TENSOR PAIRS FOR FERROTOROIDAL MOMENT

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Abstract:
The form of Physical property tensors invariant under point groups and their Subgroups can determine the basis for the classification of domain pairs in ferroic Crystals. In a ferroic Crystal containing two or more equally stable domains of the same structure but of different spatial orientation, macroscopic tensorial physical properties that are different in domains, determine a tensor distinction of the domains. In this paper, we have calculated the ferrotoroidic tensor pairs, using double Coset decomposition of all 324 ferroic Species, taking 32 grey groups as prototypic point groups.

Key Words: Ferroic point group. Ferrotoroidic, Grey group, Prototypic point group, tensor pairs , toroidal moment.

Introduction:

A Ferroic phase transition is a phase transition of a crystalline structure from a phase of higher point group symmetry G to a phase of lower point group symmetry F. In the lower symmetry phase there are n = |G| / |F|, single domain states S1, S2 …… Sn. Where |G| and |F| denote the number of elements in G and F. A ferroic Crystal contains two or more equally stable domains of volumes of the same homogeneous crystalline structure but of the different spatial orientations.
Tensor distinction had been discussed by Aizu [1] subsequently Litvin have extended and evaluated tensor distinction by magnetization, polarization and strain in a ferroic phase transition and also included tenor distinction by toroidal moment. Here ferrotoroidic tensor pairs for all 324 ferroic species by using double coset decomposition are calculated. A crystal, regarded as a thermodynamic system, any physical property can be defined by a relation between two measurable quantities.
A Crystal is an anisotropic medium, means that the response of a Crystal to an external 'force' depends not only on the magnitude of that force but also on its orientation relative to the Crystal axes. Domain states may be distinguished by the values of components of certain spontaneous macroscopic tensorial properties. Ferrotoroidic type with toroidal moment "av" for all 324 ferroic species are calculated, taking grey group as the prototypic point group since "a" have no effect on ordinary 32 point groups. So, we have consider grey groups, Here "V" denotes a polar Vector; "a" denotes zero rank tensors that change sign under time inversion. A magnetic toroidal moment represents a vector like electromagnetic multipole moment which breaks both space and time reversal symmetries simultaneously. Magnetic toroidal moment in solids have increased attention due to its potential relevance in the context of multiferroic materials and magneto electric coupling. The toroidal moment is ideally suited to discuss magneto structural or magneto electric coupling, it has been proposed as the primary order parameter for the low temperature phase transition from a ferroelectric into a simultaneously ferroelectric and ferromagnetic. This means that ferrotoroidicity is the fundamental form of ferroic order, equivalent to ferromagnetism, ferroelectricity, and ferroelasticity. A theoretical analysis of magnetic toroidal moments in periodic system, in the limit the toroidal moments are caused by a time and space reversal symmetry breaking arrangement of localized magnetic dipole moments. The toroidal property is exhibited in the crystals $BaNIF_4$, $LICOPO_4$, $GaFeo_3$, and $BIFeo_3$. All these materials have been discussed in context of multiferroics, magneto-electric coupling or ferrotoidics, by observation of ferrotoroidic domins in this material using nonlinear optical techniques. The material $LICOPO_4$ crystallizes in the oliving structure with in the orthorhombic space group Pnma, it is originally believed that magnetic moments of the four Co ions in the Unit Cell are antiferromagnetically aligned along the orthorhombic direction.
G is one of the 32 Crystallographic point group and 1^1 is a group consisting of identity and time inversion R_2. The direct product of G and 1^1, which is designated by $G1^1$ is known as grey group and 32 point groups in which R_2 does not occur explicitly nor in combination with Symmetry operations are known as ordinary point groups. The 58 groups in which R_2 Occurs implicitly are known as magnetic variants of the 32 ordinary point groups. The 32 ordinary and 58 magnetic Variants are known as magnetic point groups. Shubnikov (1951) discussed point groups are 122, where the ordinary point groups are 32, the grey groups are 32 and the magnetic point groups are 58.
Every time Symmetry point group is a prototypic point group.
Let H be a point group of an orientation state "S" and a subgroup of prototypic point group $G1^1$. Then H is called the ferroic point group. All different ways in which the elements of the ferroic point group correspond to the elements of the Prototypic point groups gives so many possible species and they are denoted by $G1^1$ FH.

Representative Tensor Pairs:

Let G be the prototypic point group of the crystal and H be the ferroic point group of the one of the domains. $H^{(i)}$, i= 1, 2 …q, denote the point groups of each of the domains with $H^{(1)} = H$. Let T denote a spontaneous physical property tensor which arises in the low symmetry phase of the crystal. Denote by $T^{(i)}$, i = 1,2………q, the specific form of the tensor T characterizing each of the q domains, and denote $T^{(1)} = T$.

All pairs of tensors having the same mutual relationship can be considered as a single class of tensor pairs and are called a class of crystallographically equivalent tensors pairs (Litvin and Wike, 1989). A single tensor pair, called a representative tensor pair is chosen from each class to represent the mutual relationship between all tensor pairs in that class.

All ordered tensor pairs could be partitioned into classes of crystallographically equivalent tensor pairs. Two tensor pairs ($T^{(i)}$, $T^{(j)}$) and ($T^{(i1)}$, $T^{(j1)}$) are said to be crystallographically equivalent with respect to G and to belong to the same class of ordered tensor pairs, if there is an element g of G such that ($T^{(i)}$, $T^{(j)}$) = ($gT^{(i1)}$, $gT^{(j1)}$) that is, if $T^{(i)} = g\,T^{(i1)}$ and $T^{(j)} = g\,T^{(j1)}$.

Let Gτ denote the stabilizer of T in G, this subgroup Gτ of G is the set of all elements g of G which leave T invariant i.e. gT=T. If Gτ =H then T is a full physical property tensor and there are $q_T = q$ distinct forms of the tensor T i.e. each of the few domains is characterized by distinct form of the tensor T. If H is a subgroup Gτ then T is a partial physical property tensor and there are $q\tau \leq q$ distinct forms of the tensor T by $T^{a}_{(d)}$ a = 1,2,……..q_τ and choose $T_d^{(1)} = T^{(1)} = T$.

All ordered distinct tensor pairs ($T_d^{(a)}$, $T_d^{(b)}$) could be partitioned into classes of crystallographically equivalent ordered distinct tensor pairs in the same manner as ($T^{(i)}$, $T^{(j)}$). The number of classes ordered distinct tensor pairs is same as the number of classes of tensor pairs (Litvin and Wike 1989).

Let G be the prototypic point group, H is the ferric point group and T is the specific form of the physical property tensor T that keeps H invariant. The number N of crystallographically equivalent ordered distinct tensor pair classes is equal to the number of double cosets in the double coset decomposition of G with respect to G_τ.

$G = G_\tau E\, G_\tau + G_\tau g_1\, G_\tau + \ldots\ldots\; G_\tau g_N\, G_\tau$

Where Gτ is the stabilizer of T in G and g_k , k= 1,2,……N are the double coset representatives . Tables of the coset and double coset decomposition of the 32 cyrystallographic point groups with respect to one of the each set of conjugate sub groups were given by janovee and Dvorakova (1974).

Litvin S.Y and Litvin (1990) have tabulated the representative tensor pairs (T, g_k T) for all classes of tensor pairs for all point groups G and sub groups H and all physical property tensor T of rank 0,1and 2.

Example: Ferrotoroidic tensor pairs for the ferroic species: $4/m1^1F2^1$

Consider the ferroic species $4/m1^1F2^1$, where $4/m1^1$ is prototypic point group and 2^1 is a ferroic point group and the stabilizer Gτ is $2^1/m$. The number of distinct tensor pair classes are 4 .The double coset decomposition of $4/m1^1$ with respect to the stabilizer $2^1/m$ is given by

$G = 4/m1^1 = (2^1/m)\, E\, (2^1/m) + (2^1/m)\, C^+_{4z}\, (2^1/m) + (2^1/m)\, R_2\, (2^1/m) + (2^1/m)\, R_2\, C^+_{4z}\, (2^1/m)$

Since form, $2^1/m$ ferroic point groups, the stabilizer is $2^1/m$ so that decomposition is w.r.t to $2^1/m$ is for these groups

Table 1: ferrotoroidic tensor pairs for ferroic species $4/m1^1$ F 2^1; $4/m1^1$ Fm; $4/m1^1$ F $2^1/m$

S No	Prototypic point group, G	Ferroic Point group, H	Stabilizer	Double coset elements	Tensor Pairs
1	$4/m1^1$	2^1 m $2^1/m$	$2^1/m$	E, C^+_{4z}, R_2, $R_2\, C^+_{4z}$	(T_1, T_2, O) (T_1, T_2, O); (T_1, T_2, O) (-T_2, T_1, O); (T_1, T_2, O) (-T_1, -T_2, O); (T_1, T_2, O) (T_2, -T_1, O)

Table 2 gives the list of all the Tensor pair representatives of toroidic physical property tensors. In the below mentioned table 2 the 2nd column represents Prototypic point group "G", 3rd column represents the ferroic point group "H" , 4th column represents the Stabilizer GT, 5th column represents the Double Coset elements and 6th column represents the Tensor Pairs.

Table 2

S No	Prototypic Point Group G	Ferroic Point Group H	Stabilizer Gτ	Double Coset elements	Tensor Pairs
1.	1^1	1	1	E , R_2	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$; $(T_1,T_2,T_3)(T_1,T_2,T_3)$
2.	$\bar{1}1^1$	1	$\bar{1}^1$	E , R_2	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3);(T_1,T_2,T_3)(T_1,T_2,T_3)$
3.	21^1	1	1	E ,C_{2z} , R_2 , $R_2\,C_{2z}$	$(T_1,T_2,T_3)(-T_1,-T_2,T_3);(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$ $(T_1,T_2,T_3)(T_1,T_2,T_3);(T_1,T_2,T_3)(T_1,T_2,-T_3)$
	21^1	2	2	E , R_2	$(0,0,T_3)(0,0,-T_3);(0,0,T_3)(0,0,T_3)$
	21^1	2^1	2^1	E , R_2	$(0,0,T_3)(0,0,-T_3);(0,0,T_3)(0,0,T_3)$
4	$m1^1$	m	m	E,R_2	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$
	$m1^1$	m^1	m^1	E,R_2	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$
5.	$2/m1^1$	1 $\bar{1}^1$	$\bar{1}^1$	E, R_2 ,C_{2z} ,$R_2\,C_{2z}$	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3);(T_1,T_2,T_3)$ $(-T_1,-T_2,-T_3)$; (T_1,T_2,T_3) $(-T_1,-T_2,T_3);(T_1,T_2,T_3)$ $(T_1,T_2,-T_3)$
	$2/m1^1$	2 m^1 $2/m^1$	$2/m^1$	E,R_2	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$
	$2/m1^1$	2^1 m $2^1/m$	$2^1/m$	E,R_2	$(T_1,T_2,0)(T_1,T_2,0)$;$(T_1,T_2,0)(-T_1,-T_2,0)$
6	2221^1	1	1	E,C_{2X},C_{2Y},C_{2Z}, R_2C_{2X},R_2C_{2Y}, R_2C_{2Z}, R_2	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3);(T_1,T_2,T_3)$ $(-T_1,T_2,-T_3)$; (T_1,T_2,T_3) $(-T_1,-T_2,T_3);(T_1,T_2,T_3);(-T_1,-T_2,-T_3)$; $(T_1,T_2,T_3);(-T_1,T_2,T_3);(T_1,T_2,T_3);(T_{-1},T_2,T_3)$; $(T_1,T_2,T_3);(T_1,T_2,-T_3);(T_1,T_2,T_3)(T_1,T_2,T_3)$
	2221^1	2^1	2^1	E,R_2,C2Y,R_2C2Y	$(T_1,T_2,0)(-T_1,-T_2,0)$; $(T_1,T_2,0)(-T_1,T_2,0)$; $(T_1,T_2,0)(-T_1,-T_2,0)$; $(T_1,T_2,0)(T_1,T_2,0)$
7	$mm21^1$	2 mm2	mm2	E,R_2	$(0,0,T_3)(0,0,-T_3);(0,0,T_3)(0,0,T_3)$
	$mm21^1$	m	m	E,C_{2Z} ,R_2,R_2C_{2Z}	$0, T_2,T_3)(0, -T_2,T_3);(0, T_2,T_3)(0, -T_2,-T_3)$; $(0, T_2,T_3)(0, T_2,-T_3);(0, T_2,T_3)(0, T_2,T_3)$
	$mm21^1$	m^1m2^1	m^1m2^1	E,R_2	$(0, T_2,0)(0, -T_2,0)$; $(0, T_2,0)(0, T_2,0)$
8.	$mmm1^1$	1 $\bar{1}^1$	$\bar{1}^1$	E,C_{2X},C_{2Y},C_{2Z}, R_2C_{2X},R_2C_{2Y}, R_2C_{2Z}	$(T_1,T_2,T_3)(T_1,-T_2,-T_3);(T_1,T_2,T_3)(-T_1,T_2,-T_3)$; $(T_1,T_2,T_3)(-T_1,-T_2,T_3);(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$; $(T_1,T_2,T_3)(-T_1,T_2,-T_3);(T_1,T_2,T_3)(T_1,-T_2,T_3)$; $(T_1,T_2,T_3)(T_1,T_2,-T_3)$
	$mmm1^1$	2 m^1 $2/m^1$ 2^12^12 mm2 mmm^1	mmm^1	E,R_2	$(0,0,T_3)(0,0,T_3)$;$(0,0,T_3)(0,0,-T_3)$
	$mmm1^1$	2^1 m			$(T_1,T_2,0)(T_1,T_2,0)$;$(T_1,T_2,0)(-T_1,-T_2,0)$

		$2^1/m$ m^1m2^1	m^1m2^1	E,R_2	
9.	41^1	1	1	E,C^+_{4z} , C^-_{4z} ,C_{2Z} R_2 $R_2C^+_{4z},R_2C^-_{4z}$ R_2C_{2Z}	$(T_1,T_2,T_3)(-T_2,T_1,T_3);(T_1,T_2,T_3)(T_2,-T_1,T_3);$ $(T_1,T_2,T_3)(-T_2,-T_1,T_3);(T_1,T_2,T_3)(-T_2,T_1,-T_3);$ $(T_1,T_2,T_3)(T_2,-T_1,-T_3);(T_1,T_2,T_3)(-T_2,T_1,-T_3);$ $(T_1,T_2,T_3)(T_1,T_2,-T_3);(T_1,T_2,T_3)(T_1,T_2,T_3);$
	41^1	2 4	4	E,R_2	$(0,0,T_3)(0,0,T_3)$; $(0,0,T_3)(0,0,-T_3)$
	41^1	2^1	2^1	E,R_2,C^+_{4z} , $R_2C^+_{4z}$	$(T_1,T_2,0)(-T_1,-T_2,0)$; $(T_1,T_2,0)(-T_2,T_1,0)$; $(T_1,T_2,0)(T_2,-T_1,0)$; $(T_1,T_2,0)(T_1,T_2,0)$
10.	$\bar{4}1^1$	1	1	E,S^-_{4Z}, S^+_{4Z} , C_{2Z} $,R_2$, $R_2S^-_{4Z}$, $R_2S^+_{4Z}$, R_2C_{2Z}	$(T_1,T_2,T_3)(T_1,T_2,T_3);$ $(T_1,T_2,T_3)(T_2,-T_1,-T_3);(T_1,T_2,T_3)(-T_2,T_1,-T_3);$ $(T_1,T_2,T_3)(-T_1,-T_2,T_3);(T_1,T_2,T_3)(-T_1,-T_2,-T_3);$ $(T_1,T_2,T_3)(-T_2,T_1,T_3);(T_1,T_2,T_3)(T_2,-T_1,T_3);$ $(T_1,T_2,T_3)(T_1,T_2,-T_3)$
	$\bar{4}1^1$	2 $\bar{4}^1$	$\bar{4}^1$	E,R_2	$(0,0,T_3)(0,0,T_3)$; $(0,0,T_3)(0,0,-T_3)$
	$\bar{4}1^1$	2^1	2^1	E,S^+_{4Z},R_2, $R_2S^+_{4Z}$	$(T_1,T_2,0)(T_1,T_2,0)$; $(T_1,T_2,0)(-T_2,T_1,0)$; $(T_1,T_2,0)(-T_1,-T_2,0)$; $(T_1,T_2,0)(T_2,-T_1,0)$
11.	$4/m1^1$	1 $\bar{1}^1$	$\bar{1}^1$	E, C^-_{4Z}, C_{4Z} C_{2Z} R_2, R_2 C^+_{4Z}, R_2 C^-_{4Z}, R_2 C_{2Z}	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$; $(T_1,T_2,T_3)(-T_2,T_1,T_3)$; $(T_1,T_2,T_3)(T_2,-T_1,-T_3);(T_1,T_2,T_3)(T_1,T_2,T_3)$; $(T_1,T_2,T_3)(T_2,-T_1,T_3);(T_1,T_2,T_3)(-T_2,T_1,-T_3)$; $(T_1,T_2,T_3)(T_1,T_2,-T_3)$, $(T_1,T_2,T_3)(-T_1,-T_2,T_3)$
	$4/m1^1$	2 m^1 $2/m^1$ 4 $\bar{4}^1$ $4/m^1$	$4/m^1$	E,R_2	$(T_1,T_2,T_3)(T_1,T_2,T_3);(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$
c.	$4/m1^1$	2^1 m $2^1/m$	$2^1/m$	E, C^+_{4z} , R_2, $R_2C^+_{4z}$	$(T_1,T_2,0)(-T_2,T_1,0)$; $(T_1,T_2,0)(T_2,-T_1,0)$; $(T_1,T_2,0)(-T_1,-T_2,0)$; $(T_1,T_2,0)(T_1,T_2,0)$;
12.	4221^1	1	1	E, C^+_{4z} , C^-_{4z} , C_{2Z} , C_{2x} , C_{2y} , C_{2a} , C_{2b} , R_2 , R_2 C^+_{4Z}, R_2 C^-_{4Z} , R_2 C_{2Z} , R_2 C_{2x}, R_2 C_{2y},R_2 C_{2a},R_2 C_{2b}	$(T_1,T_2,T_3)(T_1,T_2,T_3)$ $(T_1,T_2,T_3)(-T_2,T_1,T_3)$; $(T_1,T_2,T_3)(T_2,-T_1,T_3)$; $(T_1,T_2,T_3)(-T_1,-T_2,-T_3);(T_1,T_2,T_3)(T_1,-T_2,-T_3)$; $(T_1,T_2,T_3)(-T_1,T_2,-T_3);(T_1,T_2,T_3)(T_2,T_1,-T_3)$; $(T_1,T_2,T_3)(-T_2,-T_1,-T_3);(T_1,T_2,T_3)(T_2,-T_1,-T_3)$; $(T_1,T_2,T_3)(-T_2,T_1,-T_3);(T_1,T_2,T_3)(T_2-T_1,-T_3)$; $(T_1,T_2,T_3)(-T_1,T_2,T_3);(T_1,T_2,T_3)(T_1,-T_2,T_3)$; $(T_1,T_2,T_3)(-T_2,-T_1,T_3);(T_1,T_2,T_3)(T_2,T_1,T_3)$
	4221^1	2 2^12^1 2 4 42^12^1	42^12^1	E,R_2	$(T_1,T_2,T_3)(T_1,T_2,T_3)$;$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$
	4221^1	2^1	2^1	E, C^+_{4z} $,c_{2x},c_{2a},R_2$, $R_2C^+_{4z}$, R_2 c_{2x}, R_2 c_{2a}	$(T_1,T_2,0)$ $(T_1,T_2,0)$; $(T_1,T_2,0)(-T_2,T_1,0)$; $(T_1,T_2,0)(T_1,-T_2,0)$; $(T_1,T_2,0)(T_2,T_1,0)$; $(T_1,T_2,0)(-T_1,-T_2,0)$; $(T_1,T_2,0)(T_2,-T_1,0)$; $(T_1,T_2,0)(-T_1,T_2,0)$; $(T_1,T_2,0)(-T_2,T_1,0)$

13	$4mm1^1$	2 mm2 4 4mm	4mm	E,R_2	$(0,0,T_3)\ (0,0,T_3)\ ;(0,0,T_3)(0,0,-T_3)$
	$4mm1^1$	2^1	2^1	$E,\ C^+_{4z},\ R_2,\ R_2$ $C^+_{4z},\ R2,\ \sum_x,$ $R_2,\sum_{da.}$	$(T_1,T_2,0)(-T_2,T_1,0);\ (T_1,T_2,0)(T_2,-T_1,0);$ $(T_1,T_2,0)(-T_1,-T_2,0);\ (T_1,T_2,0)(T_1,T_2,0);$ $(T_1,T_2,0)(T_1,T_2,0)(-T_2,T_1,0)$
	$4mmm1^1$	m (x)	m	$E,\ C^+_{4z},\ C^-_{4z},\ C^-_{2z},$ $R_2,\ R_2\ C^+_{4z},\ R_2\ C^-_{4z},\ R_2\ C_{2z},$	$(0,T_2,T_3)(-T_2,0,T_3);(0,T_2,T_3)(T_2,0,T_3);$ $(0,T_2,T_3)(0,T_2,-T_3);(0,T_2,T_3)(0,-T_2,T_3);$ $(0,T_2,T_3)(T_2,0,-T_3);(0,T_2,T_3)(-T_2,0,-T_3);$ $(0,T_2,T_3)(0,T_2,T_3);(0,T_2,T_3)(0,-T_2,-T_3);$
	$4mm1^1$	$m^1\ m\ 2^1$	$m^1\ m2^1$	$E,\ C^+_{4z},\ R_2,\ R_2$ $C^+_{4z},$	$(0,T_2,0)(,-T_2,0,0);\ (0,T_2,0)(T_2,0,0);$ $(0,T_2,0)\ (0,T_2,0);\ (0,T_2,0)\ (0,-T_2,0)$
14	$\bar{4}2m1^1$	2(P) m 2^12^12(p) mm2 $\bar{4}^1$ $\bar{4}^12^1m$	$\bar{4}^12^1m$	E,R_2	$(0,0,T_3)(0,0,T_3);(0,0,T_3)(0,0,-T_3)$
	$\bar{4}2m1^1$	2(s)	2	$E,\ C_{2z}\ ,s^+_{4z},R_2,$ $R_2C_{2z},\ R_2\ s^+_{4z},$	$(T_1,0,0,)\ (-T_1,0,0);\ (T_10,0,)\ (0,T_1,0);$ $(T_1,0,0,)\ (T_1,0,0);\ (T_1,0,0,)\ (0,-T_1,0);$
	$\bar{4}2m1^1$	2^1(s)	2^1	$E,\ C_{2z}\ ,s^+_{4z},R_2,$ $R_2C_{2z},\ R_2\ s_{4z},$	$(T_1,0,0,)\ (-T_1,0,0);\ (T_10,0,)\ (0,T_1,0);$ $(T_1,0,0,)\ (T_1,0,0);\ (T_1,0,0,)\ (0,-T_1,0);$ $(0,T_2,T_3)(0,T_2,T_3);(0,T_2,T_3)(0,-T_2,T_3);$ $(0,T_2,T_3)(-T_2,0,-T_3);(0,T_2,T_3)(0,-T_2,-T_3);$ $(0,T_2,T_3)(0,T_2,-T_3);(0,T_2,T_3)(T_2,0,-T_3)$
15.	$4/mmm1^1$	1 $\bar{1}^1$	$\bar{1}^1$	$E,\ C^+_{4z},\ C^-_{4z},\ C_{2z},$ $C_{2x}\ ,C_{2y}\ ,\ C_{2a}\ ,$ $C_{2b}\ ,R_2,$ $R_2\ C^+_{4z},\ R_2\ C^-_{4z},$ $R_2\ C_{2z},\ R_2C_{2x}$ $,R_2C_{2y}\ ,\ R_2C_{2a}\ ,$ R_2C_{2b}	$(T_1,T_2,T_3)(T_1,T_2,T_3);$ $(T_1,T_2,T_3)(-T_2,T_1,T_3);(T_1,T_2,T_3)(T_2,-T_1,T_3);$ $(T_1,T_2,-T_3)(-T_1,-T_2,T_3);(T_1,T_2,T_3)(T_2,T_1,-T_3);$ $(T_1,T_2,T_3)(-T_2,-T_1,-T_3);(T_1,T_2,T_3)(-T_1,-T_2,-T_3);$ $(T_1,T_2,T_3)(T_2,-T_1,-T_3);(T_1,T_2,T_3)(-T_2,T_1,T_3);$ $(T_1,T_2,T_3)(T_1,T_2,-T_3);(T_1,T_2,T_3)(-T_2,-T_1,T_3);$ $(T_1,T_2,T_3)(T_2,T_1,T_3);(T_1,T_2,T_3)(T_1,-T_2,-T_3);$ $(T_1,T_2,T_3)(-T_1,T_2,T_3);(T_1,T_2,T_3)(-T_1,T_2,-T_3);$ $(T_1,T_2,T_3)(T_1,-T_2,T_3)$
	$4/mmm1^1$	42^12^1 4mm $\bar{4}2^1m$ $4/m^1mm$ $\bar{4}$ $4/m^1$ 2(p) m^1(p) $2/m^1$(p) 2^12^1 2(p) 2^12^1 2(s) mm2(p) m^1m2^1(ps) 4 mmm^1(p)	$4/m^1mm$	E,R_2	$(0,0,T_3)(0,0,T_3)\ ;(0,0,T_3)(0,0,-T_3)$

16.	31^1	1	1	E, C^+_3,C^-_3 $R_2 c^+_3$, $R_2 c^-_3$	$(T_1,T_2,T_3)(T_1,T_2,T_3)$;$(T_1,T_2,T_3)(-T_2,T_1,-T_2,T_3)$; $(T_1,T_2,T_3)(-T_1,+T_2,-T_1,T_3)$;$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$ $(T_1,T_2,T_3)(T_2,-T_1,+T_2,-T_3)$; $(T_1,T_2,T_3)(T_1,-T_2,+T_1,-T_3)$
	31^1	3	3	E,R_2	$(0,0,T_3)(0,0,-T_3)$;$(0,0,T_3)(0,0,T_3)$
17.	$\bar{3}1^1$	1 $\bar{1}^1$	$\bar{1}^1$	E,R_2, c^+_3, c^-_3 , R_2 c^+_3 , $R_2 c^-_3$	$(T_1,T_2,T_3)(-T_2,T_1,-T_2,T_3)$; $(T_1,T_2,T_3)(-T_1,+T_2,-T_1,T_3)$; $(T_1,T_2,T_3)(T_1,T_2,T_3)$;$(T_1,T_2,T_3)(T_2,-T_1,+T_2,-T_3)$; $(T_1,T_2,T_3)(T_1,-T_2,T_1,-T_3)$
	$\bar{3}\,1^1$	3 $\bar{3}^1$	$\bar{3}^1$	E,R_2	$(0,0,T_3)(0,0,T_3)$; $(0,0,T_3)(0,0,-T_3)$
18	321^1	2	2	E,C^+_3 ,R_2 , R_2 C^+_3	$(0, T_2 ,0,)(-T_2,-T_2,-0)$; $(0, -T_2 ,0)(T_2,T_2,-0)$; $(0, T_2 ,0,)$ $(0, T_2 ,0,)$; $(0, T_2 ,0,)$ $(0, -T_2 ,0,)$
	321^1	2^1 3 32^1	32^1	E,R_2	$(0,0,T_3)(0,0,T_3)$;$(0,0,T_3)(0,0,-T_3)$
19	$3m1^1$	m	m	E,C^+_3 ,R_2 , R_2 C^+_3	$(\frac{(T_1+T_2)}{2} ,0,T_3)$, $(0, \frac{(T_1+T_2)}{2},T_3)$; $(\frac{(T_1+T_2)}{2} ,0,T_3)$, $(\frac{-(T_1+T_2)}{2},0,-T_3)$; $(\frac{(T_1+T_2)}{2} ,0,T_3)$, $(0,-\frac{(T_1+T_2)}{2},T_3)$ $(\frac{(T_1+T_2)}{2} ,0,T_3)$, $(\frac{(T_1+T_2)}{2} ,0,T_3)$
	$3m1^1$	m^1	m^1	E,C^+_3 ,R_2 , R_2 C^+_3	$(0, T_2 ,0,)(-T_2,-T_2,-0)$; $(0, T_2 ,0)(0,-T_2,-0)$; $(0, T_2 ,0,)$ $(T_2, T_2 ,0,)$; $(0, T_2 ,0,)$ $(0, T_2 ,0,)$
	$3m1^1$	3 3m	3m	E,R_2	$(0,0,T_3)(0,0,-T_3)$;$(0,0,T_3)(0,0,T_3)$
20	$\bar{3}m1^1$	2 m^1 $2/m^1$	$2/m^1$	E,C^+_3 ,R_2 , R_2 C^+_3	$(T_1,T_2,0)(T_2, T_2,0)$; $(T_1,T_2,0)(-T_2, -T_2,0)$; $(T_1,T_2,0)(-T_1, -T_2,0)$; $(T_1,T_2,0)(T_2, T_2,0)$

Similarly, ferrotoroidic tensor pairs for all 324 ferrotoroidic Ferroic species are calculated using the above procedure, here tensor pairs for 108 ferroic spices are given table 2 and rest of the tables are available with the authors.

3. Conclusions:

D.B. Litvin has calculated tensor distinction of domains in ferroic crystals in this paper the ferrotoroidic tensor pairs are calculated using double coset decompastion for all the 324 ferrotoroidic ferroic species,where 32 grey groups are the prototypic point groups.

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