ISSN (e): 2250 – 3005 || Vol, 04 || Issue, 4 || April – 2014 ||
International Journal of Computational Engineering Research (IJCER)

Effect of First Order Chemical Reaction on Free Convection in a Vertical Double Passage Channel for Conducting Fluid

J. Prathap Kumar[1], J.C. Umavathi[2], Deena Sunil Sharanappa[3]
[1, 2, 3] Department of Mathematics, Gulbarga University, Gulbarga-585 106, Karnataka, India

ABSTRACT:
This paper reports investigation on laminar free convection in a vertical double passage channel for electrically conducting fluid in the presence of first order chemical reaction. The channel is divided into two passages by inserting a thin plane conducting baffle. After placing the baffle one of the passage is concentrated. An analytical solution has been developed for the coupled nonlinear ordinary differential equations using regular perturbation method. The results show that the thermal and mass Grashof number and Brinkman number enhances the flow where as Hartmann number and first order chemical reaction parameter suppresses the flow at all the baffle positions in both the streams.

Keywords: *Baffle, conducting fluid, first order chemical reaction, free convection, perturbation method.*

I. INTRODUCTION

The study of heat and mass transfer for an electrically conducting fluid under the influence of transverse magnetic field has attracted researchers in the recent past due to its relevance in many engineering problems such as MHD generators, heat exchangers, thermal processing, gas-cooled nuclear reactors and others. In the absence of magnetic field, references [1-3] will give some ideas about fluid flow and thermal characteristics inside a vertical channel with symmetric or asymmetric thermal boundary conditions. For an infinite vertical plate, Raptis and Kafoussias [4] studied the flow and heat transfer characteristics in the presence of magnetic field. Later Raptis [5] extended the vertical plate problem to a vertical channel problem in the presence of magnetic field. Malashetty et al. [6, 7, 8], Umavathi et al. [9], and Prathap Kumar et al., [10] studied mixed convection in a vertical channel for one and two fluid models. Umavathi et al. [11] analyzed magneto hydrodynamic free convection flow in a vertical rectangular duct for laminar, fully developed regime taking into consideration the effect of Ohmic heating and viscous dissipation. Umavathi and Sridhar [12] studied the Hartmann two-fluid Poiseuille-Couette flow in an inclined channel. Shah and London [13] have summarized the laminar forced convection heat transfer results for various channel cross sections, which were reported in the literature until 1970.

In particular process involving the mass transfer effects has been considered to be important precisely in chemical engineering equipments. The other applications include solidification of binary alloys and crystal growth dispersion of dissolved materials or particulate water in flows, drying and dehydration operations in chemical and food processing plants, evaporation at the surface of water body. The order of the chemical reaction depends on several factors. One of the simplest chemical reactions is the first order reaction in which rate of reaction is directly proportional to the species concentration. Das et al. [14] have studied the effect of mass transfer on the flow started impulsively past an infinite vertical plate in the presence of wall heat flux and chemical reaction. Muthucumaraswamy and Ganeshan [15, 16] have studied the impulsive motion of a vertical plate with heat flux/mass flux/suction and diffusion of chemically reactive species. Seddeek [17] has studied the finite element method for the effect of chemical reaction, variable viscosity, thermophoresis, and heat generation/absorption on a boundary layer hydro magnetic flow with heat and mass transfer over a heat surface. Kandasamy et al. [18, 19] have examined the effects of chemical reaction, heat and mass transfer with or without MHD flow with heat source/suction. The rate of heat transfer in a vertical channel could be enhanced by using special inserts. These inserts can be specially designed to increase the included angle between the velocity vector and the temperature gradient vector, rather than to promote turbulence. This increases the rate of heat transfer without a considerable drop in the pressure by Guo et al. [20]. A plane baffle may be used as an insert to enhance the rate of heat transfer in the channel. To avoid a considerable increase in the transverse thermal resistance into the channel, a thin and perfectly conductive baffle is used. The effect of such baffle on the laminar fully developed combined convection in a vertical channel with different uniform wall temperatures has been studied analytically by Salah El-Din [21]. Their study showed that for mixed convection the heat transfer between the walls and fluid can be significantly enhanced according to the baffle position and higher values of Nusselt number can be obtained when the baffle become as near the wall as possible.

Keeping in view the applications of chemical reaction and the increase of rate of heat transfer by introducing a baffle, motivated to investigate the effect of first order chemical reaction for electrically conducting fluid in a vertical channel. After inserting the baffle the fluid in stream-I is concentrated.

II. MATHEMATICAL FORMULATION

Consider a steady, two-dimensional laminar fully developed free convection flow in an open ended vertical channel filled with purely viscous conducting fluid. The *X*-axis is taken vertically upward, and parallel to the direction of buoyancy, and the *Y*-axis is normal to it as seen in Fig. 1. The channel walls are maintained at a constant temperature and the fluid properties are assumed to be constant. The channel is divided into two passages by means of thin, perfectly conducting plane baffle and each stream will have its own pressure gradient and hence the velocity will be individual in each stream.

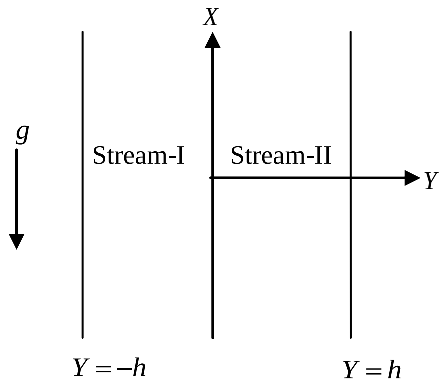

Figure1. Physical configuration.

The governing equations for velocity, temperature and concentrations are

Stream-I

$$\rho g \beta_T (T_1 - Tw_2) + \rho g \beta_C (C - C_{W2}) - \frac{dP}{dX} + \mu \frac{d^2 U_1}{dY^2} - \sigma_e B_o^2 U_1 = 0 \quad (1)$$

$$\frac{d^2 T_1}{dY^2} + \frac{\mu}{k}\left(\frac{dU_1}{dY}\right)^2 + \frac{\sigma_e}{k} B_o^2 U_1^2 = 0 \quad (2)$$

$$D\frac{d^2 C}{dY^2} - KC = 0 \quad (3)$$

Stream-II

$$\rho g \beta_T (T_2 - Tw_2) - \frac{dP}{dX} + \mu \frac{d^2 U_2}{dY^2} - \sigma_e B_o^2 U_2 = 0 \quad (4)$$

$$\frac{d^2 T_2}{dY^2} + \frac{\mu}{k}\left(\frac{dU_2}{dY}\right)^2 + \frac{\sigma_e}{k} B_o^2 U_2^2 = 0 \quad (5)$$

Subject to the boundary and interface conditions on velocity, temperature and concentration as

$$U_1 = 0, \quad T_1 = T_{W_1}, \quad C = C_{W1} \quad \text{at} \quad Y = -h$$
$$U_2 = 0, \quad T_2 = T_{W_2} \quad \text{at} \quad Y = h$$
$$U_1 = 0, \quad U_2 = 0, \quad T_1 = T_2, \quad \frac{dT_1}{dY} = \frac{dT_2}{dY}, \quad C = C_{W2} \quad \text{at} \quad Y = h^* \quad (6)$$

Introducing the following non-dimensional variables,

$$u_i = \frac{U_i}{\bar{U}_1},\ \theta_i = \frac{T_i - T_{W_2}}{T_{W_1} - T_{W_2}},\ Gr = \frac{g\beta_T \Delta T h^3}{\upsilon^2},\ Gc = \frac{g\beta_C \Delta C h^3}{\upsilon^2},\ \phi = \frac{C - C_{W2}}{C_{W1} - C_{W2}},\ \mathrm{Re} = \frac{\bar{U}_1 h}{\upsilon},\ Br = \frac{\bar{U}_1^2 \mu}{k\Delta T},\ Y* = \frac{y*}{h}$$

$$p = \frac{h^2}{\mu \bar{U}_1}\frac{dp}{dX},\ \Delta T = T_{W_2} - T_{W_1},\ \Delta C = C_{W_2} - C_{W_1},\ Y = \frac{y}{h},\ M^2 = \frac{\sigma_e B_0^{\,2} h^2}{\mu}. \tag{7}$$

One obtains the non-dimensional momentum, energy and concentration equations corresponding to stream-I and stream-II as

Stream-I

$$\frac{d^2 u_1}{dy^2} + GR_T\,\theta_1 + GR_C\,\phi - p - M^2\,u_1 = 0 \tag{8}$$

$$\frac{d^2\theta_1}{dy^2} + Br\left(\left(\frac{du_1}{dy}\right)^2 + M^2\ u_1^2\right) = 0 \tag{9}$$

$$\frac{d^2\phi}{dy^2} - \alpha^2\phi = 0 \tag{10}$$

Stream-II

$$\frac{d^2 u_2}{dy^2} + GR_T\ \theta_2 - p - M^2\,u_2 = 0 \tag{11}$$

$$\frac{d^2\theta_2}{dy^2} + Br\left(\left(\frac{du_2}{dy}\right)^2 + M^2\,u_2^2\right) = 0 \tag{12}$$

Subject to the boundary conditions,

$u_1 = 0$, $\theta_1 = 1$, $\phi = 1$, at $y = -1$

$u_2 = 0$, $\theta_2 = 0$, at $y = 1$

$$u_1 = 0,\ \ u_2 = 0,\ \ \theta_1 = \theta_2,\ \ \frac{d\theta_1}{dy} = \frac{d\theta_2}{dy},\ \phi = n,\ \text{at}\ \ y = y*, \tag{13}$$

where $GR_T = \dfrac{Gr}{\mathrm{Re}}, GR_C = \dfrac{Gc}{\mathrm{Re}}, \alpha = \dfrac{kh^2}{D},\ \ n = \dfrac{C_2 - C_{W2}}{C_1 - C_{W2}}$.

III. SOLUTIONS

Solution of equation (10) can be obtained directly using boundary condition (13) and is given by

$$\phi = B_1 Cosh(\alpha y) + B_2 Sinh(\alpha y) \tag{14}$$

Equations (8), (9), (11) and (12) are coupled non-linear differential equations. Approximate solutions can be found by using the regular perturbation method. The perturbation parameter Br is usually small and hence regular perturbation method can be strongly justified. Adopting this technique, solutions for velocity, temperature and concentration are assumed in the form

$$u_i(y) = u_{i0}(y) + Br\,u_{i1}(y) + Br^2\,u_{i2}(y) + \ldots \tag{15}$$

$$\theta_i(y) = \theta_{i0}(y) + Br\,\theta_{i1}(y) + Br^2\,\theta_{i2}(y) + \ldots \tag{16}$$

Substituting equations (15) and (16) in equations (8), (9), (11) and (12) and equating the coefficients of like power of Br to zero and one, we obtain the zero and first order equations as

Stream-I
Zeroth order equations

$$\frac{d^2u_{10}}{dy^2}+GR_T\ \theta_{10}+GR_C\ \phi-p-M^2u_{10}=0 \tag{17}$$

$$\frac{d^2\theta_{10}}{dy^2}=0 \tag{18}$$

First order equations

$$\frac{d^2u_{11}}{dy^2}+GR_T\ \theta_{11}-M^2\ u_{11}=0 \tag{19}$$

$$\frac{d^2\theta_{11}}{dy^2}+\left(\left(\frac{du_{10}}{dy}\right)^2+M^2\ {u_{10}}^2\right)=0 \tag{20}$$

Stream-II
Zeroth order equations

$$\frac{d^2u_{20}}{dy^2}+GR_T\ \theta_{20}-p-M^2\ u_{20}=0 \tag{21}$$

$$\frac{d^2\theta_{20}}{dy^2}=0 \tag{22}$$

First order equations

$$\frac{d^2u_{21}}{dy^2}+GR_T\ \theta_{21}-p-M^2\ u_{21}=0 \tag{23}$$

$$\frac{d^2\theta_{21}}{dy^2}+\left(\left(\frac{du_{20}}{dy}\right)^2+M^2{u_{20}}^2\right)=0 \tag{24}$$

The corresponding boundary conditions reduces to
Zeroth-order

$u_{10}=0$, $\theta_{10}=1$, $\phi=1$, at $y=-1$

$u_{20}=0$, $\theta_{20}=0$, at $y=1$

$$u_{10}=0,\ u_{20}=0,\ \theta_{10}=\theta_{20},\ \frac{d\theta_{10}}{dy}=\frac{d\theta_{20}}{dy},\ \phi=n,\ \text{at}\ y=y^* \tag{25}$$

First order

$u_{11}=0$, $\theta_{11}=0$ at $y=-1$

$u_{21}=0$, $\theta_{21}=0$ at $y=1$

$$u_{11}=0,\ u_{21}=0,\ \theta_{11}=\theta_{21},\ \frac{d\theta_{11}}{dy}=\frac{d\theta_{21}}{dy}\ \text{at}\ y=y^* \tag{26}$$

The solutions of zeroth and first order equations (17) to (24) using the boundary conditions as in equations (25) and (26) are

Zeroth order
Stream-I

$$\theta_{10}=z_1y+z_2 \tag{27}$$

$$u_{10}=A_1Cosh(My)+A_2Sinh(My)+r_1+r_2y+r_3Cosh(\alpha y)+r_4Sinh(\alpha y) \tag{28}$$

Stream-II

$$\theta_{20}=z_3y+z_4 \tag{29}$$

$$u_{20}=A_3Cosh(My)+A_4Sinh(My)+r_5+r_6y \tag{30}$$

First order
Stream-I

$$\theta_{11} = E_2 + E_1 y + q_1 y^2 + q_2 y^3 + q_3 y^4 + q_4 Cosh(\alpha y) + q_5 Sinh(\alpha y) + q_6 Cosh(2\alpha y) + q_7 Sinh(2\alpha y) + q_8 Cosh(2My) + q_9 Sinh(2My) + q_{10} Cosh(My) + q_{11} Sinh(My) + q_{12} yCosh(My) + q_{13} ySinh(My) + q_{14} yCosh(\alpha y) + q_{15} ySinh(\alpha y) + q_{16} Cosh\left(\alpha + M\right) y + q_{17} Cosh\left(\alpha - M\right) y + q_{18} Sinh\left(\alpha + M\right) y + q_{19} Sinh\left(\alpha - M\right) y \quad (31)$$

$$u_{11} = E_5 Cosh(My) + E_6 Sinh(My) + H_1 + H_2 y + H_3 y^2 + H_4 y^3 + H_5 y^4 + H_6 Cosh(\alpha y) + H_7 Sinh(\alpha y) + H_8 Cosh(2\alpha y) + H_9 Sinh(2\alpha y) + H_{10} Cosh(2My) + H_{11} Sinh(2My) + H_{12} yCosh(My) + H_{13} ySinh(My) + H_{14} y\cosh(\alpha y) + H_{15} ySinh(\alpha y) + H_{16} Cosh\left(\alpha + M\right) y + H_{17} Cosh\left(\alpha - M\right) y + H_{18} Sinh\left(\alpha + M\right) y + H_{19} Sinh\left(\alpha - M\right) y + H_{20} y^2 Cosh(My) + H_{21} y^2 Sinh(My) \quad (32)$$

Stream-II

$$\theta_{21} = E_4 + E_3 y + F_1 y^2 + F_2 y^3 + F_3 y^4 + F_4 Cosh(2My) + F_5 Sinh(2My) + F_6 Cosh(My) + F_7 Sinh(My) + F_8 yCosh(My) + F_9 ySinh(My) \quad (33)$$

$$u_{21} = E_7 Cosh(My) + E_8 \sinh(My) + H_{22} + H_{23} y + H_{24} y^2 + H_{25} y^3 + H_{26} y^4 + H_{27} Cosh(2My) + H_{28} Sinh(2My) + H_{29} yCosh(My) + H_{30} ySinh(My) + H_{31} y^2 Cosh(My) + H_{32} y^2 Sinh(My) \quad (34)$$

The constants appeared in the above equations are presented in the Appendix.

IV. RESULTS AND DISCUSSIONS

The problem of free convective heat and mass transfer in a vertical double passage channel filled with electrically conducting fluid is investigated. The analytical solutions are found using regular perturbation method considering Brinkman number as the perturbation parameter. The effects of governing parameters such as Hartmann number, thermal Grashof number, mass Grashof number, Brinkman number and first order chemical reaction parameter on the velocity, temperature and concentration are shown graphically.

The effect of Hartmann number on the flow is shown in Figs. 2a,b,c and 3a,b,c. As the Hartmann number increases, the velocity and temperature decreases in both the streams at all the baffle positions. As the Hartmann number increases the fluid decreases which is the classical Hartmann result. When the baffle is near the hot wall the maximum velocity is in stream-II, when the baffle is near the cold wall the maximum velocity is seen in stream-I and when the baffle is in the center of the channel the maximum velocity is seen in stream-I.

The effect of thermal Grashof number on the velocity and temperature field is shown in Figs. 4a,b,c and 5a,b,c at all three different baffle positions. As the thermal Grashof number increases, the velocity and temperature increases in both the streams at all the baffle positions. This is an expected result because increase in thermal Grashof number results in increase of bouncy force and hence increases the flow in both the streams at all the baffle positions.

The effect of mass Grashof number on the velocity and temperature field is shown in Figs. 6a,b,c and 7a,b,c respectively. As the mass Grashof number increases the flow is enhanced in both the streams at all the baffle positions. The enhancement on velocity is significant in stream-I when compared to stream-II .This is due to the fact that the fluid is concentrated only in stream-I. Though the fluid is not concentrated in stream-II still its effect is observed in stream-II when the baffle position is in the center of the channel, this is due to the reason that we have considered the baffle to be conducting. That is say that, there is heat transfer from stream-I to stream-II but there is no mass transfer from stream-I to stream-II. Due to transfer of heat from stream-I to stream-II results in increase of both thermal buoyancy force and concentration buoyancy force and hence velocity increase in stream-II slightly as mass Grashof number increases as seen in Fig. 6. As mass Grashof number increases the temperature increases significantly in stream-I when the baffle position is at the center of the cold wall. There is no much variation on the temperature field when the baffle position is near the hot wall.

The effect of Brinkman number is shown in Figs. 8a,b,c and 9a,b,c on the velocity and temperature fields respectively. As the Brinkman number increase both the velocity and temperature increases in both the streams at all the baffle positions. This is due to the fact that increase in Brinkman number increases the viscous dissipation and hence the flow is enhanced.

The effect of first order chemical reaction parameter on the velocity, temperature and concentration fields are displayed in Figs. 10a,b,c, 11a,b,c and 12a,b,c respectively. As α increases the velocity, temperature and concentration decreases in stream-I, and remains constant in stream-II. The similar result was also obtained by Srinivas and Muturajan [22] for mixed convective flow in a vertical channel. This is due to the fact that the fluid in stream-I is concentrated. The maximum value of velocity and temperature is seen in stream-II for the baffle position at $y^* = -0.8$ and in stream-I for the baffle position at $y^* = 0$ and 0.8.

V. CONCLUSIONS

The effect of first order chemical reaction in a vertical double passage channel filled with electrically conducting fluid by inserting a thin baffle is investigated. The following conclusions were drawn:
.

1. Increasing the values of Hartmann number reduces the flow field where as increase in the thermal Grashof number and mass Grashof number enhance the flow in both the streams at different baffle positions.
2. Increase in the perturbation parameter (Brinkman number) enhances the velocity and temperature in both the streams.
3. Increase in the chemical reaction parameter suppresses the velocity, temperature and concentration in stream-I and remains invariant in stream-II.

REFERENCES

[1] S. Habchi and Archarya. Laminar mixed convection in a symmetrically or asymmetrically heated vertical channel. Numer. Heat Transfer, 9:605-618, 1986.

[2] W. Aung and G. Worku. Developing flow and reversal in a vertical channel with asymmetric wall temperatures. J. Heat Transfer, 108:299-304, 1986.

[3] M. Iqbal, B.D. Aggarwala and A.G. Fowlere. Laminar combined free and forced convection in a vertical non-circular ducts under uniform heat flux. Int. J. Heat Mass Transfer, 12:1123-1139, 1969.

[4] N. Raptis, Kafoussias. Heat transfer in a flow through a porous medium bounded by an infinite vertical plane under the action of magnetic field, Energy Res., 6:241-245, 1982.

[5] Raptis. Flow through a porous medium in the presence of magnetic field. Energy Res., 10:97-100, 1986.

[6] M.S. Malashetty, J.C. Umavathi and J. Prathap Kumar. Two fluid magnetoconvection flow in an inclined channel. Int. J. Transport Phenomena, 3:73-84, 2000.

[7] M.S. Malashetty, J.C. Umavathi and J. Prathap Kumar. Convective magnetohydrodynamic two fluid flow and heat transfer in an inclined channel. Heat and Mass Transfer, 37:259-264, 2001.

[8] M.S. Malashetty, J.C. Umavathi and J. Prathap Kumar. Magnetoconvection of two immiscible fluids in a vertical enclosure, Heat and Mass Transfer, 42:977-993, 2006

[9] J.C. Umavathi, B. Mallikarjun Patil and I. Pop. On laminar mixed convection flow in a vertical porous stratum with symmetric wall heating conditions. Int. J. Transport Phenomena, 8:1-14, 2006.

[10] J. Prathap Kumar, J.C. Umavathi and Basavaraj M Biradar. Mixed convection of composite porous medium in a vertical channel with asymmetric wall heating conditions. J. Porous Medium, 13:271-285, 2010.

[11] J.C. Umavathi, I.C. Liu, J. Prathap Kumar and I. Pop. Fully developed magneto convection flow in a vertical rectangular duct. Heat and Mass Transfer, 47:1-11, 2011.

[12] J.C. Umavathi and K.S.R. Sridhr. Hartmann two fluid Poisulle-Couette flow in an inclined channel, Int. J. Appl. Mech. Engg., 14:539-557, 2009.

[13] R.K. Shah and A. London. Laminar flow forced convection heat transfer and flow friction in straight and curved ducts-A summary of analytical solutions. Stanford University, Stanford, California, USA, Tech. Rep.vol. 75, 1971.

[14] U.N. Das, R.K. Deka and V.M. Soundalgekar. Effects of mass transfer on the flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Forsch Ingenieurwes, 60:284-287, 1994.

[15] R. Muthucumaraswamy and P. Ganesan. On impulsive motion of a vertical plate with heat flux and diffusion of chemically reactive species. Forsch Ingenieurwes, 66:17-23, 2000.

[16] R. Muthucumaraswamy and P. Ganesan. First order chemical reaction on flow past impulsively stratified vertical plate with uniform heat and mass flux. Acta Mech., 147:45-57, 2001.

[17] M.A. Seddek. Finite element method for the effects of chemical reaction, variable viscosity, thermophoresis and heat generation/absorption on a boundary layer hydro magnetic flow with heat and mass transfer over a heat surface. Acta Mech., 177:1-18, 2005.

[18] R. Kandasamy, K. Perisamy and K.K. Sivagnana Prabhu. Effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection. Int. J. Heat Mass Transfer, 48:1388-1394, 2005.

[19] R. Kandasamy, K. Perisamy and K.K. Sivagnana Prabhu. Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects. Int. J. Heat Mass Transfer, 48:4557-4561, 2005.

[20] Z.Y. Guo, D.Y. Li and B.X. Wang. A novel concept for convective heat transfer enhancement. Int. J. Heat Mass Transfer,.41:2221-2225, 1998.

[21] M.M. Salah El-Din. Fully developed laminar convection in a vertical double-passage channel. Appl Energy, 47:69-75, 1994.

[22] S. Srinivas, R. Muthuraj. Effect of chemical reaction and space porosity on MHD mixed convective flow in a vertical asymmetric channel with peristalsis. Mathematical and Computer Modeling, 54:1213-1227, 2011.

NOMENCLATURE

B_0 magnetic field
Br Brinkman number
C_1 concentration in Stream-I
C_0 reference concentration
C_p specific heat at constant pressure
c_p dimensionless specific heat at constant pressure
D diffusion coefficients
E_0 applied electric field
E electric field load parameter
h channel width
h^* width of passage
g acceleration due to gravity
Gr Grashoff number
Gr Grashoff number
G_c modified Grashoff Number
GR_T, GR_C dimensionless parameters
k thermal conductivity of fluid
M Hartmann number
p nondimensional pressure gradient
Re Reynolds number
T_1, T_2 dimensional temperature distributions
T_{w_1}, T_{w_2} temperatures of the boundaries
$\overline{U_1}$ reference velocity
u_1, u_2 nondimensional velocities in Stream-I, Stream-II
U_1, U_2 dimensional velocity distributions
y^* baffle position

GREEK SYMBOLS

α chemical reaction parameters
β_T coefficients of thermal expansion
β_C coefficients of concentration expansion
σ_e electrical conductivity
$\Delta T, \Delta C$ difference in temperatures & concentration
θ_i non-dimensional temperature
γ kinematics viscosity
ϕ non-dimensional concentrations
ρ density
μ viscosity

SUBSCRIPTS

i refer quantities for the fluids in Stream-I and Stream-II, respectively.

VI. ACKNOWLEDGEMENTS

The authors greatly acknowledge the funding support from the Major Research project supported by University Grants Commission India (Grant No. 41-774/2012 (SR)).

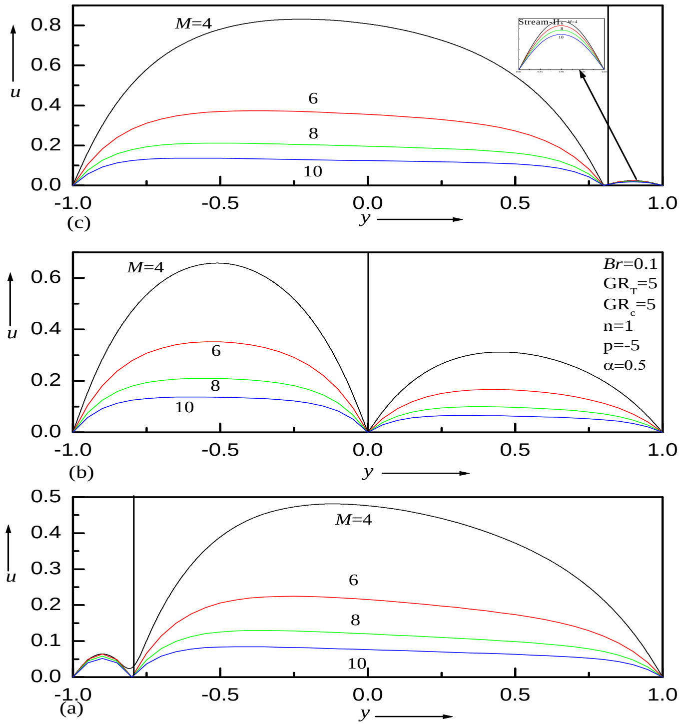

Figure2.Velocity profiles for differnt values of Hartmann number M
(a) $y^*=-0.8$ (b) $y^*=0$ (c) $y^*=0.8$

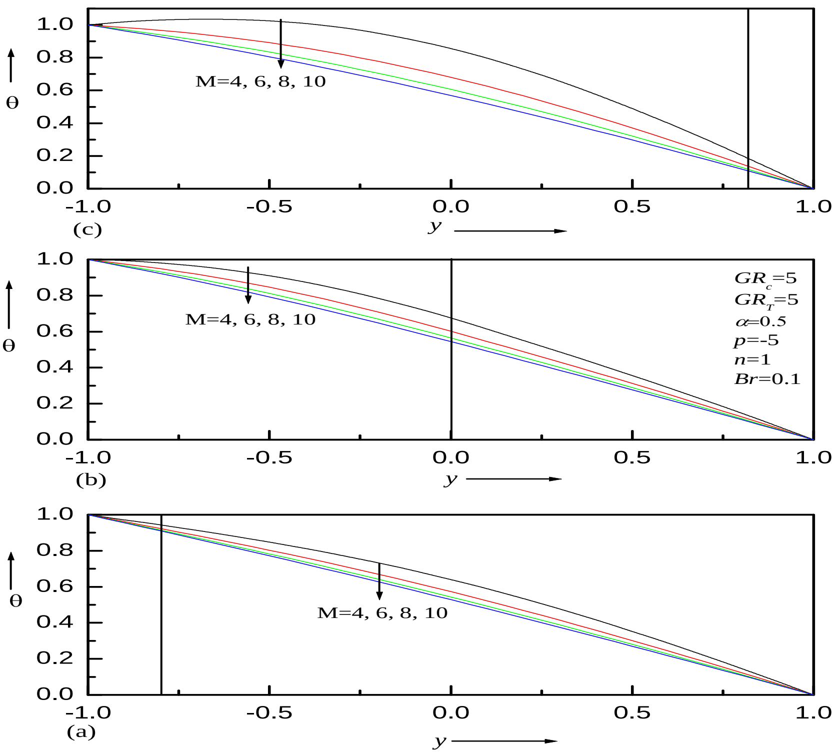

Figure3.Temperature profiles for differnt values of Hartmann number M
(a) $y^*=-0.8$ (b) $y^*=0$ (c) $y^*=0.8$

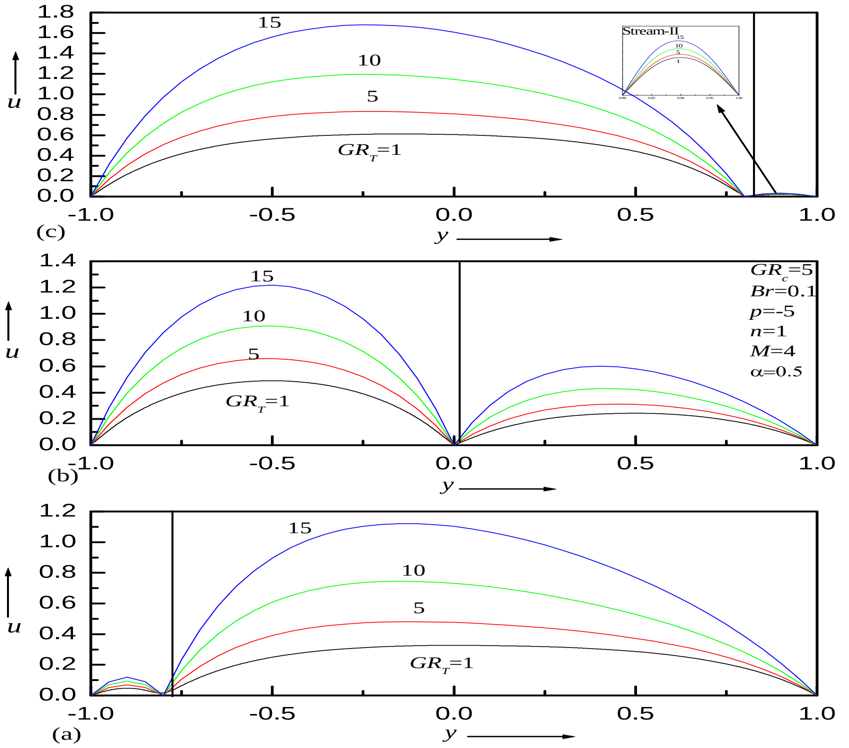

Figure4. Velocity profiles for different values of thermal Grashoff number GR_T at (a) y^*=-0.8 (b) y^*=0.0 (c) y^*=0.8

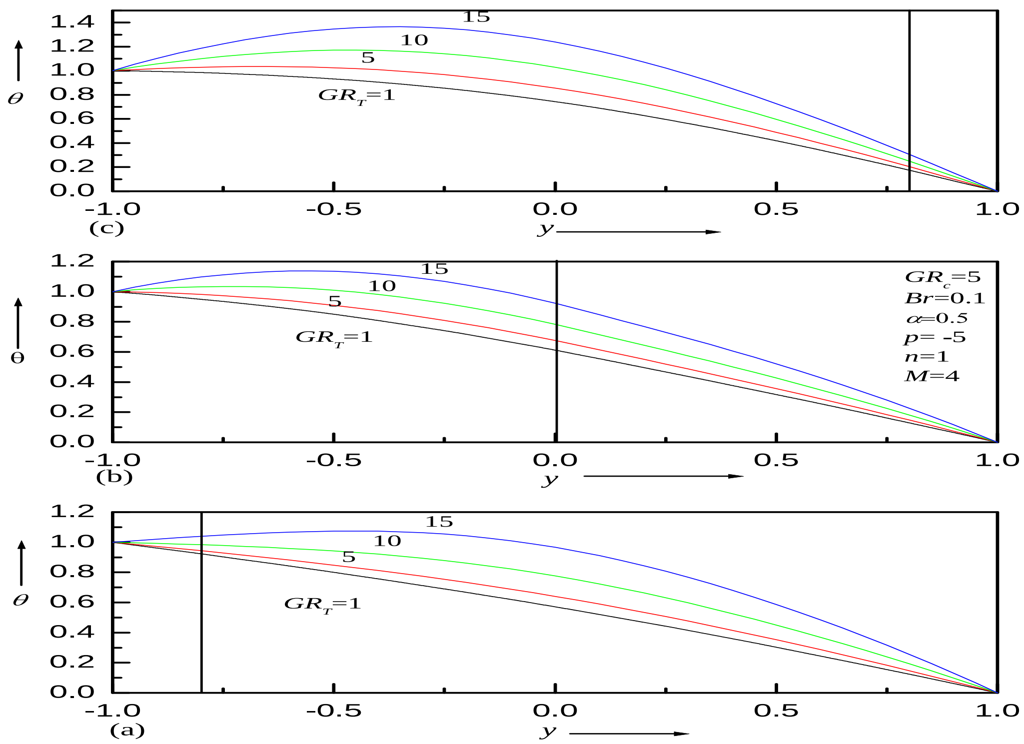

Figure5.Temperature profiles for different values of thermal Grashof number GR_T at (a) y^*=-0.8 (b) y^*=0 (c) y^*=0.8

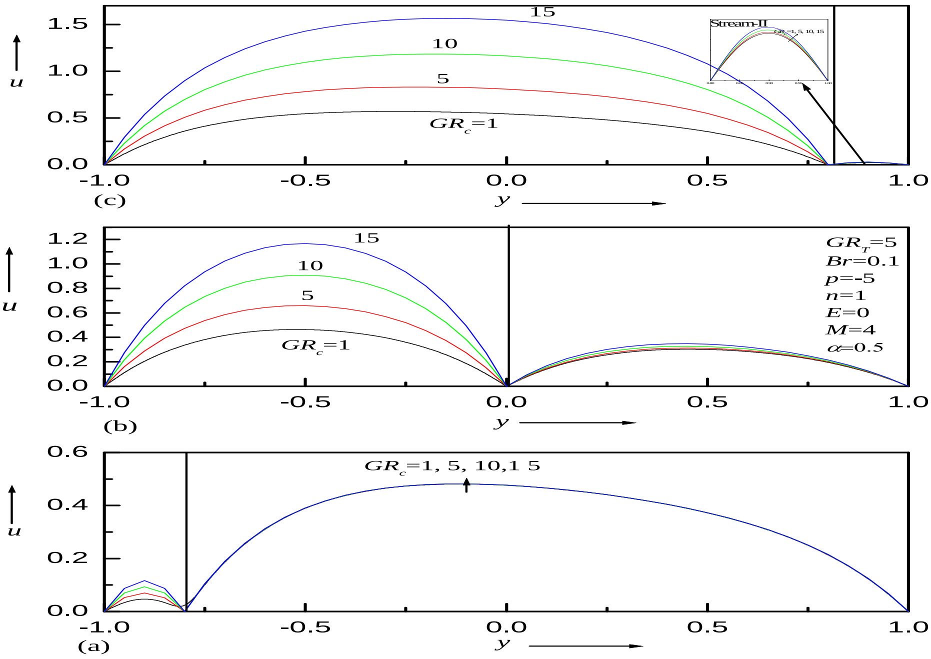

Figure6.Velocity profiles for different values of concentration Grashof number GR_C at (a) y^*=-0.8 (b) y^*=0 (c) y^*=0.8

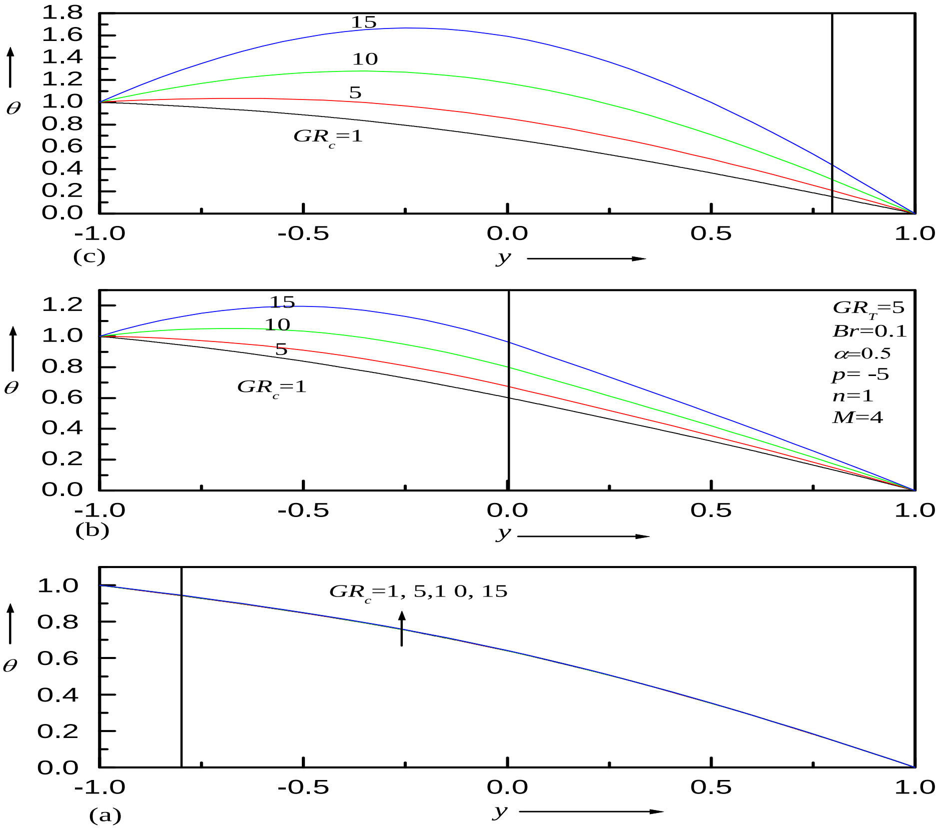

Figure7.Temperature profiles for different values of concentration Grashoff number GR_C at (a) y^*=-0.8 (b) y^*=0 (c) y^*=0.8

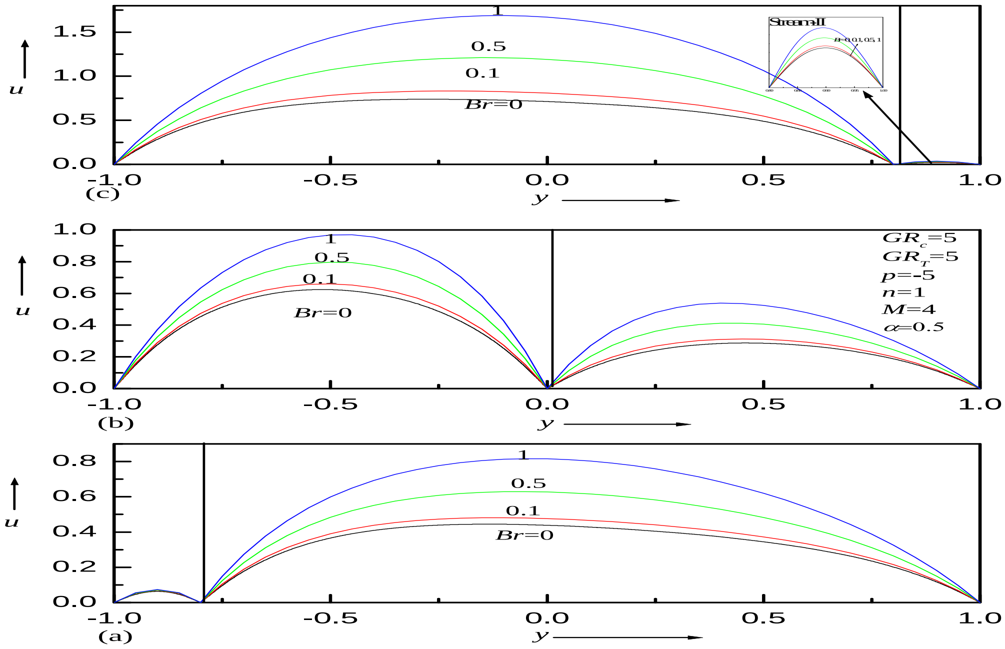

Figure8.Velocity profiles for different values of Brinkman number *Br* (a) y^{*}=-0.8 (b) y^{*}=0 (c) y^{*}=0.8

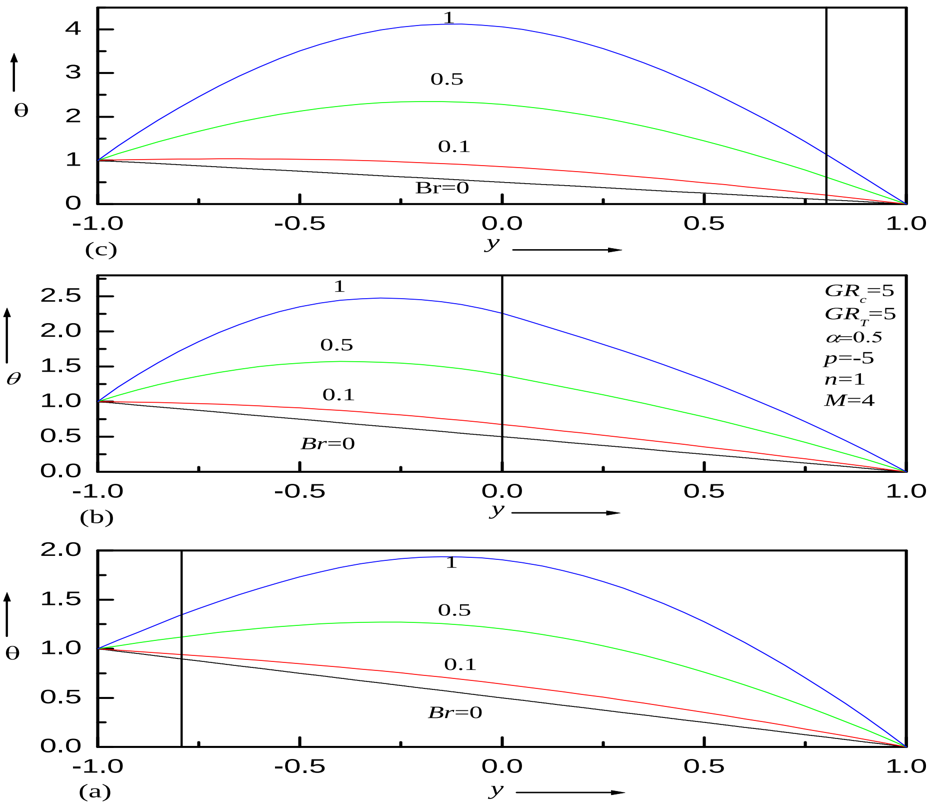

Figure9.Temperature profiles for differeent values of Brinkman number *Br* (a) y^{*}=-0.8 (b) y^{*}=0 (c) y^{*}=0.8

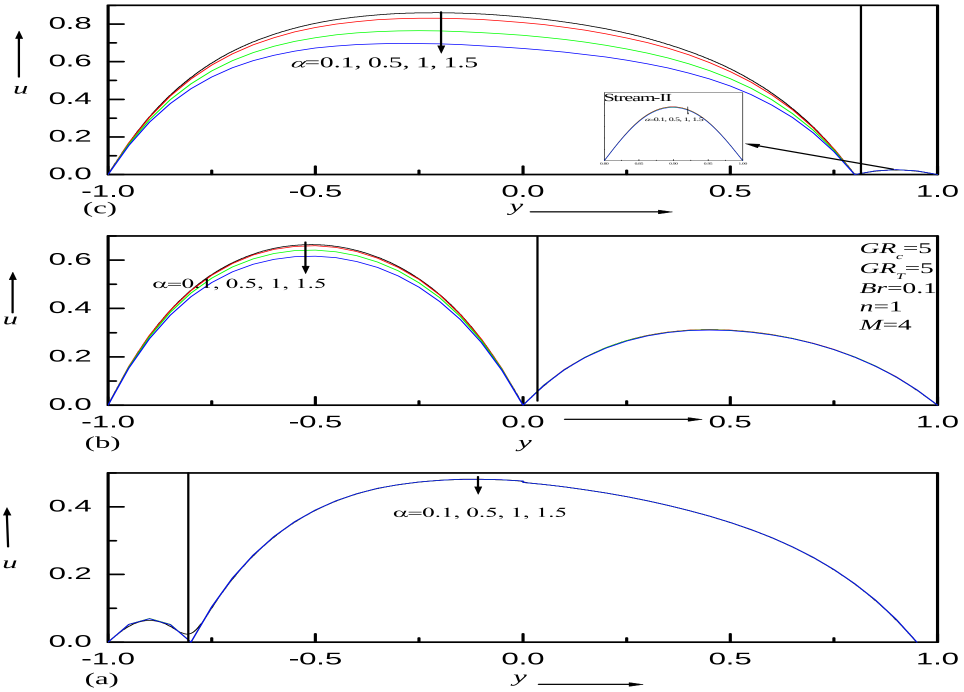

Figure10.Velocity profiles for different values of chemical reaction parameter α (a) y^*=-0.8 (b) y^*=0 (c) y^*=0.8

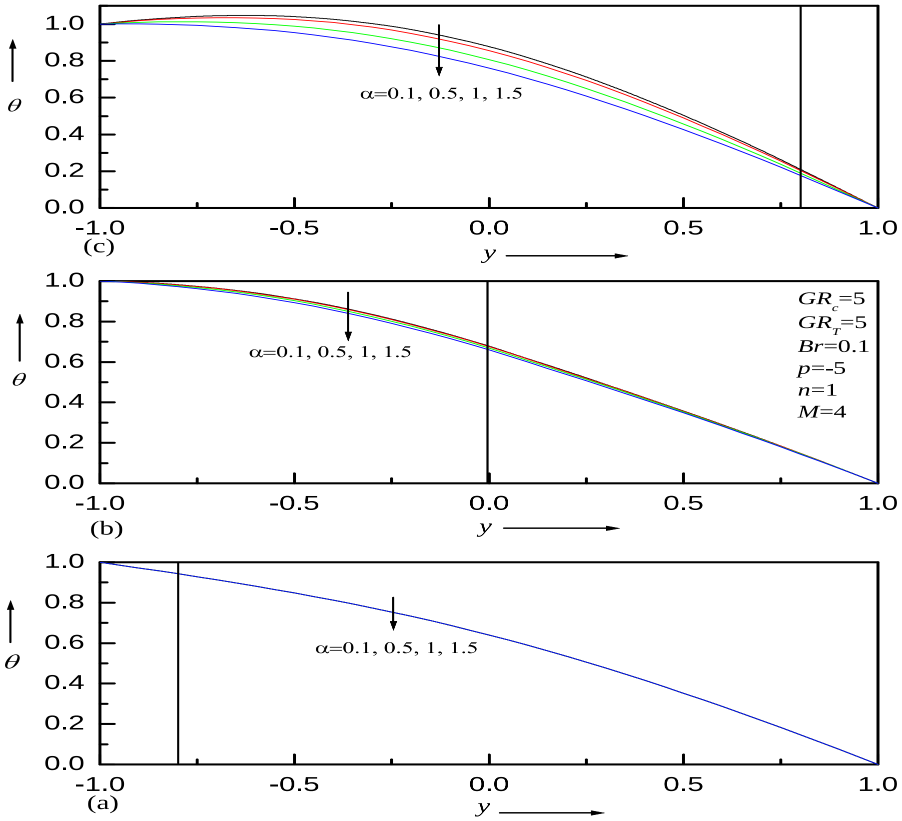

Figure11.Temperature profiles for different values of chemical reaction parameter α (a) y^*=-0.8 (b) y^*=0 (c) y^*=0.8

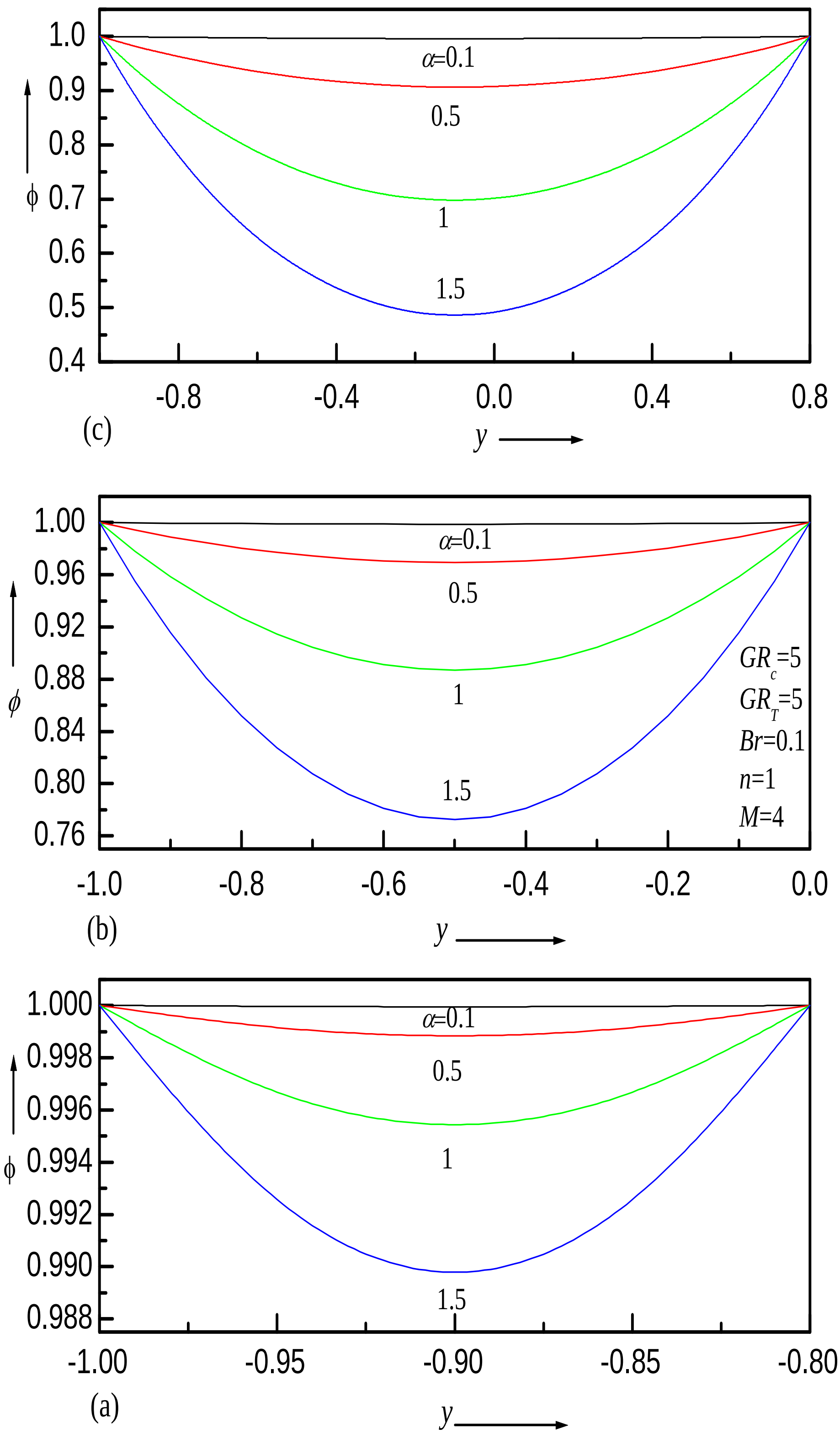

Figure12. Concentration profiles for different values of chemical reaction parameter α at (a) y*=-0.8 (b)y*=0 (c) y*=0.8

Appendix

$$B_1 = \frac{Sinh(\alpha y^*) + nSinh(\alpha)}{Cosh(\alpha)Sinh(\alpha y^*) + Cosh(\alpha y^*)Sinh(\alpha y^*)},\ B_2 = \frac{nCosh(\alpha y^*) - Cosh(\alpha y^*)}{Cosh(\alpha)Sinh(\alpha y^*) + Cosh(\alpha y^*)Sinh(\alpha y^*)},\ z_1 = -\frac{1}{2},$$

$$z_2 = \frac{1}{2},\ z_3 = \frac{1}{2},\ z_4 = \frac{-1}{2},\ r_1 = \frac{GR_TC_2 - P}{M^2},\ r_2 = \frac{GR_TC_1}{M^2},\ r_3 = -\frac{GR_CB_1}{\alpha^2 - M^2},\ r_4 = -\frac{GR_CB_2}{\alpha^2 - M^2},\ r_5 = \frac{GR_TC_4 - P}{M^2},$$

$$r_6 = \frac{GR_TC_3}{M^2},\ T_1 = -r_1 + r_2 - r_3\cosh(\alpha) + r_4\sinh(\alpha),\ T_3 = -r_5 - r_6,\ T_4 = -r_5 - r_6 y^*,$$

$$T_2 = -(r_1 + r_2 y^* + r_3\cosh(\alpha y^*) + r_4\sinh(\alpha y^*)),\ \ A_1 = \frac{T_1Sinh(My^*) + T_2Sinh(M)}{Cosh(M)Sinh(My^*) + Cosh(My^*)Sinh(M)},$$

$$A_2 = \frac{T_2Cosh(M) - T_1Cosh(My^*)}{Cosh(M)Sinh(My^*) + Cosh(My^*)Sinh(M)},\ A_3 = \frac{T_3Sinh(My^*) - T_4Sinh(M)}{Cosh(M)Sinh(My^*) - Cosh(My^*)Sinh(M)},$$

$$A_4 = \frac{T_3 - A_3Cosh(M)}{Sinh(M)},\ p_1 = -\frac{2M^2r_1^2 + 2r_2^2 + \left(r_3^2 - r_4^2\right)\left(M^2 - \alpha^2\right)}{2},\ p_2 = -2M^2r_1r_2,\ p_3 = -M^2r_2^2,$$

$$p_4 = -\left(2M^2r_1r_3 + 2r_2r_4\alpha\right),\ p_5 = 2M^2r_1r_4 + 2r_2r_3\alpha,\ p_6 = -\frac{\left(r_3^2 + r_4^2\right)\left(\alpha^2 + M^2\right)}{2},\ p_7 = -\left(M^2r_3r_4 + r_3r_4\alpha^2\right),$$

$$p_8 = -\left(A_1^2M^2 + A_2^2M^2\right),\ p_9 = -2A_1A_2M^2,\ p_{10} = -\left(2A_1M^2r_1 + 2A_2Mr_2\right),\ p_{11} = -\left(2A_2M^2r_1 + 2A_1Mr_2\right),$$

$$p_{12} = -2A_1r_2M^2,\ p_{13} = -2A_2M^2r_2,\ p_{14} = -2M^2r_2r_3,\ p_{15} = -2M^2r_2r_4,$$

$$p_{16} = -\left(A_1r_3M^2 + A_2Mr_4\alpha + A_2M^2r_4 + A_1M\alpha r_3\right),\ p_{17} = -\left(A_1M^2r_3 + A_2M\alpha r_4 - A_2M^2r_4 - A_1M\alpha r_3\right),$$

$$p_{18} = -\left(A_2r_3M^2 + A_1Mr_4\alpha + A_1M^2r_4 + A_2M\alpha r_3\right),\ p_{19} = -\left(A_1M^2r_4 + A_2M\alpha r_3 - A_2M^2r_3 - A_1M\alpha r_4\right),\ q_1 = \frac{p_1}{2},$$

$$q_2 = \frac{p_2}{6},\ q_3 = \frac{p_3}{12},\ q_4 = \frac{p_4\alpha - 2p_{15}}{\alpha^3},\ q_5 = \frac{p_5\alpha - 2p_{14}}{\alpha^3},\ q_6 = \frac{p_6}{4\alpha^2},\ q_7 = \frac{p_7}{4\alpha^2},\ q_8 = \frac{p_8}{4M^2},\ q_9 = \frac{p_9}{4M^2},$$

$$q_{10} = \frac{p_{10}M - 2p_{13}}{M^3},\ q_{11} = \frac{p_{11}M - 2p_{12}}{M^3},\ q_{12} = \frac{p_{12}}{M^2},\ q_{13} = \frac{p_{13}}{M^2},\ q_{14} = \frac{p_{14}}{\alpha^2},\ q_{15} = \frac{p_{15}}{\alpha^2},\ q_{16} = \frac{p_{16}}{(\alpha + M)^2},$$

$$q_{17} = \frac{p_{17}}{(\alpha - M)^2},\ q_{18} = \frac{p_{18}}{(\alpha + M)^2},\ q_{19} = \frac{p_{19}}{(\alpha - M)^2},\ R_1 = -\left(M^2r_5^2 + M^2r_6^2\right),\ R_2 = -2M^2r_5r_6,\ R_3 = -M^2r_6^2,$$

$$R_4 = -A_3^2M^2 - A_4^2M^2,\ R_5 = -2A_3A_4M^2,\ R_6 = -\left(2A_3M^2r_5 + 2A_4M^2r_6\right),\ R_7 = -\left(2A_4M^2r_5 + 2A_3M^2r_6\right),$$

$$F_1 = \frac{R_1}{2},\ F_2 = \frac{R_2}{6},\ F_3 = \frac{R_3}{12},\ F_4 = \frac{R_4}{4M^2},\ F_5 = \frac{R_5}{4M^2},\ F_6 = \frac{R_6M - 2R_9}{M^3},\ F_7 = \frac{R_7M - 2R_8}{M^3},\ F_8 = \frac{R_8}{M^2},\ F_9 = \frac{R_9}{M^2},$$

$$T_5 = -\begin{pmatrix} q_1 - q_2 + q_3 + q_4Cosh(\alpha) - q_5Sinh(\alpha) + q_6Cosh(2\alpha) - q_7Sinh(2\alpha) + q_8Cosh(2M) - q_9Sinh(2M) \\ +q_{10}Cosh(M) - q_{11}Sinh(M) - q_{12}Cosh(M) + q_{13}Sinh(M) - q_{14}Cosh(\alpha) + q_{15}Sinh(\alpha) \\ +q_{16}Cosh(\alpha + M) + q_{17}Cosh(\alpha - M) - q_{18}Sinh(\alpha + M) - q_{19}Sinh(\alpha - M) \end{pmatrix},$$

$$T_6 = -\left(F_1 + F_2 + F_3 + F_4Cosh(2M) + F_5Sinh(2M) + F_6Cosh(M) + F_7Sinh(M) + F_8Cosh(M) + F_9Sinh(M)\right),$$

$$\begin{aligned} T_7 = {} & F_1y^{*2} + F_2y^{*3} + F_3y^{*4} + F_4\cosh(2My^*) + F_5Sinh(2My^*) + F_6Cosh(My^*) + F_7Sinh(My^*) + F_8y^*Cosh(My^*) \\ & +F_9y^*Sinh(my^*) - (q_1y^{*2} + q_2y^{*3} + q_3y^{*4} + q_4Cosh(\alpha y^*) + q_5Sinh(\alpha y^*) + q_6Cosh(2\alpha y^*) + q_7Sinh(2\alpha y^*) \\ & +q_8Cosh(2My^*) + q_9Sinh(2My^*) + q_{10}Cosh(My^*) + q_{11}Sinh(My^*) + q_{12}y^*Cosh(My^*) + q_{13}ySinh(My^*) \\ & +q_{14}yCosh(\alpha y^*) + q_{15}ySinh(\alpha y^*) + q_{16}Cosh(\alpha + M)y^* + q_{17}Cosh(\alpha - M)y^* \\ & +q_{18}Sinh(\alpha + M)y^* + q_{19}Sinh(\alpha - M)y^*) \end{aligned},$$

$$\begin{aligned} T_8 = {} & 2F_1y^* + 3F_2y^{*2} + 4F_3y^{*3} + 2MF_4Sinh(2My^*) + 2MF_5Cosh(2My^*) + F_6MSinh(My^*) + F_7MCosh(My^*) \\ & +F_8(y^*MSinh(My^*) + Cosh(My^*)) + F_9(y^*MCosh(My^*) + Sinh(My^*)) - (2q_1y^* + 3q_2y^{*2} + 4q_3y^{*3} \\ & +q_4\alpha Sinh(\alpha y^*) + q_5\alpha Cosh(\alpha y^*) + 2\alpha q_6Sinh(2\alpha y^*) + 2q_7\alpha Cosh(2\alpha y^*) + 2Mq_8Sinh(2My^*) \\ & +2Mq_9Cosh(2My^*) + q_{10}MSinh(My^*) + q_{11}MCosh(My^*) + q_{12}(y^*MSinh(My^*) + Cosh(My^*)) \\ & +q_{13}(y^*MCosh(My^*) + Sinh(My^*)) + q_{14}(y^*\alpha Sinh(\alpha y^*) + Cosh(\alpha y^*)) + q_{15}(y^*\alpha Cosh(\alpha y^*) \\ & +Sinh(\alpha y^*)) + q_{16}(\alpha + M)Sinh(\alpha + M)y^* + q_{17}(\alpha - M)Sinh(\alpha - M)y^* + q_{18}(\alpha + M)Cosh(\alpha + M)y^* \\ & +q_{19}(\alpha - M)Cosh(\alpha - M)y^*) \end{aligned},$$

$$E_1 = \frac{\left(-T_5 + T_6 + T_7 + T_8 - T_8 y^*\right)}{2},\ E_2 = \frac{\left(T_5 + T_6 + T_7 + T_8 - T_8 y^*\right)}{2},\ E_3 = \frac{\left(-T_5 + T_6 + T_7 - T_8 - T_8 y^*\right)}{2},$$

$$E_4 = \frac{\left(T_5 + T_6 - T_7 + T_8 + T_8 y^*\right)}{2},\ H_1 = \frac{GR_T\left(E_2 M^4 + 2q_1 M^2 + 24q_3\right)}{M^6},\ H_2 = \frac{GR_T\left(E_1 M^2 + 6q_2\right)}{M^4},$$

$$H_3 = \frac{GR_T\left(q_1 M^2 + 12q_3\right)}{M^4},\ H_4 = \frac{GR_T q_2}{M^2},\ H_5 = \frac{GR_T q_3}{M^2},\ H_6 = \frac{2\alpha q_{15} GR_T - q_4 GR_T\left(\alpha^2 - M^2\right)}{\left(\alpha^2 - M^2\right)^2},$$

$$H_7 = \frac{2q_{14}\alpha GR_T - q_5 GR_T\left(\alpha^2 - M^2\right)}{\left(\alpha^2 - M^2\right)^2},\ H_8 = -\frac{q_6 GR_T}{4\alpha^2 - M^2},\ H_9 = -\frac{q_7 GR_T}{4\alpha^2 - M^2},\ H_{10} = -\frac{q_8 GR_T}{3M^2},\ H_{11} = -\frac{q_9 GR_T}{3M^2},$$

$$H_{12} = \frac{q_{12} GR_T - 2q_{11} GR_T M}{4M^2},\ H_{13} = \frac{q_{13} GR_T - 2q_{10} GR_T M}{4M^2},\ H_{14} = -\frac{q_{14} GR_T}{\alpha^2 - M^2},\ H_{15} = -\frac{q_{15} GR_T}{\alpha^2 - M^2},$$

$$H_{16} = -\frac{q_{16} GR_T}{\left(\alpha + M\right)^2 - M^2},\ H_{17} = -\frac{q_{17} GR_T}{\left(\alpha - M\right)^2 - M^2},\ H_{18} = -\frac{q_{18} GR_T}{\left(\alpha + M\right)^2 - M^2},\ H_{19} = -\frac{q_{19} GR_T}{\left(\alpha - M\right)^2 - M^2},$$

$$H_{20} = -\frac{q_{13} GR_T}{4M},\ H_{21} = -\frac{q_{12} GR_T}{4M},\ H_{22} = \frac{GR_T\left(E_4 M^4 + 2F_1 M^2 + 24F_3\right)}{M^6},\ H_{23} = \frac{GR_T\left(E_3 M^2 + 6F_2\right)}{M^4},$$

$$H_{24} = \frac{GR_T\left(F_1 M^2 + 12F_3\right)}{M^4},\ H_{25} = \frac{GR_T F_2}{M^2},\ H_{26} = \frac{GR_T F_3}{M^2},\ H_{27} = -\frac{GR_T F_4}{3M^2},\ H_{28} = -\frac{GR_T F_5}{3M^2},$$

$$H_{29} = \frac{F_8 GR_T - 2F_7 GR_T M}{4M^2},\ H_{30} = \frac{F_9 GR_T - 2F_6 GR_T M}{4M^2},\ H_{31} = -\frac{F_9 GR_T}{4M},\ H_{32} = -\frac{F_8 GR_T}{4M},$$

$$\begin{aligned} T_9 = {} & H_1 - H_2 + H_3 - H_4 + H_5 + H_6 Cosh(\alpha) - H_7 Sinh(\alpha) + H_8 Cosh(2\alpha) - H_9 Sinh(2\alpha) + H_{10}\cosh(2M) \\ & - H_{11} Sinh(2M) - H_{12} Cosh(M) + H_{13} Sinh(M) - H_{14} Cosh(\alpha) + H_{15} Sinh(\alpha) + H_{16} Cosh\left(\alpha + M\right) \\ & + H_{17} Cosh\left(\alpha - M\right) - H_{18} Sinh\left(\alpha + M\right) - H_{19} Sinh\left(\alpha - M\right) + H_{20} Cosh(M) - H_{21} Sinh(M) \end{aligned},$$

$$\begin{aligned} T_{10} = {} & H_1 + H_2 y^* + H_3 y^{*2} + H_4 y^{*3} + H_5 y^{*4} + H_6 Cosh(\alpha y^*) + H_7 Sinh(\alpha y^*) + H_8 Cosh(2\alpha y^*) + H_9 Sinh(2\alpha y^*) \\ & + H_{10} Cosh(2My^*) + H_{11} Sinh(2My^*) + H_{12} y^* Cosh(My^*) + H_{13} y^* Sinh(My^*) + H_{14} y^* Cosh(\alpha y^*) \\ & + H_{15} y^* Sinh(\alpha y^*) + H_{16} Cosh\left(\alpha + M\right) y^* + H_{17} Cosh\left(\alpha - M\right) y^* + H_{18} Sinh\left(\alpha + M\right) y^* + H_{19} Sinh\left(\alpha - M\right) y^* \\ & + H_{20} y^{*2} Sinh(My^*) + H_{21} y^{*2} Sinh(My^*) \end{aligned},$$

$$\begin{aligned} T_{11} = {} & H_{22} + H_{23} y^* + H_{24} y^{*2} + H_{25} y^{*3} + H_{26} y^{*4} + H_{27} Cosh(2My^*) + H_{28} Sinh(2My^*) + H_{29} y^* Cosh(My^*) \\ & + H_{30} y^* Sinh(My^*) + H_{31} y^{*2} Cosh(My^*) + H_{32} y^{*2} Sinh(My^*) \end{aligned},$$

$$\begin{aligned} T_{12} = {} & H_{22} + H_{23} + H_{24} + H_{25} + H_{26} + H_{27} Cosh(2M) + H_{28} Sinh(2M) + H_{29} Cosh(M) + H_{30} Sinh(M) \\ & + H_{31} Cosh(M) + H_{32} Sinh(M) \end{aligned},$$

$$E_5 = \frac{\left(T_9 Sinh(My^*) + T_{10} Sinh(M)\right)}{Cosh(M) Sinh(My^*) + Cosh(My^*) Sinh(M)},\ E_6 = \frac{\left(T_{10} Cosh(M) - T_9 Cosh(My^*)\right)}{Cosh(M) Sinh(My^*) + Cosh(My^*) Sinh(M)},$$

$$E_7 = \frac{\left(T_{12} Sinh(My^*) - T_{11} Sinh(M)\right)}{Cosh(M) Sinh(My^*) - Cosh(My^*) Sinh(M)},\ E_8 = \frac{\left(T_{11} Cosh(M) - T_{12} Cosh(My^*)\right)}{Cosh(M) Sinh(My^*) - Cosh(My^*) Sinh(M)}.$$