# A New Analytical Method for the Sizing and Siting of DG in Radial System to Minimize Real Power Losses

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**Abstract**

To minimize line losses of power systems, it is crucially important to define the size and location of local generation to be placed. On account of some inherent features of distribution systems, such as radial structure, large number of nodes, a wide range of X/R ratios; the conventional techniques developed for the transmission systems generally fail on the determination of optimum size and location of distributed generations. In this study, a loss sensitivity factor, based on the equivalent current injection, is formulated for the distribution systems. The formulated sensitivity factor is employed for the determination of the optimum size and location of distributed generation so as to minimize total power losses by an analytical method without use of admittance matrix, inverse of admittance matrix or Jacobian matrix. The proposed method is in close agreement with the classical grid search algorithm based on successive load flows.

**Keywords:** Distributed generation, Equivalent current injection, Loss sensitivity factor, Optimum location, Optimal size, Radial system, Power losses.

## 1. Introduction:

One of the most important motivation for the studies on the integration of distributed resources to the grid is the exploitation of the renewable resources such as; hydro, wind, solar, geothermal, biomass and ocean energy, which are naturally scattered around the country and also are smaller in size. Accordingly, these resources can only be tapped through integration to the distribution system by means of distributed generation. Although there is no consensus on the exact definition of distributed generation (DG), there are some significant attempts, in the literature [1, 2], to define the concept. Meanwhile DG, which generally consists of various types of renewable resources, can best be defined as electric power generation within distribution networks or on the customer side of the system [1, 2], in general. This definition is preferred along this paper.DG affects the flow of power and voltage conditions on the system equipment. These impacts may manifest themselves either positively or negatively depending on the distribution system operating conditions and the DG characteristics. Positive impacts are generally called ‘system support benefits’, and include voltage support, loss reduction, transmission and distribution capacity release ,improved utility system reliability and power quality. On account of achieving above benefits, the DG must be reliable, dispatch able, of the proper size and at the proper locations[3,4] Energy cost of renewable-based distributed generation when compared to the conventional generating plants is generally high because the factors of social and environmental benefits could not be included in the cost account. Accordingly, most of the studies to determine the optimum location and size of DG could not consider the generation cost, directly. Although one of the most important benefits of DG is reduction on the line losses, it is crucially important to determine the size and the location of local generation to be placed. For the minimization of system losses, there have been number of studies to define the optimum location of DG. The various approaches on the optimum DG placement for minimum power losses can be listed as the classical approach: second-order algorithm method[5], the meta-heuristics approaches [6, 8] : genetic algorithm and Hereford Ranch algorithm [6], fuzzy-GA method [7], tabu search [8], and the analytical approaches [9,13]. In the analytical studies [9,11], optimal place of the DGs are determined exclusively for the various distributed load profiles such as; uniformly, increasingly, centrally in radial systems to minimize the total losses. Additionally, in [12], optimal size and place of DG is obtained and analyzed by considering the effects of static load models. These analytical studies are generally based on phasor current injection method which has unrealistic assumptions such as; uniformly, increasingly, centrally distributed load profiles. These assumptions may cause erroneous solution for the real systems. In [13] the optimal size and location of DG is calculated based on exact loss formula and compared with successive load flows and loss sensitivity methods. The method is computationally less demanding for radial and networked systems, however, it requires the calculation of the bus impedance matrix, Zbus, the inverse of the bus admittance matrix, Ybus. It should be noted that due to the size, complexity and specific characteristics of distribution networks, the method could not be directly applied to distribution systems. It fails to meet the requirements in robustness aspects in the distribution system environments [14].It is already pointed out that although the heuristic methods are

intuitive, easy to understand and simple to implement as compared to analytical and numerical programming methods, the results produced by heuristic algorithms are not guaranteed to be optimal [15].
In this study, the optimum size and location of distributed generation will be defined so as to minimize total power losses by an analytical method based on the equivalent current injection technique and without the use of impedance or Jacobian matrices for radial systems. The optimum size of DG and placement for loss minimization are determined by the proposed method and validated using the 34 bus radial distribution system these results are close related to the classical grid search algorithm .The proposed method is easy to be implemented, faster and more accurate than the classical method, meta-heuristic methods and early analytical methods . It is more suitable for radial systems of considerable sizes than the analytical method proposed earlier of Nareshacharya's paper. Since the proposed method is an analytical method and exploits the topological characteristics of a distribution system, there is no need for the Jacobian matrix, the bus admittance matrix, Ybus, or the bus impedance matrix, Zbus. Therefore the proposed method can achieve the advantages of computation time reduction and, accuracy improvement. The derived sensitivity factor ($\partial Ploss/\partial P$) can be also used for various purposes such as; network planning, network reconfiguration, optimal power flow and reactive power dispatch, etc.

## 2. Optimum Size And Location Of DG:

The proposed method is based on the equivalent current injection that uses the bus-injection to branch-current (*BIBC*) and branch-current to bus-voltage (*BCBV*) matrices which were developed based on the topological structure of the distribution systems and is widely implemented for the load flow analysis of the distribution systems. The details of both matrices can be found in [16].The method proposed here requires only one base case load flow to determine the optimum size and location of DG.

### 2.1. Theoretical analysis

In this section, the total power losses will be formulated as a function of the power injections based on the equivalent current injection. The formulation of total power losses will be used for determining the optimum size of DG and calculation of the system losses.

At each bus $k$, the corresponding equivalent current injection is specified by

$$I_k = \left(\frac{p_k + jQ_k}{V_k}\right)^* k = 1, 2, 3, ........n \qquad (1)$$

where $Vk$ is the node voltage, $Pk + jQk$ is the complex power at each bus $k$, $n$ is the total number of buses, '*' symbolizes the complex conjugate of operator

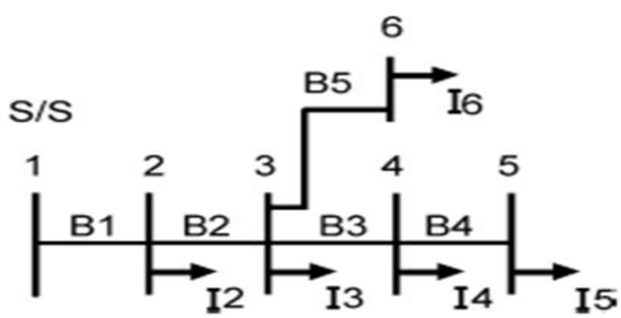


**Fig.1. A Simple Distribution System**

The equivalent current injection of bus $k$ can be separated into real and imaginary parts by (2):

$$re(I_k) = \frac{P_k \cos(\theta_k) + Q_k \sin(\theta_k)}{|V_k|}, im(I_k) = \frac{P_k \sin(\theta_k) - Q_k \cos(\theta_k)}{|V_k|} \qquad (2)$$

where $\theta k$ is the angle of $k$th node voltage. The branch current $B$ is calculated with the help of *BIBC* matrix. The *BIBC* matrix is the result of the relationship between the bus current injections and branch currents. The elements of *BIBC* matrix consist of '0's or '1's:

$$[B]_{nbX1} = [BIBC]_{nbX(n-1)} . [I]_{(n-1)X1} \qquad (3)$$

Where $nb$ is the number of the branch, [$I$] is the vector of the equivalent current injection for each bus except the reference bus. Branch currents of a simple distribution system given in Fig 1are obtained by *BIBC* matrix as in (4).While the rows of *BIBC* matrix concern with the branches of the network, on the other hand, the columns of the matrix are related with the bus current injection except the reference bus. Detailed description of *BIBC* matrix's building algorithm can be found in [16].

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} \tag{4}$$

The total power losses can be expressed as a function of the bus current injections:

$$ploss = \sum_{k=1}^{nb} | B_k |^2 .R_k = [R]^T \, |[BIBC].[I]|^2 \tag{5}$$

where $Rk$ is the $k$th branch resistance and the branch resistance vector is given in (6):

$$[R]_{nbX1} = [R_1 \quad R_2 \quad R_3 \quad R_4 \quad R_5 \quad R_6 \quad R_7 \quad R_8 \quad R_9 .......... \quad R_{nb}]^T \tag{6}$$

The total power losses can be written as a function of the real and imaginary parts of the bus current injection:

$$Ploss = [R]^T \, |[BIBC.[I]]|^2 = [R]^T \, |[BIBC].[re(I)] + j[BIBC].[im(I)]|^2 \tag{7}$$

where $[re(I)]$ and $[im(I)]$ are the vectors that consist of real and imaginary parts of the bus current injection:

$$ploss = [R]^T \, (([BIBC].[re(I)])^2 + ([BIBC].[im(I)])^2) \tag{8}$$

By substituting the equivalent bus injection expression(2) into (8), the total power losses can be rewritten

$$ploss = [R]^T \left( [BIBC].\left[ \frac{P\cos(\theta) + Q\sin(\theta)}{|V|} \right] \right)^2 + [R]^T \left( [BIBC].\left[ \frac{P\sin(\theta) - Q\cos(\theta)}{|V|} \right] \right)^2 \tag{9}$$

at $l$th branch the power loss can be obtained as

$$ploss_j = R_j.\left[ \left( \sum_{k=2}^{n} BIBC(j,k-1) \frac{P_k\cos(\theta_k) + Q_k\cos(\theta_k)}{|V_k|} \right)^2 + \left( \sum_{k=2}^{n} BIBC(j,k-1) \frac{P_k\sin(\theta_k) - Q_k\cos(\theta_k)}{|V_k|} \right)^2 \right] \tag{10}$$

The total power losses are the sum of the each branch power losses:

$$ploss = \sum_{l=1}^{nb} R_l \left[ \left( \sum_{k=2}^{n} BIBC(j,m-1) \frac{P_m\cos(\theta_m) + Q\sin(\theta_m)}{|V_m|} \right)^2 + \left( \sum_{m=2}^{n} BIBC(j,m-1) \frac{p_m\sin(\theta_m)}{|V_m|} \right)^2 \right] \tag{11}$$

The voltage drop from each bus to the reference bus is obtained with *BCBV* and *BIBC* matrices as

$$[\Delta V]_{(n-1)X1} = [BCBV].[BIBC].[I] \tag{12}$$

where *BCBV* matrix is responsible for the relations between branch currents and bus voltages. The elements of *BCBV* matrix consist of the branch impedances. Building algorithm of *BCBV* matrix can be found in [16]. In addition, building algorithm of *BCBV* matrix is provided as follows for convenience

:• Step 1. Read BIBC matrix, Zb branch impedance vector
.• Step 2. Convert Zb vector to a diagonal matrix Z by setting off diagonal elements to zero; (Z  diag(Zb)).
• Step 3. Multiply transpose of *BIBC* matrix with *Z* matrix; (*BCBV* = *BIBC*T *Z*).

The voltage drop of a simple distribution system given in Fig.1 is obtained as

$$[\Delta V] = [BCBV].[BCBV].[I]$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} - \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_{\partial 6} \end{bmatrix} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 \\ Z_{12} & Z_{23} & 0 & 0 & Z_{36} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} \tag{13}$$

### 2.2. The loss sensitivity factor

The derivation of the $j$th branch power loss per $i$th bus injected real power $\partial Plossl/\partial Pk$ can be obtained from (10) as

$$\frac{\partial ploss_l}{\partial P_k} = 2R_l.\left(\sum_{m=2}^{n} BIBC(l,m-1).\frac{P_m\cos(\theta_m)+Q_m\sin(\theta_m)}{|V_m|}\right).XBIBC(l,k-1).\frac{\cos(\theta_k)}{|V_k|} + 2R_l.\left(\sum_{m=2}^{n} BIBC(l,m-1).\frac{P_m\sin(\theta_m)-Q_m\cos(\theta_m)}{|V_m|}\right).XBIBC(l,k-1).\frac{\sin(\theta_k)}{|V_k|} \quad (14)$$

Sum of the above expression leads to the derivation of the total power losses per $k$th bus injected real power, $\partial Ploss/\partial Pk$:

$$\frac{\partial ploss_l}{\partial P_k} = 2\sum_{l=1}^{nb} R_l.\left(\sum_{m=2}^{n} BIBC(l,m-1).re(I_m)\right).XBIBC(l,k-1).\frac{\cos(\theta_k)}{|V_k|} + 2\sum_{l=1}^{nb} R_l.\left(\sum_{m=2}^{n} BIBC(j,k-1).im(I_k)\right).XBIBC(l,k-1).\frac{\sin(\theta_k)}{|V_k|} \quad (15)$$

If the $i$th bus is not connected the $j$th branch then the elements of $BIBC$ matrix is zero($BIBC(j,i-1) = 0$) and the derivative of the corresponding element is equated to zero ($\partial Plossj/\partial Pi = 0$). Accordingly, the derivative of the total power losses per $i$th bus injected real power gives the sensitivity factor and can be expressed as

$$\frac{\partial ploss}{\partial P_k} = 2\sum_{l=1}^{nb} R_l \sum_{m=2}^{n} dPBIBC_k(l,m-1).\left[\frac{\cos(\theta_k)}{|V_k|}.re(I_m) + \frac{\sin(\theta_k)}{|V_k|}.im(I_m)\right] \quad (16)$$

The sensitivity factor with the above relation can be shown in matrix form as

$$\frac{\partial ploss}{\partial P_k} = 2[R]^T\left[[dPBIBC_k].[re(I)]\frac{\cos(\theta_k)}{|V_k|}\right] + \left[[dPBIBC_k].[im(I)]\frac{\sin(\theta_k)}{|V_k|}\right] \quad (17)$$

where [d$PBIBCi$] matrix is constructed by a simple algorithm given step by step as follows:

- Step 1. Read BIBC matrix, i bus number for DG.
- Step 2. Set dBIBCi matrix, dBIBCi = BIBC.
- Step 3. Find the row with zero elements for the (i−1)th column of dBIBCi matrix; (zerorow= find(dBIBCi(:, i−1) = 0)).
- Step 4. Convert all non zero elements of these zero;(d$BIBCi$($zerorow$,:)=$zeros$($length$($zerorow$), $n$−1)).

To better explain [d$PBIBCi$]matrix building algorithm, [d$PBIBC$4] matrix which belongs to the sensitivity factor of the 4th bus, $\partial Ploss/\partial P4$, is given in (18) for the distribution system in Fig 1.

$$dBIBC_4 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (18)$$

### 2.3. Determination of optimal size:

The goal is to determine the optimum size of DG at any location so as to minimize total power losses. To determine the optimum size of DG, the derivative of the total power losses per each bus injected real powers are equated to zero as

$$\frac{\partial Ploss}{\partial p_k} = 0 \quad (19)$$

The expression of (16) can be shown in detail as

$$\frac{\partial Ploss}{\partial P_i} = 2\sum_{j=1}^{nb} R_j.\sum_{\substack{k=2\\k\neq i}}^{n} dpBIBC_i(j,k-1).X\left[\frac{\cos(\theta_i)}{|V_i|}.re(I_k) + \frac{\sin(\theta_i)}{|V_i|}.im(I_k)\right] + 2\sum_{j=1}^{nb} R_j dpBIBC_i(j,i-1).X\left[\frac{P_i\cos^2(\theta_i)+Q_i\sin(\theta_i)\cos(\theta_i)+P_i\sin^2(\theta_i)-Q_i\cos(\theta_i)\sin(\theta_i)}{|V_i|^2}\right] \quad (20)$$

The optimal size of the added DG is extracted from(20) by equating the right hand side to zero:

$$\frac{\partial Ploss}{\partial P_k} = 2\sum_{l=1}^{nb} R_l.\sum_{\substack{m=2\\m\neq i}}^{n} dPBIBc_k(l,m-1).X\left[\frac{\cos(\theta_k)}{|V_k|}.re(I_m) + \frac{\sin(\theta_k)}{|V_k|}.im(I_m)\right] + 2\sum_{l=1}^{nb} R_l.dPBIBC_k(l.k-1).\frac{P_k}{|V_k|^2} = 0 \quad (21)$$

The real power injection at the bus $k$, $Pi$ is extracted from as (21)

$$p_i = -\frac{|V_i|\sum_{j=1}^{nb} R_j \sum_{\substack{k=2\\k\neq i}}^{n} dPBIBC_i(j,k-1)[\cos(\theta_i).re(I_k)+\sin(\theta_k).im(I_k)]}{\sum_{j=1}^{nb} R_j dPBIBC_i(j,i-1)} \quad (22)$$

The minus sign in (22) indicates that *Pk* should be injected to the system. To facilitate a practical computation, can be written in matrix format by omitting the minus sign as

$$P_k = \frac{|V_k|[R]^T[dPBIBC_k](\cos(\theta_k)[redI_k] + \sin(\theta_k)[imdI_k])}{[R]^T dPBIBC_k(:, k-1)} \qquad (23)$$

where two new terms, [*re* d*Ik*] and [*im* d*Ik*], are constructed by equating *i*th elements the real and imaginary part of the bus current injection vector, [*re*(*I*)] and [*im*(*I*)] to zero. To illustrate the concept [*re* d*I*4] and [*im* d*I*4] vectors that belong to the 4th bus real power injection of the simple distribution system in Fig.1, are provided in (24):

$$[redI_4] = \begin{bmatrix} re(I_2) & re(I_3) & 0 & re(I_5) & re(I_6) \end{bmatrix}^T, \qquad (24)$$
$$[imdI_4] = \begin{bmatrix} im(I_2) & im(I_3) & 0 & im(I_5) & im(I_6) \end{bmatrix}^T$$

The optimum size of added DG at bus *i* can be obtained by

$$P_{dg_k} = P_k + pload_k \qquad (25)$$

**2.4. Determination of optimal size and placement for DG**

The objective is to minimize power losses, *Ploss*, in the system by injected power, *Pdg*. The main constraints are to restrain the [voltages along the radial system within 1±0.05 pu. The proposed method to determine the optimal size and placement of DG is give step by step as follows and also as a general flowchart in Fig.2.

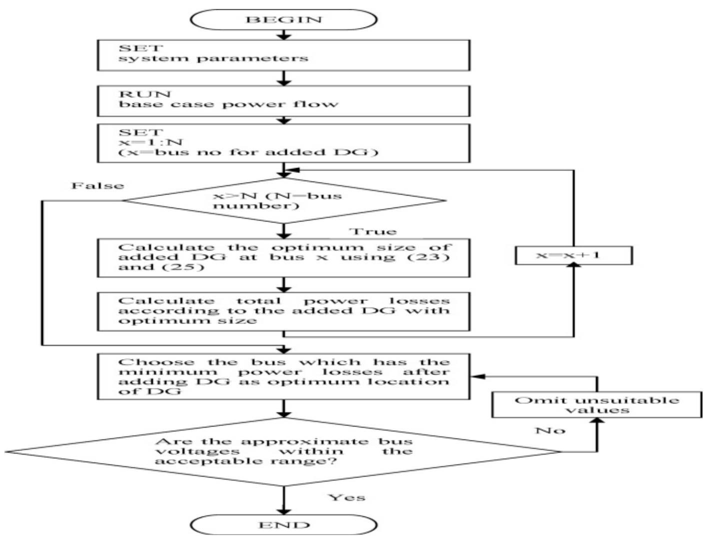


Fig. 2. General flowchart of the proposed method.

- Step 1. Run the base case power flow.
- Step 2. Find the optimum size of added DG for each bus except the reference bus using (23) and (25).
- Step 3. Calculate total power losses from(5) for each bus by placing optimum size of power to the bus.
- Step 4. Choose the bus which has the minimum power losses after adding DG as optimum location.
- Step 5. Check whether the approximate bus voltages are within the acceptable range by (12).
- Step 6. If the bus voltages are not within the acceptable range then omit DG from bus and return to Step 4.

## 3. The Results Of Simulations And Analysis

In order to evaluate the proposed algorithm described in Section2, 34 bus test system, taken from the literature, are used. Accordingly, optimum size and place of DG for the 34 bus distribution test system [17] are determined with the proposed method. The classical grid search algorithm is too costly because of computation time, that takes hours even days depending upon size of the system and power steps. By using variable step size, successive load flows as also known sequential load flows could be employed instead of grid search algorithm. In this case, computation time will reduce significantly.

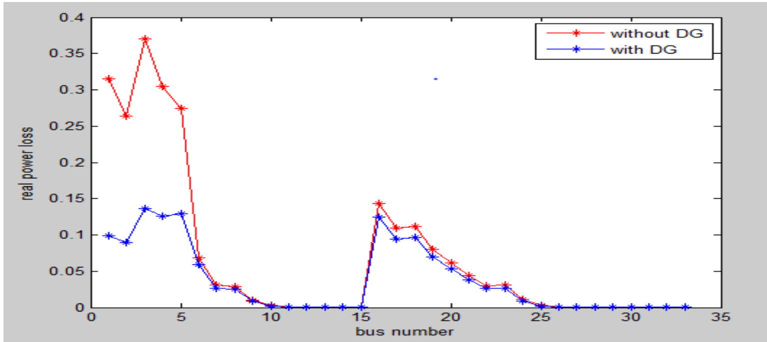


Fig 3:Variation of power loss with and without DG

The above graph in Fig 3 represents the power loss without Distribution generation(DG) & with Distributed generation(DG),In without DG the real power loss is 229.76 and after inclusion of DG real power loss is reduced to 108.6, usage of BIBC & BCBV algorithm for load flow solution and for the location of Distribution generation The loss sensitivity factor is employed &sizing is employed with the Analytical method The below graph in Fig 4 represents the Voltage variation of without Ditributed generation & with Distributed generation. Voltage profile is improved for with installation Distributed generation.

| Bus number | Reakpower (Without DG) | Realpower (With DG) | %Loss reduction |
|---|---|---|---|
| 34 | 229.76 | 108.6 | 52.7 |

Table 1: Result of 34 Bus Test System

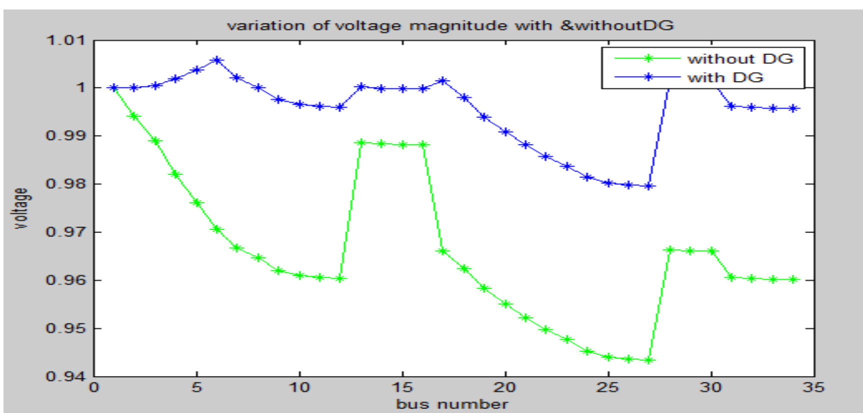


**Fig 4:Variation of Voltage with and without DG**

## 4. Conclusion

This study presents and evaluates an analytical method which can be used to determine the optimal placement and sizing of DG without use of admittance, impedance or Jacobian matrix with only one power flow for radial systems. The method is easy to be implemented and faster for given accuracy. The derived sensitivity factor is more suitable for distribution systems and could be utilized by means of simple matrix algebra. The optimal size and location of the DG, which is determined by the method, is also evaluated against Acharya's method and the classical grid search algorithm. It is found that the proposed method is in close agreement with Acharya's method and the grid search algorithm. It is appeared that the proposed method is faster than other methods in the computation time and it is appropriate for the distribution systems.

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