). This framing could be related to any of the perspectives in Kaiser and Sriraman's (2006) classification.

A proposal in this debate is to consider elements of authentic modelling within a task. Vos (2011) suggested a definition for authenticity in which components of a task, instead of the task itself, include objects that are "clearly not created for educational purposes" (p. 721). In this sense, many tasks within different perspectives have authentic elements of mathematical modelling. Indeed, many reports in the consulted literature do not include the whole modelling cycle or process due to limitations in implementation. In other cases, the instructional approach does not include the whole process of modelling, or does not start from the 'real world.' For instance, Silva Soares (2015) suggested model analysis as a teaching approach in which students analyse an already existing model instead of creating a model from realdata.

Most recently, Carreira and Baioa (2017) addressed the concept of 'credibility' rather than 'authenticity' in mathematical tasks. While authors have argued that real life situations have the potential to make the learning experience more attractive, this focus on credibility places the relevance at a personal level forstudents.

Finally, many publications focused on simulations using computer systems. Simula- tions are used as a part of the modelling cycle involving real data (e.g. Niss, 2015). However, simulations are also used as models to teach specific content within and beyond mathematics (e.g. Gomes Neves, Carvalho Silva, & Duarte Teodoro, 2011). While students may not engage with real data when using a simulator, they can experi- ment within the model and learn both mathematical and extra-mathematical content.

Regarding the purposes for mathematical modelling, we identified a list, summarized in Table 1, that extends the purposes considered in Kaiser and Sriraman (2006). Many of these might be included in one or more of the perspectives in this classification. In particular, awareness of social and global issues, participatory attitude, and culture of innovation could be considered purposes within both the realistic and thesocio-critical approaches, if we extend their description. For the realistic approach, for instance, the purpose can involve an "ultimate goal," as suggested by Carreira and Baioa(2017).

Purposes To:	Examples
Learnmathematicscontent	Algebra, Geometry, Calculus, Statistics
Applymathematics	Problemsolving
Learnotherdisciplines	Chemistry, Biology, Finances, Heath Care
Conductresearch	Research on learning in virtualenvironments
Design learning environments	Design simulators and virtual environments for learning purposes
Develop modellingcompetencies	Elements of modelling; criteria for quality in mathematical modelling
Developlearningskills	Generalize the solution of a problem to othersimilar problems
Generate mathematical theory	Conceptual understanding; mathematical proof
Developcriticalthinking	Judge models used in daily life; questionpurpose and assumptions of different models

Generate mathematical theory	Conceptual understanding; mathematical proof
Developcriticalthinking	Judge models used in daily life; questionpurpose and assumptions of different models
Understand mathematics as a discipline	Historical, social and political aspects of mathematics as a discipline
Develop awareness of social and global issues	Create and critique models used to predict: economic growth; global warming; tax revenue
Promote a participatory attitude	Engage in addressing real problems and decision-making within the community
Promote a culture of innovation	Create something for a customer; program software for an audience
Engage in emancipation strategies	Decolonization; cultural practices in mathematics and mathematical modelling

Table 1: Purposes for mathematical modelling in education, with examples.

While developing mathematical modelling competencies is a common purpose within the literature, other aspects of modelling are barely considered, such as the quality criteria proposed by Perrenet, Zwaneveld, Overveld and Borghuis (2017), comprising: genericity, scalability, specialization, audience, convincingness, distinctiveness, surprise, and impact. In fact, most of the modelling tasks do not include constructing or experimenting with physical objects, creating something for a customer, writing computer code, or making a decision that will affect the local community. Niss (2015)proposed the term prescriptive modellingthat involves designing, prescribing, organizing or structuring certain aspects of the real world. Papers that included these activities could be considered as promoters of a culture of innovation and a parti- cipatory attitude. For example, Orey and Rosa (2017) reported a task addressing the issue of tariffs in public transportation.

Finally, a few papers addressed mathematical modelling for research or for developing learning environments, which are not considered as purposes in Kaiser and Sriraman's (2006) classification. Campbell (2011), for example, addressed the use of virtual reality, which requires mathematical modelling to create virtual spaces and objects. In contrast to the other purposes for modelling, students may not engage in elements of modelling in the corresponding learning, or research,environments.

Latin American trends

As we reviewed articles written in Spanish, we identified two main trends on mathematical modeling for Latin America, namely: the number of publications, and the innovative aspects in their approaches. These contributions strongly emphasise the social and cultural influences of modellingeducation.

Mathematical modelling research from Spanish speaking countries produced a modest number of papers in the publications from Springer. However, the review revealed activity in mathematical modelling in Latin America since the 1990s, and in the case of Brazil, since the 1970s (SalettBiembengut, 2016). This brings into contention Blum and Niss's (1991) claims that mathematical modelling was initially developed in regions such as Germany and theUK.

While the number of publications from Spanish speaking countries has increased modestly in the last few years, publications from Brazil are conspicuous in documents published by Springer. Particularly, the 16 ICTMA Conference, held in Brazil in 2013, resulted in an increased number of authors from the host country.

Regarding innovative aspects in Latin American approaches, Stillman, Blum, &Biembengut (2015) identified elements of "a unique Latin American perspective to modelling" in the work of Brazilian author, UbiratanD'Ambrosio, who discusses knowledge generation (cognition), its individual and social organization (episte- mology) and the way it is confiscated, institutionalised and given back to the people who generated it (politics). His perspective on mathematical modeling extends the socio-critical perspective and is a strategy for building up systems of knowledge in different cultural environments.

Another Latin American modelling trend corresponds to research reported as socio-epistemological (see for instance Arrieta Vera and Díaz Moreno, 2016; Quiroz Rivera & Rodríguez Gallegos, 2015). This approach understands mathematical modelling in terms of social practices, both in school and in formal mathematics.

Concluding Remarks

This paper complements other reviews of the state of the art regarding the debate on authenticity, the purposes

of modelling, and Latin American trends. The discussion on authenticity and the identification of purposes for mathematical modellingproblema- tize Kaiser and Sriraman's (2006) classification of perspectives on mathematical modelling. Perhaps, while this classification has been useful in the past, it may be appropriate to pay closer attention to the purposes of mathematical modelling on an individual basis, and consider the elements of authenticity in tasks, as suggested by Vos (2011). These elements of authenticity may vary based on the mathematical con- tent addressed in each task. For instance, modelling with statistics may involve ele- ments of authenticity (e.g. using real data) that differ from the elements of authenticity for modelling with calculus (e.g. usingsimulations).

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