Numerical Solution of Heat and Mass Transfer with thermal radiation and MHD Boundary layer flow over a Stretching Surface with Suction/Injection

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Abstract

In this research the researchers studied and made an analysis to the effects of suction/Injection, MHD and thermal radiation on mass transfer characteristics over stretching surface. The magneto hydrodynamic flow over a stretching surface with heat and mass transfer, chemical reaction, radiation has been studied. The system of non-linear differential equations has been obtained and transformed in to set of ordinary differential equations with the help of similarity transformation for the governing flow. This set of ODEs has been solved and results have carried out for different values of the various physical parameters involved in the problem. The results showing the effect of various physical parameters on velocity temperature and concentration profiles have been obtained and presented graphically. The velocity decreases with increasing effect of magnetic parameter while with the increase in radiation parameter, the velocity increases. The mass and momentum transport in laminar boundary layer on moving, stationary and linearly stretching surface has important applications in polymer industry and electrochemistry.

Key Words: Heat and Mass Transfer, MHD flow, chemical reaction, Radiation, Runge-Kutta method with shooting echnique.

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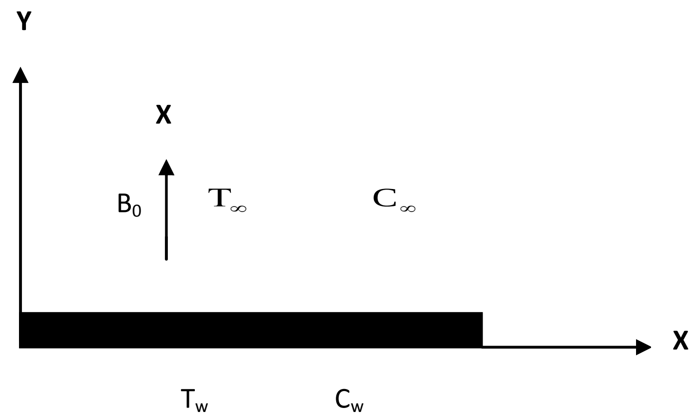
Introduction

The boundary-layer flow over a continuously stretching surface moving with a certain velocity in an otherwise quiescent fluid medium is an often-encountered flow in many engineering processes. There are lots of applications in industries such as the hot rolling, wire drawing, glass fiber production and so on [1]–[3]. The pioneer work in this area was done by Sakiadis [4], [5]. He described the boundary layer assumptions and governing equations of the problem, and the boundary-layer flow on a continuously stretching surface with a certain speed was investigated. His work was further verified by Tsou et al. [6] experimentally. At the same time, the thermal boundary layer for this flow configuration with constant wall temperature was also discussed [6]. For these investigations, the boundary conditions on the surface were extended by other researchers [7]–[11].Fang [12] studied the influence of property variation on the boundary layers of a stretching surface. The effect of thermal radiation on the heat transfer over a nonlinearly stretching sheet immersed in an otherwise quiescent fluid has been studied by Bataller [13]. Exact solution of mass transfer over a stretching surface with chemical reaction and suction/ injection has been studied by Hassan [14]. Joshi and Kumar [15] described the combined effect of chemical reaction, radiation and MHD on mixed convection heat and mass transfer along a vertical moving surface. The purpose of this investigation is to study the effects of MHD, convection and radiation over a stretching surface.

Mathematical Formulation

The steady, laminar, incompressible and viscous fluid on a continuous stretching surface with MHD, convection and radiation has been considered. The fluid properties are assumed to be constant in a limited temperature range. The concentration of diffusing species is very small in comparison to other chemical species, the concentration of species far from the surface. The chemical reactions are taking place in the flow and all physical properties are assumed to be constant. The x-axis runs along the continuous surface in the direction of the motion and the
y-axis is perpendicular to it. Under Boussinesq approximation the equations governing the flow are:



$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \qquad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_1\left(C-C_\infty\right) \qquad (4)$$

where u, v are the velocity components along x and y directions, ν is the kinematics viscosity, k is the thermal diffusivity, β is the volumetric coefficient of expansion for heat transfer, ρ is the density, σ is the electrical conductivity of the fluid, g is acceleration due to gravity, T is the temperature, T_∞ is the temperature of the fluid far away from the surface, C_∞ is the concentration of the fluid far away from the surface, D is the molecular diffusivity, k_1 is the first order chemical reaction, with the boundary conditions:

$$u=Ax \qquad v=v_w \qquad T=T_w \qquad C=C_w \qquad \text{at } y=0 \qquad (5)$$

$$u=0 \qquad T=T_\infty \qquad C=C_\infty \qquad \text{as } y=\infty \qquad (6)$$

Introducing the similarity variables and non-dimensional parameters:

$$\eta=y\sqrt{\frac{A}{\nu}} \qquad \psi=\sqrt{A\nu}\,xf(\eta) \qquad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \qquad \varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

Where , $v=-\sqrt{A\nu}f(\eta)$

$$M=\frac{\sigma B_0^2}{\rho A}, \qquad u=Axf'(\eta), \qquad S_c=\frac{\nu}{D},$$

$$P_r=\frac{\nu}{\alpha}, \qquad R=\frac{4\sigma\sigma_\infty^3}{kx}, \qquad L=\frac{k_1 S_c}{A} \qquad (7)$$

Where Pr is Prandal number, L is chemical reaction parameter, M is magnetic parameter and Sc is Schmidt number. The values of Grashof number for heat transfer and mass transfer are Gr and Gc respectively.
Using equation (7), equations (2), (3) and (4) reduce to:

$$f''' + ff'' - (f')^2 - Mf' = 0 \qquad (8)$$

$$\frac{1}{P_r}\left(1+\frac{4}{3}R\right)\theta'' + f\theta' = 0 \qquad (9)$$

$$\varphi'' + fS_c\varphi' - L\varphi = 0 \qquad (10)$$

The corresponding initial and boundary conditions are:

$$f' = 1,\quad f = f_w,\quad \theta = 1,\quad \varphi = 1, \quad \text{at} \quad \eta = 0$$
$$f' = 0,\quad \theta = 0,\quad \varphi = 0 \quad \text{as} \quad \eta = \infty \qquad (11)$$

Results and discussion

The non-linear ordinary differential equations (8)-(10) with boundary conditions (11) have been solved by using fourth order Runge-Kutta method with shooting technique for various values of physical parameters. The effects of these parameters on the velocity, temperature and concentration profiles have been analyzed with the help of graphical representation through figures 1-8. Figures 1, 2 and 3 show that the velocity decreases with an increase in magnetic parameter, Schmidt number and Prandal number respectively. Figures 4 shows that the velocity increases with an increase in radiation parameter and heat and mass transfer parameter.The temperature increases with an increase in the radiation parameter as shown in figure 5. Figures 6 and 7 show that the concentration decreases with an increase in chemical reaction parameter, and Schmidt number. The temperature decreases with increasing values of the Prandal number as shown in figure 8. There is no significant effect seen in velocity for chemical reaction parameter and in temperature for Schmidt number, chemical reaction parameter and magnetic parameter.

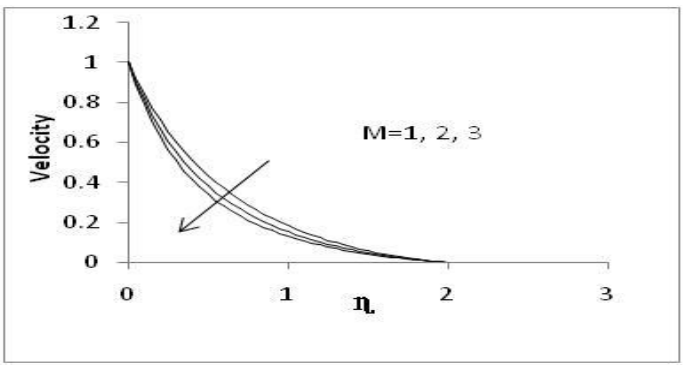


Figure. 1. Velocity profile with Magnetic parameter for Gr=1, Gc=1, R=0.5, Pr=0.71, Sc=0.2, L=0.2

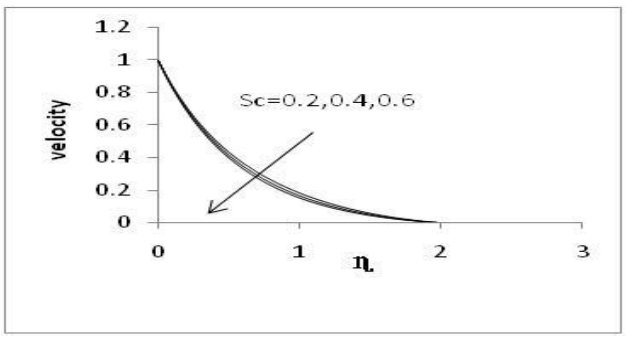


Figure. 2. Velocity profile with Schmidt number for M=1, Gr=1, Gc=1, R=0.5, Pr=0.71, L=0.2

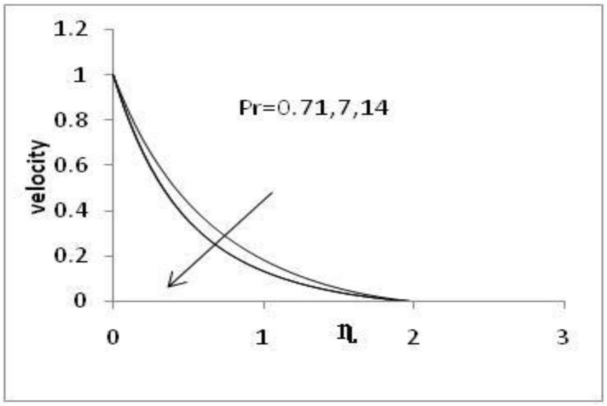


Figure3.Velocity profile with Prandtl

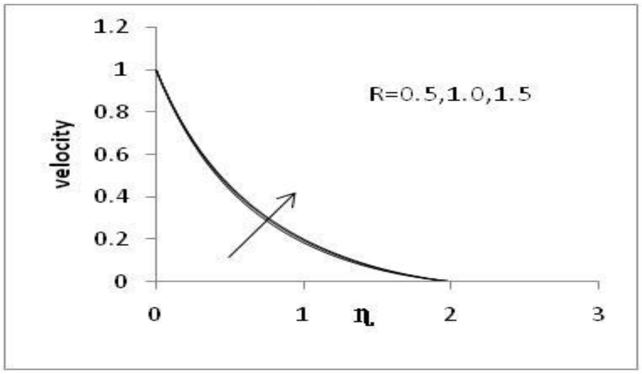


Figure4. Velocity profile with

number for M=1, Gr=1, Gc=1, R=0.5, Sc=0.2, L=0.2

Radiation parameter for M=1, Gr=1, Gc=1, Pr=0.71, Sc=0.2, L=0.2

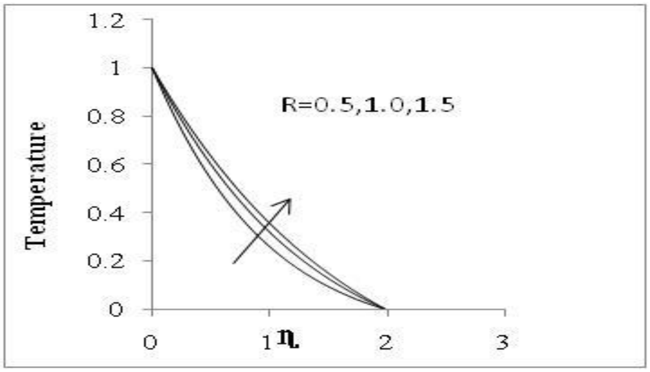


Figure5.Temperature profile with Radiation parameter for M=1, Gr=1,Gc=1, Pr=0.71, Sc=0.2, L=0.2

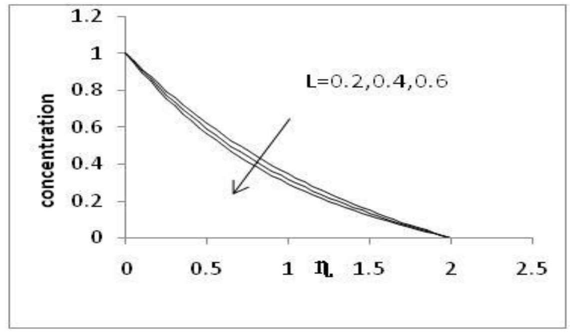


Figure6. Concentration profile with Chemical reaction parameter for M=1, Gr=1,Gc=1,R=0.5, Pr=0.71, Sc=0.2,

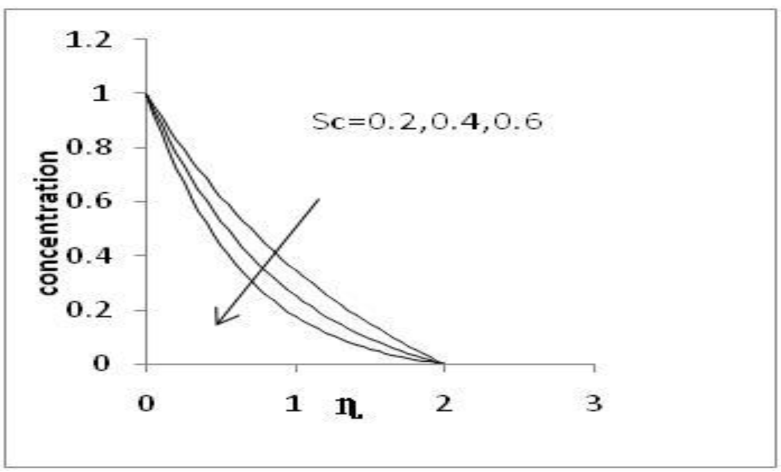


Figure7. Concentration profile with Schmidt number for M=1, Gr=1,Gc=1, R=0.5, Pr=0.71, L=0.2,

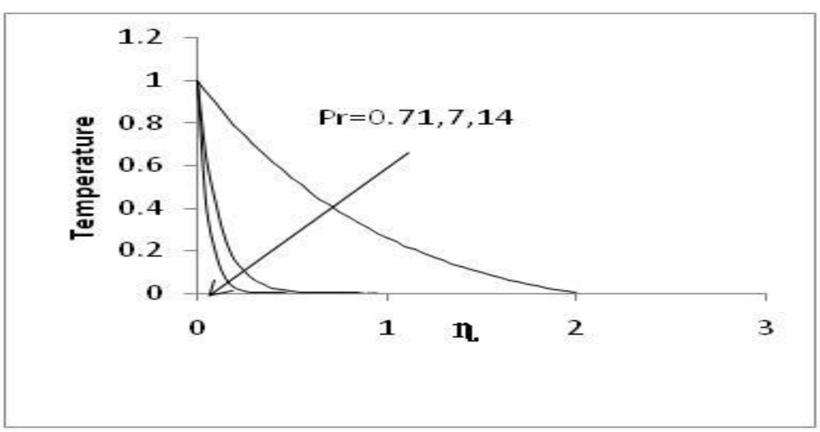


Figure8. Temperature profile with Prandal number for M=1,Gr=1,Gc=1, R=0.5,Sc=0.2, L=0.2,

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