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Bending Analysis of Piezolaminated Composite Plates Using HSDT

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ABSTRACT

Smart materials are the materials where in one or more properties can be significantly changed in a controlled fashion by external stimuli, such as temperature, stress, moisture etc. Among various types of smart materials, piezoelectric materials are mostly use because, theoretical analysis of this material has been so well developed. In this work an attempt has been made to analyze the piezolaminated composite plates subjected to electromechanical loading using higher order shear deformation theory (HSDT) for more accurate results. For analysis two displacement models are to be considered i.e., model-1 accounts for strain in thickness direction is zero whereas in model-2 in-plane displacements are expanded as cubic functions of the thickness coordinate. The solutions are obtained using Navier's method for anti-symmetric cross ply and angle ply composite laminated plates attached with piezoelectric layer with a specific type of simply supported boundary conditions.
Keywords: Angle ply, cross ply, HSDT, Navier's method, piezolaminated composite plates and smart materials.

I. INTRODUCTION

Smart materials have an inbuilt property of controlling the performance of structure. Using smart materials as actuators shape, size, stability and vibration of the laminated composite plates can be controlled. Piezoelectric materials are materials that produce a voltage when stress is applied. Since this effect also applies in the reverse manner, the voltage across the sample will produce stress within the sample. Structures made from smart materials can be bent, expanded or contracted when a voltage is applied. Piska Raghu et al [1], presented analytical solutions for composite laminated plates using nonlocal third order shear deformation theory considering surface effects. The theory developed by them was based on Eringen's theory of nonlocal continuum mechanics and thrid order plate theory of Reddy. They presented the analytical solutions for bending and vibration of simply supported laminated plates using new formulation to demonstrate nonlocal effects on deflection and vibration frequencies for different length to thickness ratios. V.M. Sreehari et al [2], developed finite element formulation for conduct of buckling and bending analysis of smart composite plates based on inverse hyperbolic shear deformation theory. They derived the governing equation of piezolaminated composite plate using Hamilton's variational principle. Matlab programme has been developed by them using the finite element formulation. Significant numerical examples for enhancement of buckling load and deflection control for composite plates with internal flaw. M. Filippi et al [3], proposed one-dimensional layer wise theories which make use of higher-order zig-zag functions over mathematical layers of cross-sectional area. Using piecewise continuous power series expansions they obtained the variable kinematics theories. They applied virtual displacements and Carrera unified formulation and solved using finite element formulation for obtaining governing equations. S. M. Shiyekar et al [4] presented an analytical solution for cross ply piezolaminated composite laminated plates subjected to bi-directional bending. Equations of equilibrium have been obtained by them using principle of minimum potential energy. The obtained results by them were compared with exact solution.

An analytical solution for cross-ply laminated composite plates incorporated with piezoelectric fiber reinforced composite (PFRC) actuators under bidirectional bending is offered by Kant et al [5]. They used higher order shear and normal deformation theory (HOSNT12) for analyzing the smart materials subjected to electromechanical loading. Two model problems were considered by Alden C. Cook et al [6] for multiscale analysis procedure. In their first model problem they considered a simply-supported sandwich plate consisting of a piezoceramic fibre and bottom surfaces. In second model a cantilever graphite substrate is considered with segmented piezoceramic fiber composite extension actuators attached at its top. Kant et al [7] developed analytical solutions for the cylindrical flexure of piezoelectric plates based on higher order shear deformation theory. They analyzed a unidirectional composite plate attached with distributed actuator and sensor layers under mechanical and electrical loading conditions. Fariborz Heidary et al [8] have outlined linear response of piezothermoelastic plate based on Hamilton's principle and finite element methods. Results obtained of piezolaminated composite plates subjected to thermomechanical loadings are presented. They suppressed the vibrations, using the difference in electric potential between piezoelectric layers.

II. FORMULATION OF HSDT

In formulating the higher-order shear deformation theory, a composite plate of $0 \le x \le a$; $0 \le y \le b$ attached with an actuator and is simply supported along four sides of the plate is considered. In order to approximate 3D-elasticity plate problem to a 2D one, the displacement components $u(x, y, z, t), v(x, y, z, t)$ and $w(x, y, z, t)$ at any point in the plate are expanded in terms of the thickness coordinate. The elasticity solution indicates that the transverse shear stress varies parabolically through the plate thickness. This requires the use of a displacement field, in which the in-plane displacements are expanded as cubic functions of the thickness coordinate. The displacement field which assumes w (x, y, z) constant through the plate thickness and thus setting $\varepsilon_z = 0$ is expressed as [4]:

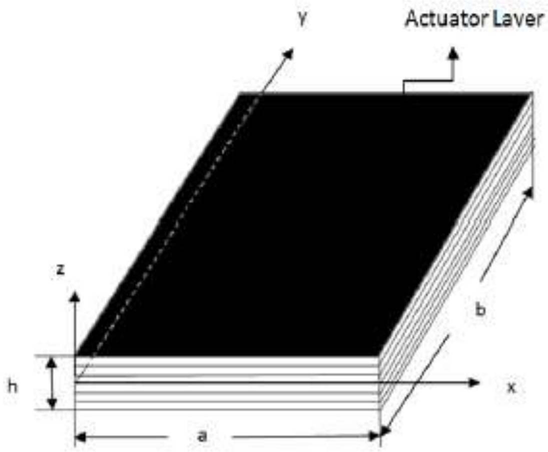

Fig. 1. Composite laminated plate attached with piezoelectric layer [5]

$$\left.\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2 u_o^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2 v_o^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_o(x,y) \end{aligned}\right\} \quad \ldots\ldots (1)$$

The displacement field in which the in-plane displacements are expanded as cubic functions of the thickness coordinate in addition the transverse normal strain may vary nonlinearly through the plate thickness is expressed as [5]:

$$\left.\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2 u_o^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2 v_o^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_o(x,y) + z\theta_z(x,y) + z^2 w_o^*(x,y) + z^3\theta_z^*(x,y) \end{aligned}\right\} \quad \ldots\ldots (2)$$

Where the parameters u_0, v_0 are the in plane displacements and w_o is the transverse displacement of a point (x, y) on the mid plane. The functions θ_x, θ_y are rotations of the normal to the midplane about y and x–axes, respectively. The parameters u_0^*, v_0^*, w_0^*, θ_x^*, θ_y^*, and θ_z^* are the corresponding higher–order deformation terms and they represent higher–order transverse cross sectional deformation modes.

Constitutive relations for smart materials:

The constitutive relation for smart materials of a single piezoelectric layer couples the elastic and electric fields are [7]:

$$\{\sigma\} = [Q]\{\varepsilon\} - [e]\{E\} \quad \ldots\ldots (3)$$

Where σ, Q, ε, e and E are stress vector, elastic constant matrix, strain vector, piezoelectric constant matrix and electric field intensity vector respectively.

Eq. 3 can be represented in two components of stresses**.** One is elastic stress component (es) and other is piezoelectric stress component (pz) and written as [7]:

$$\{\sigma\} = \{\sigma\}^{es} - \{\sigma\}^{pz} \quad \ldots\ldots. (4)$$

The governing equations of displacement model will be derived using Hamilton's principle [8]:

$$\int_0^T (\delta U + \delta V - \delta K)\, dt = 0 \quad \ldots\ldots (5)$$

Integrating Eq. 5 through the thickness of the laminate, and rewriting in-plane force, moment resultants, transverse force resultants and inertias in matrix, it is obtained as:

$$\begin{Bmatrix} F_i \\ F_i^* \\ -- \\ M_r \\ M_r^* \\ -- \\ F_t \\ F_t^* \end{Bmatrix} = \begin{bmatrix} A & | & B & | & 0 \\ \hline B^t & | & D_b & | & 0 \\ \hline 0 & | & 0 & | & D_s \end{bmatrix} \begin{Bmatrix} \in_0 \\ \in_0^* \\ -- \\ L \\ L^* \\ -- \\ \phi \\ \phi^* \end{Bmatrix} \quad \ldots (6)$$

III. RESULTS AND DISCUSSIONS

The material properties of graphite/epoxy used for each orthotropic layer of the substrate are [4]:

$$\frac{E_1}{E_2} = 25,\ E_2 = E_3 = 7\,Gpa,\ G_{13} = G_{12} = 0.5E_2,$$

$$G_{23} = 0.2E_2,\ \mu_{12} = \mu_{23} = \mu_{13} = 0.25$$

Material properties for PFRC layer are [5]:

$$C_{11} = 32.6\,GPa;\ C_{12} = C_{21} = 4.3\,GPa;\ C_{13} = C_{31} = 4.76\,GPa;\ C_{22} = C_{33} = 7.2\,GPa;$$

$$C_{23} = 3.85\,GPa;\ C_{44} = 1.05\,GPa;\ C_{55} = C_{66} = 1.29\,GPa;\ e_{31} = -6.76\,C/m^2;$$

$$g_{11} = g_{22} = 0.037\,E-9\,C/Vm;\ g_{33} = 10.64-9\,C/Vm.$$

Fig. 2 & 3 shows the variation of non-dimensionalized centre deflection (w) against modulus ratio of simply supported anti-symmetric cross-ply laminated composite plate with and without piezo layer for model-1 and model-2. The piezo layer actuation is more in case of 8 layer laminated plate than 2 and 4 laminated plates. From the Fig. 2 it is found that the maximum percentage variation in non-dimensionalized centre deflection is 10.2 for composite laminated plate with and without piezo layer. From the figure it is observed that the deflections become smaller due to the piezoelectric effect. It is because the plate is deformed by the external load and the electric charges are generated in the piezo layer in opposite direction. As a result a force is generated through the converse piezoelectric effect and actively controls the static response of the laminated plate.

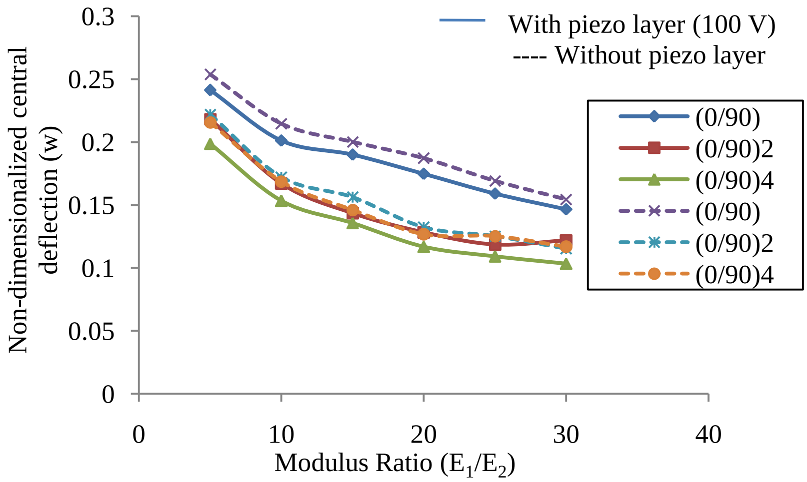

Fig. 2: Non-dimensionalized central deflection (w) vs modulus ratio (E_1/E_2) for simply supported anti-symmetric cross-ply piezolaminated plate for model-1

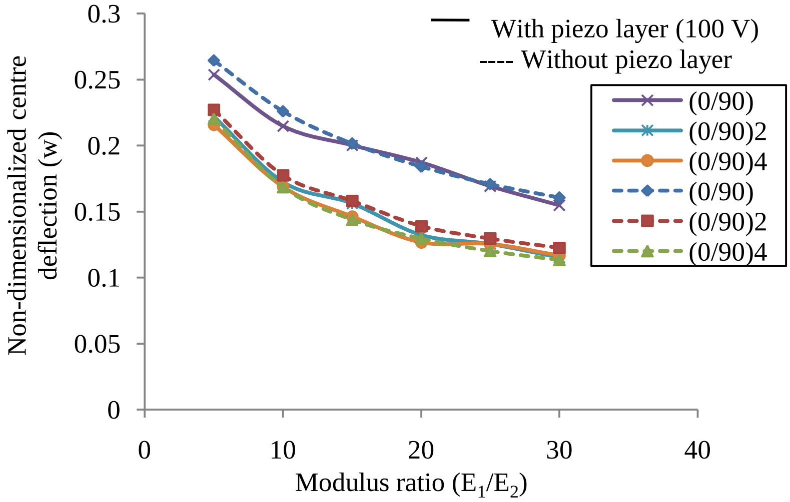

Fig. 3: Non-dimensionalized central deflection (w) vs modulus ratio (E_1/E_2) of simply supported anti-symmetric cross-ply piezolaminated plate for model-2

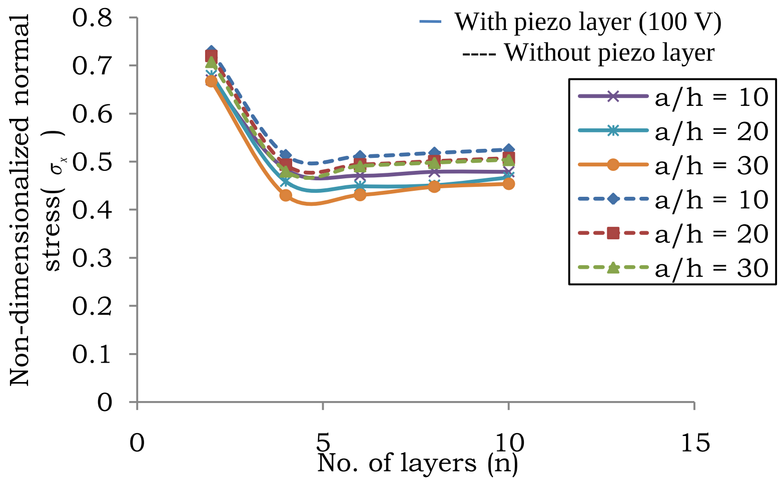

Fig. 4: Non-dimensionalized normal stress (σ_x) vs no. of layers (n) of simply supported anti-symmetric cross-ply piezolaminated plate for model-1

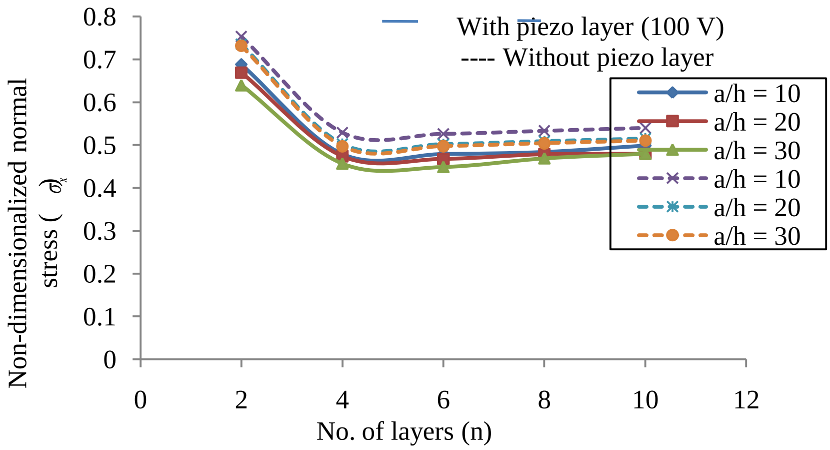

Fig. 5: Non-dimensionalized normal stress (σ_x) vs no. of layers (n) of simply supported anti-symmetric cross-ply piezolaminated plate for model-2

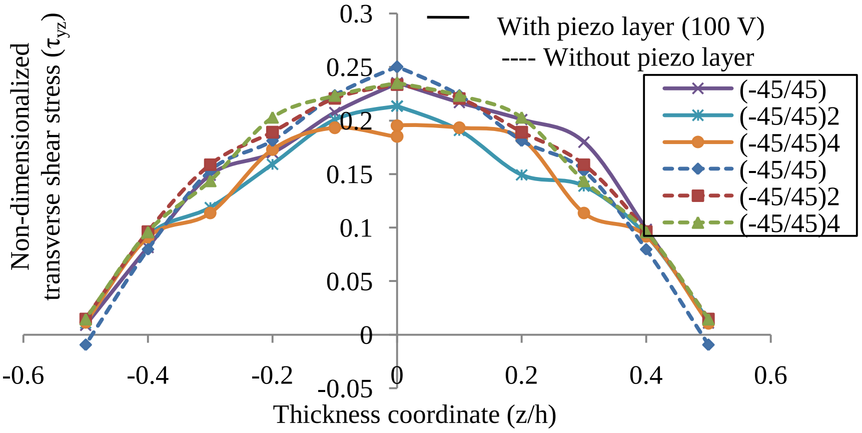

Fig. 6: Non-dimensionalized transverse shear stress (τ_{yz}) vs thickness coordinate (z/h) of simply supported anti-symmetric angle-ply piezolaminated plate for model-1

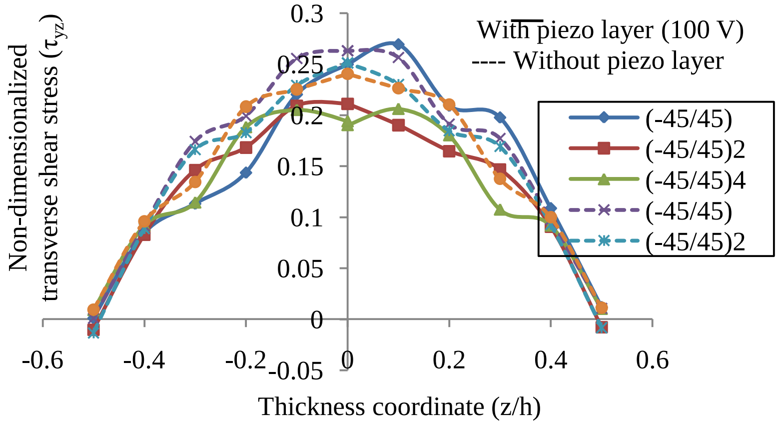

Fig. 7: Non-dimensionalized transverse shear stress (τ_{yz}) thickness coordinate (z/h) of simply supported anti-symmetric angle-ply piezolaminated plate for model-2

Fig. 4 & 5 shows the variation of non-dimensionalized normal stresses (σ_x) against no. of layers for anti-symmetric cross-ply laminated composite plates with and without piezo layer for model-1 and model-2 under electromechanical loading. In 2 layered laminated plates the stresses induced are more compared to those of 4, 6 and 8 layered piezolaminated plates. The effect of induced electrical voltage is more in a 2 layered piezolaminated composite plate causes increase in magnitude of stresses. The maximum percentage variation between with and without piezo layer is 8.96 and 9.44 for model-1 and model-2 respectively. Fig. 6 & 7 shows the variation of non-dimensionalized transverse shear stress (τ_{yz}) against thickness coordinate of simply supported angle-ply laminated composite plate with and without piezo layer for model-1 and model-2. From figures it is noticed that the distribution of shear stress across thickness coordinates increases significantly in composite layer in contact with piezo layer and decreases in other layers. The maximum percentage variation in shear stress between with and without piezo layer for 2 layered laminated plates is 7.72 in model-1 and 8.21 in model-2.

IV. CONCLUSIONS

Analytical procedure has been developed for model-1 and model-2 for analyzing piezolaminated composite plates subjected to electromechanical loading. Higher order shear deformation theory is used to model elastic substrate response to voltages. The non-dimensionalized transverse displacement, normal and shear stresses are obtained for various modulus ratios, no. of layers and thickness coordinates subjected to with and without electrical loading. From the study it is found that, shear deformation effects has the impact on piezoelectric laminates, and cannot be ignored while modeling them. It can also be concluded from the results that the actuating effects are more in case of thick than thin laminates.

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NOMENCLATURE

A - Extension stiffness matrix.
B - Bending-Extension coupling matrix.
D_S - Elasticity matrix relating shear force and shear strains.
D_b - Elasticity matrix relating moments and bending strains.
E_i - Young's modulus of elasticity in the i^{th} direction.
I - Moment of inertia.
F_i - In-plane force resultants.
M_r - Moment resultants.
F_t - Transverse force resultants.
σ - Stress vector.
Q - Elastic constant matrix.
ε - Strain vector.
e - Piezoelectric constant matrix.
E - Electric field intensity vector.
δU - Virtual strain energy.
δV - Virtual work done by applied forces.
δK - Virtual kinetic energy.
z - Distance of a point along the z-axis.
x, y, z - Cartesian co-ordinates.
u, v, w - Components of deformation in x, y, z axes.
$\varepsilon_o, \varepsilon_o^*$ - Strain components.
L, L^* - Bending curvatures.
φ , φ * - Transverse shear strains.

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