A Characterization of Magneto-Thermal Convection in Rivlin-Ericksen Viscoelastic Fluid in a Porous Medium

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Abstract:

A layer of Rivlin-Ericksen viscoelastic fluid heated from below in a porous medium is considered in the presence of uniform vertical magnetic field. Following the linearized stability theory and normal mode analysis, the paper mathematically established the conditions for characterizing the oscillatory motions which may be neutral or unstable, for any combination of perfectly conducting, free and rigid boundaries at the top and bottom of the fluid. It is established that all non-decaying slow motions starting from rest, in a Rivlin-Ericksen viscoelastic fluid of infinite horizontal extension and finite vertical depth in a porous medium, which is acted upon by uniform vertical magnetic field opposite to gravity and a constant vertical adverse temperature gradient, are necessarily non-oscillatory, in the regime

$$\left(\frac{Qp_2}{\pi^2}\right)\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\le 1 ,$$

where Q is the Chandrasekhar number, F is the viscoelasticity parameter, the porosity ε , P_l is the medium permeability and p_2 is the magnetic Prandtl number. The result is important since it hold for all wave numbers and for any combination of perfectly conducting dynamically free and rigid boundaries and the exact solutions of the problem investigated in closed form, are not obtainable.

Key Words: Thermal convection; Rivlin-Ericksen Fluid; Magnetic field; PES; Rayleigh number; Chandrasekhar number.

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1. Introduction

Stability of a dynamical system is closest to real life, in the sense that realization of a dynamical system depends upon its stability. Right from the conceptualizations of turbulence, instability of fluid flows is being regarded at its root. The thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside plays an important role in Geophysics, interiors of the Earth, Oceanography and Atmospheric Physics, and has been investigated by several authors and a detailed account of the theoretical and experimental study of the onset of Bénard Convection in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [1] in his celebrated monograph. The use of Boussinesq approximation has been made throughout, which states that the density changes are disregarded in all other terms in the equation of motion except the external force term. There is growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry. Bhatia and Steiner [2] have considered the effect of uniform rotation on the thermal instability of a viscoelastic (Maxwell) fluid and found that rotation has a destabilizing influence in contrast to the stabilizing effect on Newtonian fluid. In another study Sharma [3] has studied the stability of a layer of an electrically conducting Oldroyd fluid [4] in the presence of magnetic field and has found that the magnetic field has a stabilizing influence. There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's [4] constitutive relations. Two such classes of fluids are Rivlin-Ericksen's and Walter's (model B') fluids. Rivlin-Ericksen [5] has proposed a theoretical model for such one class of elastico-viscous fluids. Kumar et al. [6] considered effect of rotation and magnetic field on Rivlin-Ericksen elastico-viscous fluid and found that rotation has stabilizing effect; where as magnetic field has both stabilizing and destabilizing effects. A layer of such fluid heated from below or under the action of magnetic field or rotation or both may find applications in geophysics, interior of the Earth, Oceanography, and the atmospheric physics. With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable.

In all above studies, the medium has been considered to be non-porous with free boundaries only, in general. In recent years, the investigation of flow of fluids through porous media has become an important topic due to the recovery of crude oil from the pores of reservoir rocks. When a fluid permeates a porous material, the gross effect is represented by the

Darcy's law. As a result of this macroscopic law, the usual viscous term in the equation of Rivlin-Ericksen fluid motion is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)q\right]$, where μ and μ' are the viscosity and viscoelasticity of the Rivlin-Ericksen fluid, k_1 is the medium permeability and q is the Darcian (filter) velocity of the fluid. The problem of thermosolutal convection in fluids in a porous medium is of great importance in geophysics, soil sciences, ground water hydrology and astrophysics. Generally, it is accepted that comets consist of a dusty 'snowball' of a mixture of frozen gases which, in the process of their journey, changes from solid to gas and vice-versa. The physical properties of the comets, meteorites and interplanetary dust strongly suggest the importance of non-Newtonian fluids in chemical technology, industry and geophysical fluid dynamics. Thermal convection in porous medium is also of interest in geophysical system, electrochemistry and metallurgy. A comprehensive review of the literature concerning thermal convection in a fluid-saturated porous medium may be found in the book by Nield and Bejan [7]. Sharma et al [8] studied the thermosolutal convection in Rivlin-Ericksen rotating fluid in porous medium in hydromagnetics with free boundaries only.

Pellow and Southwell [9] proved the validity of PES for the classical Rayleigh-Bénard convection problem. Banerjee et al [10] gave a new scheme for combining the governing equations of thermohaline convection, which is shown to lead to the bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries and, Banerjee and Banerjee [11] established a criterion on characterization of non-oscillatory motions in hydrodynamics which was further extended by Gupta et al. [12]. However no such result existed for non-Newtonian fluid configurations in general and in particular, for Rivlin-Ericksen viscoelastic fluid configurations. Banyal [13] have characterized the oscillatory motions in couple-stress fluid.

Keeping in mind the importance of Rivlin-Ericksen viscoelastic fluids and magnetic field, as stated above, this article attempts to study Rivlin-Ericksen viscoelastic fluid heated from below in a porous medium in the presence of uniform magnetic field, with more realistic boundaries and it has been established that the onset of instability in a Rivlin-Ericksen viscoelastic fluid heated from below in a porous medium, in the presence of uniform vertical magnetic field, cannot manifest itself as oscillatory motions of growing amplitude if the Chandrasekhar number Q, the viscoelasticity parameter F, the porosity ε, the medium permeability P_l and the magnetic Prandtl number p_2, satisfy the inequality, $\left(\frac{Qp_2}{\pi^2}\right)\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\leq 1$, for all wave numbers and for any combination of perfectly conducting dynamically free and rigid boundaries.

2. Formulation Of The Problem And Perturbation Equations

Here we Consider an infinite, horizontal, incompressible electrically conducting Rivlin-Ericksen viscoelastic fluid layer, of thickness d, heated from below so that, the temperature and density at the bottom surface z = 0 are T_0 and ρ_0, and at the upper surface z = d are T_d and ρ_d respectively, and that a uniform adverse temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained. The gravity field $\vec{g}(0,0,-g)$ and a uniform vertical magnetic field pervade on the system $\vec{H}(0,0,H)$.This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability k_1.

Let p, ρ, T, α, g, η, μ_e and $\vec{q}(u,v,w)$ denote respectively the fluid pressure, fluid density temperature, thermal coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability and filter velocity of the fluid. Then the momentum balance, mass balance, and energy balance equation of Rivlin-Ericksen fluid and Maxwell's equations through porous medium, governing the flow of Rivlin-Ericksen fluid in the presence of uniform vertical magnetic field (Rivlin and Ericksen [5]; Chandrasekhar [1] and Sharma et al [6])are given by

$$\frac{1}{\varepsilon}\left[\frac{\partial\vec{q}}{\partial t}+\frac{1}{\varepsilon}\left(\vec{q}.\nabla\right)\vec{q}\right]=-\left(\frac{1}{\rho_0}\right)\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\nu+\nu'\frac{\partial}{\partial t}\right)\vec{q}+\frac{\mu_e}{4\pi\rho_o}(\nabla\times\vec{H})\times\vec{H}, \quad (1)$$

$$\nabla.\vec{q} = 0, \quad (2)$$

$$E\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \kappa\nabla^2 T, \quad (3)$$

$$\varepsilon\frac{d\vec{H}}{dt} = (\vec{H}.\nabla)\vec{q} + \varepsilon\eta\nabla^2\vec{H}, \quad (4)$$

$$\nabla.\vec{H} = 0, \quad (5)$$

Where $\frac{d}{dt} = \frac{\partial}{\partial t} + \varepsilon^{-1}\vec{q}.\nabla$, stand for the convective derivatives. Here

$E = \varepsilon + (1-\varepsilon)\left(\frac{\rho_s c_s}{\rho_0 c_i}\right)$, is a constant and while ρ_s, c_s and ρ_0, c_i, stands for the density and heat capacity of the solid (porous matrix) material and the fluid, respectively, ε is the medium porosity and $\vec{r}(x, y, z)$.

The equation of state is

$$\rho = \rho_0[1-\alpha(T-T_0)], \quad (6)$$

Where the suffix zero refer to the values at the reference level z = 0. In writing the equation (1), we made use of the Boussinesq approximation, which states that the density variations are ignored in all terms in the equation of motion except the external force term. The kinematic viscosity ν, kinematic viscoelasticity ν', magnetic permeability μ_e, thermal diffusivity κ, and electrical resistivity η, and the coefficient of thermal expansion α are all assumed to be constants.

The steady state solution is

$$\vec{q} = (0,0,0), \rho = \rho_0(1+\alpha\beta z), T = -\beta z + T_0, \quad (7)$$

Here we use the linearized stability theory and the normal mode analysis method. Consider a small perturbations on the steady state solution, and let $\delta\rho$, δp, θ, $\vec{q}(u,v,w)$ and $\vec{h} = (h_x, h_y, h_z)$ denote respectively the perturbations in density ρ, pressure p, temperature T, velocity $\vec{q}(0,0,0)$ and the magnetic field $\vec{H} = (0,0,H)$. The change in density $\delta\rho$, caused mainly by the perturbation θ in temperature is given by

$$\delta\rho = -\rho_0(\alpha\theta). \quad (8)$$

Then the linearized perturbation equations of the Rinlin-Ericksen fluid reduces to

$$\frac{1}{\varepsilon}\frac{\partial\vec{q}}{\partial t} = -\frac{1}{\rho_0}(\nabla\delta p) - \vec{g}(\alpha\theta) - \frac{1}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{q} + \frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H}, \quad (9)$$

$$\nabla.\vec{q} = 0, \quad (10)$$

$$E\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta, \quad (11)$$

$$\varepsilon\frac{\partial\vec{h}}{\partial t} = \left(\vec{H}.\nabla\right)\vec{q} + \varepsilon\eta\nabla^2\vec{h}. \quad (12)$$

And $$\nabla.\vec{h} = 0, \quad (13)$$

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

3. Normal Mode Analysis

Analyzing the disturbances into two-dimensional waves, and considering disturbances characterized by a particular wave number, we assume that the Perturbation quantities are of the form

$$[w, \theta, h_{z,}] = [W(z), \Theta(z), K(z)] \exp\left(ik_x x + ik_y y + nt\right), \qquad (14)$$

Where k_x, k_y are the wave numbers along the x- and y-directions, respectively, $k = \left(k_x^2 + k_y^2\right)^{\frac{1}{2}}$, is the resultant wave number, n is the growth rate which is, in general, a complex constant; and $W(z), K(z) and \Theta(z)$ are the functions of z only.

Using (14), equations (9)-(13), within the framework of Boussinesq approximations, in the non-dimensional form transform to

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+\sigma F)\right]\left(D^2 - a^2\right)W = -Ra^2\Theta + Q\left(D^2 - a^2\right)DK, \qquad (15)$$

$$\left(D^2 - a^2 - p_2\sigma\right)K = -DW, \qquad (16)$$

And

$$\left(D^2 - a^2 - Ep_1\sigma\right)\Theta = -W, \qquad (17)$$

Where we have introduced new coordinates (x', y', z') = (x/d, y/d, z/d) in new units of length d and $D = d/dz'$. For convenience, the dashes are dropped hereafter. Also we have substituted $a = kd, \sigma = \frac{nd^2}{\nu}$, $p_1 = \frac{\nu}{\kappa}$ is the thermal Prandtl number; $p_2 = \frac{\nu}{\eta}$ is the magnetic Prandtl number; $P_l = \frac{k_1}{d^2}$ is the dimensionless medium permeability, $F = \frac{\nu'}{d^2}$ is the dimensionless viscoelasticity parameter of the Rivlin-Ericksen fluid; $R = \frac{g\alpha\beta d^4}{\kappa\nu}$ is the thermal Rayleigh number and $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0 \nu\eta\varepsilon}$ is the Chandrasekhar number. Also we have Substituted $W = W_\oplus, \Theta = \frac{\beta d^2}{\kappa}\Theta_\oplus$ and $K = \frac{Hd}{\varepsilon\eta}K_\oplus$ and $D_\oplus = dD$, and dropped $(\oplus)$ for convenience.

We now consider the cases where the boundaries are rigid-rigid or rigid-free or free-rigid or free-free at z = 0 and z = 1, as the case may be, and are perfectly conducting. The boundaries are maintained at constant temperature, thus the perturbations in the temperature are zero at the boundaries. The appropriate boundary conditions with respect to which equations (15) -- (17), must possess a solution are

$W = 0 = \Theta$, on both the horizontal boundaries,

DW = 0, on a rigid boundary,

$D^2 W = 0$, on a dynamically free boundary,

K = 0, on both the boundaries as the regions outside the fluid are perfectly conducting, (18)

Equations (15)--(17), along with boundary conditions (18), pose an eigenvalue problem for σ and we wish to characterize σ_i, when $\sigma_r \geq 0$.

We first note that since W and K satisfy $W(0) = 0 = W(1)$ and $K(0) = 0 = K(1)$, in addition to satisfying to governing equations and hence we have from the Rayleigh-Ritz inequality Schultz [14]

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz \text{ And } \int_0^1 |DK|^2 dz \geq \pi^2 \int_0^1 |K|^2 dz, \qquad (19)$$

4. Mathematical Analysis

We prove the following lemma:

Lemma 1: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 \left\{ |DK|^2 + a^2 |K|^2 \right\} dz \le \frac{1}{\pi^2} \int_0^1 |DW|^2 dz$$

Proof: Multiplying equation (16) by K^* (the complex conjugate of K), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times and making use of boundary conditions on K namely $K(0) = 0 = K(1)$, it follows that

$$\int_0^1 \left\{ |DK|^2 + a^2 |K|^2 \right\} dz + \sigma_r p_2 \int_0^1 |K|^2 dz = \text{Real part of} \left\{ \int_0^1 K^* DW dz \right\} \le \left| \int_0^1 K^* DW dz \right| \le \int_0^1 |K^* DW| dz,$$

$$\le \int_0^1 |K^*||DW| dz \le \int_0^1 |K||DW| dz \le \left\{ \int_0^1 |K|^2 dz \right\}^{\frac{1}{2}} \left\{ \int_0^1 |DW|^2 dz \right\}^{\frac{1}{2}}, \qquad (20)$$

(Utilizing Cauchy-Schwartz-inequality),

This gives that

$$\int_0^1 |DK|^2 dz \le \left\{ \int_0^1 |K|^2 dz \right\}^{\frac{1}{2}} \left\{ \int_0^1 |DW|^2 dz \right\}^{\frac{1}{2}} \qquad (21)$$

Inequality (20) on utilizing (19), gives

$$\left\{ \int_0^1 |K|^2 dz \right\}^{\frac{1}{2}} \le \frac{1}{\pi^2} \left\{ \int_0^1 |DW|^2 dz \right\}^{\frac{1}{2}}, \qquad (22)$$

Since $\sigma_r \ge 0$ and $p_2 \rangle 0$, hence inequality (20) on utilizing (22), give

$$\int_0^1 \left(|DK|^2 + a^2 |K|^2 \right) dz \le \frac{1}{\pi^2} \int_0^1 |DW|^2 dz, \qquad (23)$$

This completes the proof of lemma.

We prove the following theorem:

Theorem 1: If $R \rangle 0$, $F \rangle 0$, $Q \rangle 0$, $P_l \rangle 0$, $p_1 \rangle 0$, $p_2 \rangle 0$, $\sigma_r \ge 0$ and $\sigma_i \ne 0$ then the necessary condition for the existence of non-trivial solution (W, Θ, K) of equations (15) – (17), together with boundary conditions (18) is that

$$\left(\frac{Q p_2}{\pi^2} \right) \left(\frac{\varepsilon P_l}{P_l + \varepsilon F} \right) \rangle 1 .$$

Proof: Multiplying equation (15) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l} (1 + \sigma F) \right] \int_0^1 W^* \left(D^2 - a^2 \right) W dz = -R a^2 \int_0^1 W^* \Theta dz + Q \int_0^1 W^* D \left(D^2 - a^2 \right) K dz, \qquad (24)$$

Taking complex conjugate on both sides of equation (17), we get

$$\left(D^2 - a^2 - E p_1 \sigma^* \right) \Theta^* = -W^*, \qquad (25)$$

Therefore, using (25), we get

$$\int_0^1 W^* \Theta dz = -\int_0^1 \Theta \left(D^2 - a^2 - E p_1 \sigma^* \right) \Theta^* dz, \qquad (26)$$

Also taking complex conjugate on both sides of equation (16), we get

$$\left[D^2 - a^2 - p_2 \sigma^* \right] K^* = -DW^*, \qquad (27)$$

Therefore, equation (27), using appropriate boundary condition (18), we get

$$\int_0^1 W^* D(D^2 - a^2)K dz = -\int_0^1 DW^*(D^2 - a^2)K dz = \int_0^1 K(D^2 - a^2)(D^2 - a^2 - p_2\sigma^*)K^* dz, \quad (28)$$

Substituting (26) and (28), in the right hand side of equation (24), we get

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+\sigma F)\right]\int_0^1 W^*(D^2 - a^2)W dz = Ra^2\int_0^1 \Theta(D^2 - a^2 - Ep_1\sigma^*)\Theta^* dz$$

$$+Q\int_0^1 K^*(D^2 - a^2)^2 K dz - Qp_2\sigma^*\int_0^1 K^*(D^2 - a^2)K dz, \quad (29)$$

Integrating the terms on both sides of equation (29) for an appropriate number of times and making use of the appropriate boundary conditions (18), we get

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+\sigma F)\right]\int_0^1 \left(|DW|^2 + a^2|W|^2\right)dz = Ra^2\int_0^1\left(|D\Theta|^2 + a^2|\Theta|^2 + Ep_1\sigma^*|\Theta|^2\right)dz$$

$$-Q\int_0^1\left(|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2\right)dz - Qp_2\sigma^*\int_0^1\left(|DK|^2 + a^2|K|^2\right)dz, \quad (30)$$

Now equating the imaginary parts on both sides of equation (30), and cancelling $\sigma_i (\neq 0)$ throughout, we get

$$\left[\frac{1}{\varepsilon} + \frac{F}{P_l}\right]\int_0^1\left(|DW|^2 + a^2|W|^2\right)dz = \left[-Ra^2 Ep_1\int_0^1|\Theta|^2 dz + Qp_2\int_0^1\left(|DK|^2 + a^2|K|^2\right)dz\right], \quad (31)$$

Now R $\rangle$ 0, $\varepsilon \rangle 0$ and $Q \rangle 0$, utilizing the inequalities (23), the equation (31) gives,

$$\left[\left(\frac{1}{\varepsilon} + \frac{F}{P_l}\right) - \left(\frac{Qp_2}{\pi^2}\right)\right]\int_0^1 |DW|^2 dz + I_1 \langle 0, \quad (32)$$

Where

$$I_1 = \left(\frac{1}{\varepsilon} + \frac{F}{P_l}\right)a^2\int_0^1 |W|^2 dz + Ra^2 Ep_1\int_0^1|\Theta|^2 dz,$$

Is positive definite, and therefore, we must have

$$\left(\frac{Qp_2}{\pi^2}\right)\left(\frac{\varepsilon P_l}{P_l + \varepsilon F}\right) \rangle 1. \quad (33)$$

Hence, if

$\sigma_r \geq 0$ and $\sigma_i \neq 0$, then $\left(\frac{Qp_2}{\pi^2}\right)\left(\frac{\varepsilon P_l}{P_l + \varepsilon F}\right) \rangle 1$. (34)

And this completes the proof of the theorem.

Presented otherwise from the point of view of existence of instability as stationary convection, the above theorem can be put in the form as follow:-

Theorem 2: The sufficient condition for the onset of instability as a non-oscillatory motions of non-growing amplitude in a Rivlin-Ericksen viscoelastic fluid in a porous medium heated from below, in the presence of uniform vertical magnetic field is that, $\left(\frac{Qp_2}{\pi^2}\right)\left(\frac{\varepsilon P_l}{P_l + \varepsilon F}\right) \leq 1$, where Q is the Chandrasekhar number, ε is the porosity, P_l is the medium permeability and F is the viscoelasticity parameter, for any combination of perfectly conducting dynamically free and rigid boundaries.

or

The onset of instability in a Rivlin-Ericksen viscoelastic fluid in a porous medium heated from below, in the presence of uniform vertical magnetic field, cannot manifest itself as oscillatory motions of growing amplitude if the Chandrasekhar number Q, the porosity ε, the medium permeability P_l and the viscoelasticity parameter F, satisfy the inequality $\left(\frac{Qp_2}{\pi^2}\right)\left(\frac{\varepsilon P_l}{P_l + \varepsilon F}\right) \leq 1$, for any combination of perfectly conducting dynamically free and rigid boundaries.

The sufficient condition for the validity of the 'PES' can be expressed in the form:

Theorem 3: If $(W, \Theta, K, Z, X, \sigma)$, $\sigma = \sigma_r + i\sigma_i$, $\sigma_r \geq 0$ is a solution of equations (15) – (17), with R $\rangle$ 0 and,

$$\left(\frac{Qp_2}{\pi^2}\right)\left(\frac{\varepsilon P_l}{P_l + \varepsilon F}\right) \leq 1,$$

Then $\sigma_i = 0$.

In particular, the sufficient condition for the validity of the 'exchange principle' i.e., $\sigma_r = 0 \Rightarrow \sigma_i = 0$ if $\left(\frac{Qp_2}{\pi^2}\right)\left(\frac{\varepsilon P_l}{P_l + \varepsilon F}\right) \leq 1$.

In the context of existence of instability in 'oscillatory modes' and that of 'overstability' in the present configuration, we can state the above theorem as follow:-

Theorem 4: The necessary condition for the existence of instability in 'oscillatory modes' and that of 'overstability' in a Rivlin-Ericksen viscoelastic fluid in a porous medium heated from below, in the presence of uniform vertical magnetic field, is that the Chandrasekhar number Q, the porosity ε, the viscoelasticity parameter of the fluid F and the medium permeability P_l, must satisfy the inequality $\left(\frac{Qp_2}{\pi^2}\right)\left(\frac{\varepsilon P_l}{P_l + \varepsilon F}\right) \rangle 1$, for any combination of perfectly conducting dynamically free and rigid boundaries

5. Conclusions

This theorem mathematically established that the onset of instability in a Rivlin-Ericksen viscoelastic fluid in the presence of uniform vertical magnetic field, cannot manifest itself as oscillatory motions of growing amplitude if the Chandrasekhar number Q, the porosity ε, the viscoelasticity parameter of the fluid F and the medium permeability P_l, satisfy the inequality $\left(\frac{Qp_2}{\pi^2}\right)\left(\frac{\varepsilon P_l}{P_l + \varepsilon F}\right) \leq 1$, for any combination of perfectly conducting dynamically free and rigid boundaries

The essential content of the theorem, from the point of view of linear stability theory is that for the configuration of Rivlin-Ericksen viscoelastic fluid of infinite horizontal extension heated form below, for any combination of perfectly conducting dynamically free and rigid boundaries at the top and bottom of the fluid, in the presence of uniform vertical magnetic field, parallel to the force field of gravity, an arbitrary neutral or unstable modes of the system are definitely non-oscillatory in character if $\left(\frac{Qp_2}{\pi^2}\right)\left(\frac{\varepsilon P_l}{P_l + \varepsilon F}\right) \leq 1$, and in particular PES is valid.

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