

# Effect of Chemical Reaction on Mass Transfer over a Stretching Surface Embedded In a Porous Medium

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## ABSTRACT

An analysis is made to investigate the mass transfer over a stretching surface embedded in a porous medium in the presence of first order chemical reaction. Using similarity transformation, the governing partial differential equations are transformed into a set of ordinary differential equations which solved by shooting method. Comparisons with previously published work on special cases of the problem are performed and the results are found to be in excellent agreement. It is observed that the local mass transfer and concentration profile are very sensitive to change in the values of reaction rate parameter, permeability parameter and Schmidt number.

**KEYWORDS:** Mass transfer – Stretching surface – Porous medium- Chemical reaction.

## I. INTRODUCTION

The boundary layer flows due to stretching surface in porous medium are relevant to many engineering problems such as paper production, preparing plastic and metal sheets etc. The dynamics of the boundary layer flow over a stretching surface originated from the pioneering work of Sakiadis [1, 2] who initiated the study of boundary layer flow over a continuous solid surface moving with constant speed. Crane [3] extended it to analyze the steady two dimensional boundary layer flow caused by the stretching of elastic flat surface which moves in its plane with velocity varying linearly with distance from a fixed point. Many authors presented some mathematical results, and good amount of references can be found in the papers by Ali [4] and [5], Elbashbeshy [6], Ishak et al. [7] and Elbashbeshy and Bazid [8]. The studies carried out in these papers in the case steady state flow. The unsteady state problem over a stretching surface, which is stretched with a velocity that depends on time is considered by Anderson et al. [9], Elbashbeshy and Bazid [10] and Ishak et al. [11]. The effects of chemically reactive solute distribution on fluid flow due to a stretching surface also bear equal importance in engineering researches. The chemical reaction effects were studied by many researchers on several physical aspects. The diffusion of a chemically reactive species in a laminar boundary layer flow over a flat plate was demonstrated by Chambre and Young [12]. The effect of transfer of chemically reactive species in the laminar flow over a stretching sheet explained by Andersson et al. [13]. Takhar et al. [14] analyzed the flow and mass transfer on a stretching sheet with a magnetic field and chemically reactive species with n-th order reaction.

The mass transfer in boundary layer flow due to a stretching surface in porous medium also has important applications in many industrial problems. The effect of mass transfer in laminar flow over a stretching surface was investigated by Radwan and Elbashbeshy [15]. Radwan and Elbashbeshy [15] analyzed the flow and mass transfer on a stretching surface with a magnetic field. Akyildiz et al. [16] reported a solution for diffusion of chemically reactive species in a flow of a non-Newtonian fluid over a stretching sheet immersed in a porous medium. El-Aziz [17] explained unsteady flow due to a stretching sheet with mass and heat transfer. Recently, Krishnendu [18] studied the boundary layer flow with first order chemical reaction over a porous flat plate. Krishnendu [19] studied the mass transfer on a continuous flat plate moving in a parallel or reversely to a free stream in the presence of a chemical reaction. Ferdows et al. [20] investigated the effects of order of chemical reaction on mass transfer over a linearly stretching surface. Based on the above-mentioned investigations and applications, this paper is concerned with two-dimensional steady, incompressible, laminar boundary layer flow of a fluid over a linearly stretching surface.

In this paper we investigate numerically the effects of chemical reaction on the steady laminar two-dimensional boundary layer flow and mass transfer over a stretching surface embedded in porous medium. The method of solutions based on the well-known similarity analysis together with shooting method.

## II. Formulation of the problem

Consider two dimensional steady, laminar boundary layer flow of a fluid over a linearly stretching surface (i.e. stretched with a velocity proportional to  $x$ ) embedded in porous medium with velocity  $u_w$  and concentration  $C_w$  moving axially through a stationary fluid. The fluid is viscous incompressible with constant physical properties. We assume that the fluid far away from the surface is at rest and at concentration  $C_\infty$ . The  $x$  – axis runs along the continuous surface in the direction of motion and  $y$  – axis is perpendicular to it. The continuity, momentum and reactive concentration equations for governing the flow and concentration distribution in the boundary layer region along the stretching surface may be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k} u \tag{2}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - R(C - C_\infty) \tag{3}$$

The boundary conditions are given by

$$y = 0 : u = \lambda x, v = 0, C = C_\infty + Ax^n \tag{4}$$

$$y \rightarrow \infty : u = 0, C = C_\infty$$

where  $u$  and  $v$  are the velocity components in the  $x$  and  $y$  directions, respectively,  $k$  is the permeability of porous medium.  $C$  is the concentration of the fluid,  $\nu$  is the kinematic viscosity,  $R$  is a constant of first-order chemical reaction rate,  $n$  is a power-law exponent, which signifies the change of amount of solute in the  $x$ -direction,  $D$  is the effective diffusion coefficients and  $A$  and  $\lambda$  are constants. The equation of continuity is satisfied if we choose a dimensionless stream function  $\psi(x, y)$  such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}.$$

In order to solve Equations (1)-(4), we introduce the following similarity transformation and dimensionless concentration.

$$\eta = y \sqrt{\frac{n}{\nu}}, \quad f(\eta) = \frac{\psi(x, y)}{x \sqrt{n \nu}}, \quad \theta(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

where  $\eta$  is the similarity variable,  $\psi(x, y)$  is the dimensionless stream function,  $\theta(\eta)$  is the dimensionless concentration. Consequently, equations (2)-(3) and the boundary conditions (4) can be written in the following form

$$f''' + ff'' - f'^2 - Kf' = 0 \tag{5}$$

$$\theta'' + Sc(f\theta' - n f' \theta) - ScR_c \theta = 0 \tag{6}$$

$$\eta = 0 : f = 0, f' = 1, \theta = 1 \tag{7}$$

$$\eta \rightarrow \infty : f' = 0, \theta = 0$$

Where prime denotes differentiation with respect to  $\eta$ .  $K = \frac{v}{nk}$  is a permeability parameter,  $R_c = \frac{R}{n}$  is a reaction rate parameter of the solute and  $Sc = \frac{v}{D}$  is Schmidt parameter.

The mass transfer coefficient in terms of the Sherwood number  $Sh$  is given by

$$Sh = -\frac{x\left(\frac{\partial C}{\partial y}\right)}{C_w - C_\infty} = -x\sqrt{\frac{n}{v}}\theta'(0)$$

$$\frac{Sh}{\sqrt{R_e}} = -\theta'(0)$$

where  $R_e = \frac{u_w x}{v}$  is the local Reynolds number.

### III. Numerical method

The above Eqs. (5) and (6) along with boundary conditions (7) are solved by converting them to an initial value problem. We set

$$f' = z, \quad z' = p, \quad \theta' = q$$

$$p' = Kz + z^2 - fp \tag{8}$$

$$q' = Sc [R\theta + nz\theta - fq] \tag{9}$$

with the boundary conditions

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 \tag{10}$$

In order to integrate (8) and (9) as an initial value problem we require a value for  $p(0)$  i.e.  $f''(0)$  and  $\theta'(0)$  but no such values are given in the boundary. The most important factor of shooting method is to choose the appropriate finite values of  $\eta_\infty$ . In order to determine  $\eta_\infty$  for the boundary value problem stated by Eqs. (8)– (9), we start with some initial guess value for some particular set of physical parameters to obtain  $f''(0)$  and  $\theta'(0)$ . The solution procedure is repeated with another large value of  $\eta_\infty$  until two successive values of  $f''(0)$  and  $\theta'(0)$  differ only by the specified significant digit. The last value of  $\eta_\infty$  is finally chosen to be the most appropriate value of the limit  $\eta_\infty$  for that particular set of parameters. The value of  $\eta_\infty$  may change for another set of physical parameters. Once the finite value of  $\eta_\infty$  is determined then the integration is carried out. We compare the calculated values for  $f'$  and  $\theta$  at  $\eta = 10$  (say) with the given boundary conditions  $f'(10) = 0$  and  $\theta(10) = 0$  and adjust the estimated values,  $f''(0)$  and  $\theta'(0)$ , to give a better approximation for the solution. We take the series of values for  $f''(0)$  and  $\theta'(0)$ , and apply the fourth order classical Runge–Kutta method with step-size  $h = 0.01$ . The above procedure is repeated until we get the results up to the desired degree of accuracy,  $10^{-5}$ .

#### IV. RESULTS AND DISCUSSION

The set of non-linear ordinary differential equations (5) and (6) satisfying the boundary conditions (7) have been solved numerically using the Mathematica method for several values of the involved parameters, namely permeability parameter, power-law exponent, reaction rate parameter of the solute and Schmidt parameter. In case  $n=0$  and  $K = 0$  our results are very similar to those considered by Uddin et al. [21], Takhar et al. [14] and Andersson et al. [13]. To validate the numerical method used in this study, the case  $n=0$  and  $K = 0$ , was considered and the results for concentration gradient are compared with the numerical solution which reported in Uddin et al. [21], Takhar et al. [14] and Andersson et al. [13]. The quantitative comparison is shown in Table 1, and found to be in excellent agreement.

Table1 Comparison of the values of  $-\theta'(0)$  with that of Takhar et al. [14], Andersson et al. [13] and Uddin et al. [21] for  $n=0$  and  $K = 0$ .

Sc	$R_c$	Present Study	Takhar et al. [14]	Andersson et al. [13]	Uddin et al.[21]
0.1	0.1	0.149083	0.15042	0.14900	0.15057
1	0.1	0.668754	0.67044	0.66900	0.66873
1	1	1.176401	1.17761	1.17700	1.17679
10	1	3.871327	3.87469	3.88000	3.87347
10	10	10.241185	10.24283	10.25000	10.24535

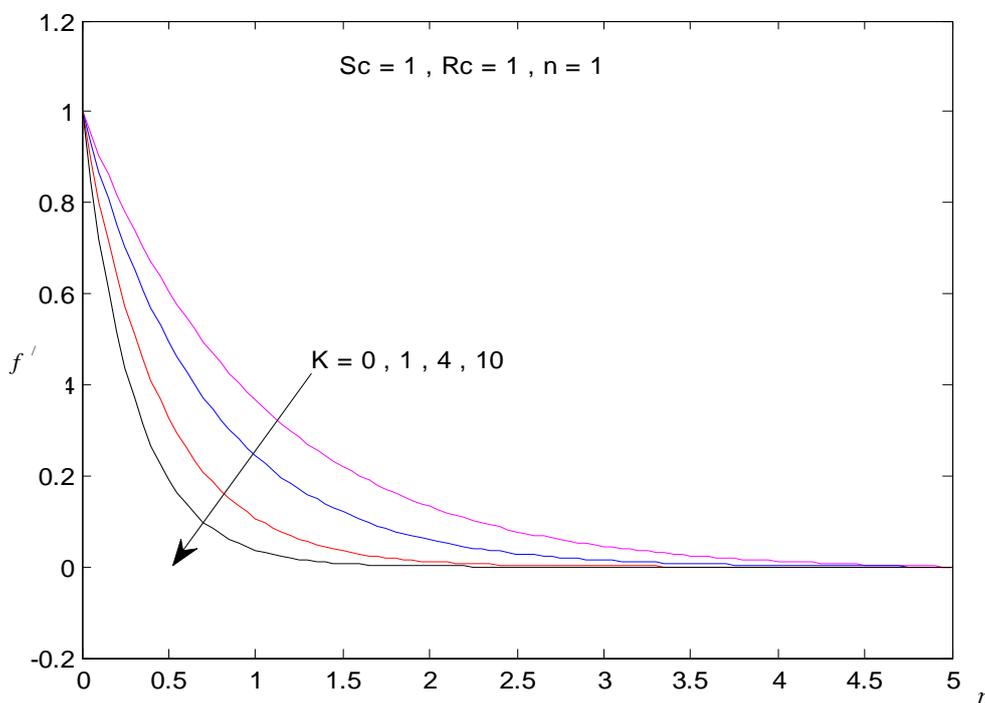


Fig. 1: Velocity profiles  $f'(\eta)$  for various values of K

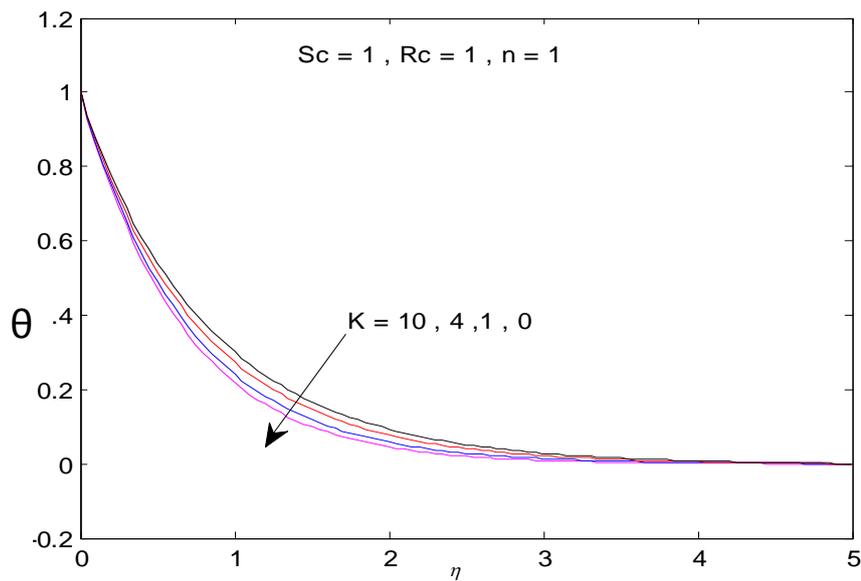


Fig.2: Concentration profiles  $\theta(\eta)$  for various values of  $K$

The numerical solution of velocity has presented for various values of the permeability parameter  $K$ . The reactant solute equation is solved numerically and the results are shown graphically. The velocity profiles for various values of the permeability parameter  $K$  have been plotted in Fig. 1. From the figure it is noted that with increase of  $K$ , the velocity for any fixed value of  $\eta$  decreases.

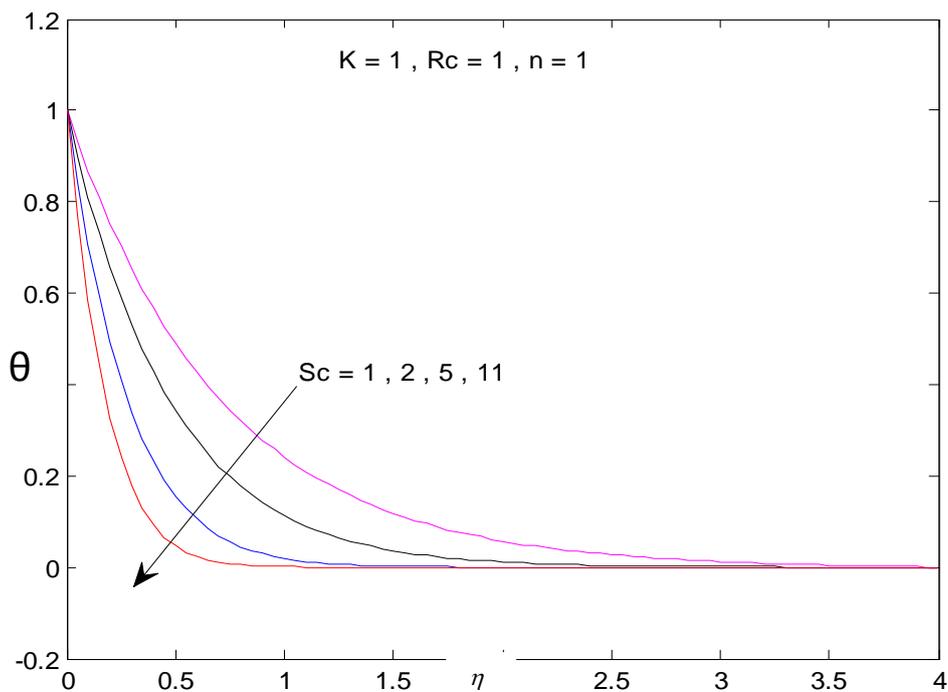


Fig. 3. Concentration profiles  $\theta(\eta)$  for various values of  $Sc$

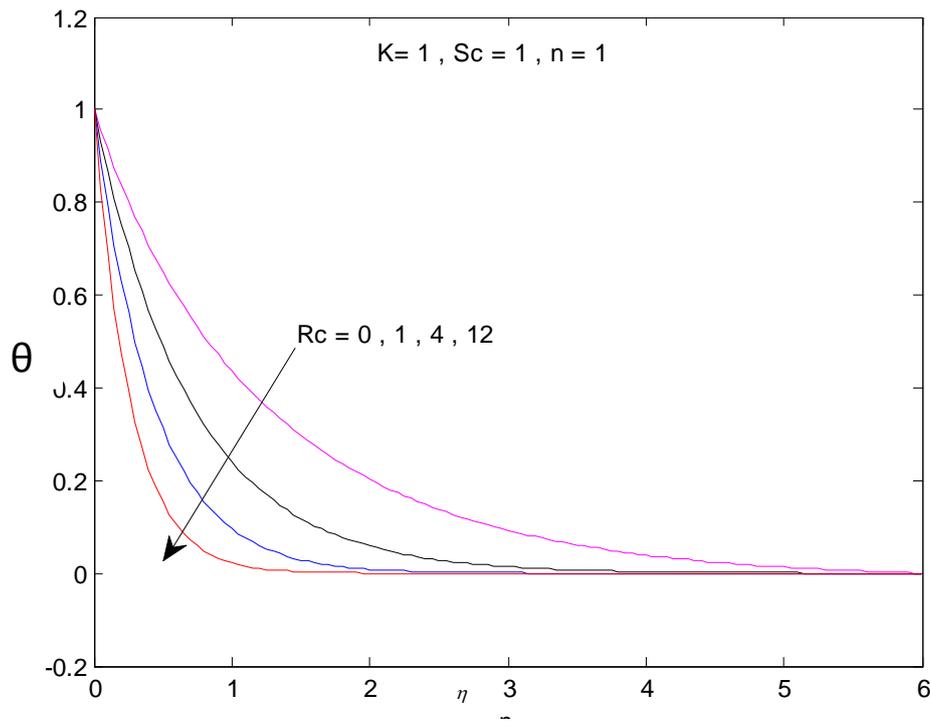


Fig4. Concentration profiles  $\theta(\eta)$  for various values of  $R_c$

Thus it is clear that the permeability parameter  $K$  opposes motion. Consequently, the momentum boundary layer thickness reduces with the increase  $K$  in and this fact is also seen from wall shear stress behaviour. Exhibits concentration profiles for various values of  $K$ . The value of contaminate solute at particular value of  $\eta$  increases with the increase of the permeability parameter  $K$  and also the concentration boundary layer thickness increases. This implies that the permeability parameter acts to enhance the distribution of the reaction solute on the stretching surface. This result may be useful, in the situation where the enhancement of solute transfer from the surface is the prime important.

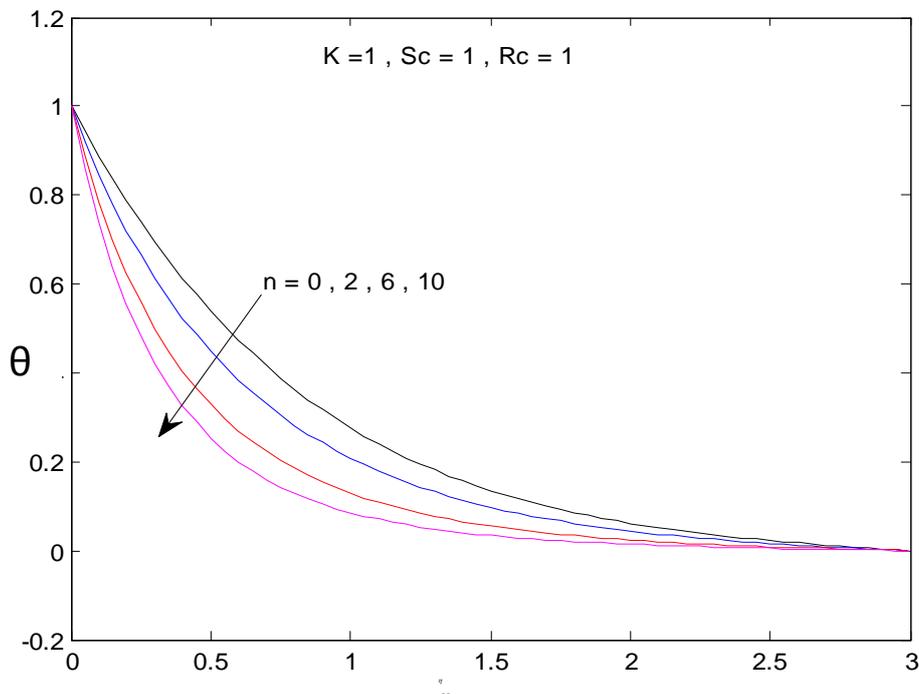


Fig. 5. Concentration profiles  $\theta(\eta)$  for various values of  $n(\geq 0)$ . with  $K=Sc=R_c = 1$

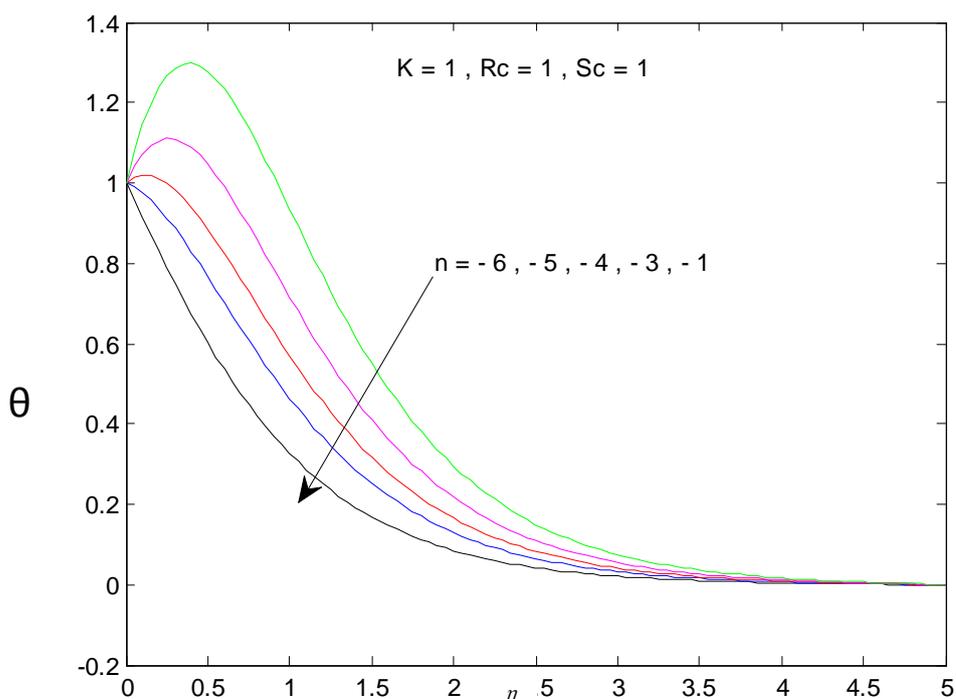


Fig. 6. Concentration profiles  $\theta(\eta)$  for various values of  $n(<0)$ . with  $K=Sc=R_c=1$

Now, we concentrate on variation the solute curves for different values of Schmidt number  $Sc$ . The curves are drawn in the Fig. 3. The Schmidt number has major effects on the distribution of solute. The concentration boundary layer thickness as well as the concentration at a fixed point decreases quickly with increasing values of  $Sc$ . This is due to the fact that the rate of solute transfer from the surface increases when the Schmidt number increases. Figure 4 is the graphical representation of concentration profiles for various values of reaction rate parameter  $R_c$ . It has been found that the reaction rate parameter affect the solute profiles in similar way as that of the Schmidt number i.e., the increase of  $R_c$  reduces both the solute boundary layer thickness and value of the solute at fixed  $\eta$ . So, in case of the distribution of reactive solute, the reaction rate parameter is a decelerating agent.

Finally, Figs. 5 and 6 exhibit the concentration profiles in the boundary layer flow region for different values of power-law exponent  $n$ . It is noticed from Fig. 5 that for the increasing values of  $n$  with  $n>0$ , the curve representing the distribution of solute for specific value of  $\eta$  decreases. While, in Fig. 6 the concentration profile increases with increase in the magnitude of  $n$  with  $n<0$  and for large negative values of  $n$ , the overshoot of solute is observed near the surface. Thus, the effect of increase of  $n$  when the surface concentration is  $C_w = C_\infty + Ax^n$  is completely opposite to the effect of increase  $n$  when the surface concentration is  $C_w = C_\infty + A/x^n$  where  $A$  is constant positive. Note that, the wall concentration is constant when  $n=0$ .

Table 2

Values of skin-friction coefficient  $f''(0)$  and  $-\theta'(0)$  for various values of permeability parameter  $K$ .

K	$f''(0)$	$-\theta'(0)$
0	-1.00000	1.45942
1	-1.414214	1.414214
4	-2.236068	1.3433
10	-3.316625	1.2781

Table 3: Values of  $-\theta'(0)$  for various values of Schmidt number  $Sc$ 

Sc	$-\theta'(0)$
1	1.414214
2	2.063933
5	3.366092
11	5.085476

Table 4: Values of  $-\theta'(0)$  for various values of reaction rate parameter  $R_c$ 

$R_c$	$-\theta'(0)$
0	0.892503
1	1.414214
4	2.2692285
12	3.641534

Table 5: Values of  $-\theta'(0)$  for various values of  $n$ 

n	$-\theta'(0)$
0	1.154956
2	1.649948
6	2.434269
10	3.061963

Tables 2-5 summarize the calculated values of local skin friction coefficient  $f''(0)$  and rate of mass transfer  $-\theta'(0)$  for  $1 \leq Sc \leq 11$ ,  $0 \leq n \leq 10$  and  $0 \leq R_c \leq 12$ . These tables indicate that  $-\theta'(0)$  increases as permeability parameter  $K$  increases. Since  $K$  does not occur explicitly in the diffusion equation, its effect on mass transfer  $-\theta'(0)$  is small. Also, these tables indicate that  $-\theta'(0)$  increases as  $n$ ,  $R_c$  and  $Sc$  increase.

## V. CONCLUSIONS

In this investigation, an analysis is made to find the behaviour of the distribution of reactive solute undergoing a first order reaction in steady boundary layer flow incompressible fluid over a stretching surface embedded in porous medium with variable surface concentration. Using the similarity transformation a set of ordinary differential equations is obtained from the governing equations. The momentum equation and the equation of reactive solute are solved numerically. The results show that the permeability parameter tends to reduce the rate of flow from the wall and is broadening the solute layer. The Schmidt number and the reaction rate parameter reduce the solute boundary layer thickness. Most importantly, the effects of initial variable solute distribution over a stretching surface is interesting i.e. for the increase in magnitude of  $n$ , the concentration decreases when  $n > 0$  whereas increases when  $n < 0$ .

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