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A Finite Capacity Markovian Queueing System With Catastrophic Effects And Bernoulli Feedbacks

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ABSTRACT
In this paper, we develop and introduce the concept of Bernoulli feedbacks in a single server finite capacity Markovian queueing system with catastrophic effects having faster and slower arrival rates. When the service of a customer is completed, the customer may depart forever from the system or may immediately join as a feedback customer for receiving additional service. This may be repeated any number of times until the customer is satisfied with the service. The system suffers random catastrophe that, once they occur, instantly removes all customers from the system and then server undergoes for a repair process. Service resumes as soon as the server returns from the repair facility. For this model, the steady- state solution and different measures of effectiveness are derived. Finally sensitivity analysis is also carried out to justify the validity of the model and relevant conclusion is

presented.
KEYWORDS:*single-server, finite capacity,faster and slower rate of arrivals, catastrophic effects, Bernoulli feedbacks, sensitivity analysis*
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I. INTRODUCTION

Queueing is a prevalant phenomenon in our daily lives. At this time, queueing theory is very important in studying scheduling and system performance, it is also an all powerfull tool to solve various problems in many complex systems, such as computer systems, telecommunication systems, call centres, flexible manufacturing systems and service systems. During the past few decades, there has been increasing interest in studying queueing systems.On recent years it has been a rapid growth in the literature on queues with server breakdowns and feedback due to their widely applications in manufacturing systems, public telephone booths of coin box type etc.No system is found to be perfect in the real world, since all the devices fail more or less frequently. Thus, the random failures and systematic repair of components of a manufacturing system have a siginificant impact on the output and the productivity of the machining system.In this paper we adopt the more realistic approach of unrelaible server, that is, we admit the possibility that the server is subjected to breakdowns (arrival of virus to the CPU, hardware breakdowns, preventive maintenance, spare replacement etc.)Feedback in queueing literature represents customer dissaitisfaction because of inappropriate quality of service. In case of feedback, after getting partial or incomplete service, customer retries for service.The feedback phenomenon in the queueing system occur in many practical situations such as the problem involving hospital emergency wards handling critical patients and unsatisfied customers in public telephone booths of coin box type etc.The above two concepts (Unreliable server and feedback) can be successfully modeled as a finite capacity Markovian queueing system with catastrophic effects and Bernoulli feedbacks.This work is an extension of the effect of catastrophes on a single server queueing system with finite capacity having faster and slower rate of arrivals.

II. REVIEW OF LITERATURE SURVEY

Queueing models with catastrophes have found more applications. Many authors have been studied certain systems and assuming that they may be subject to catastrophes. The study of the queueing models with service interruption dates back to 1950s. Kalidass and Kasturi[10] was studied a new class of queues with working breakdowns. Avi-Itzhak and Mitrany[2] obtained a steady state M/M/N queueing system where each server is subject to random breakdowns of exponentially distributed duration.Considerable effors have been devoted to study the Bernoulli feedback system by a good number of authors. Takacs [21] was the first to study feedback queueing models, where the customers who completed their service are feedback instantaneously to join the tail of the queue with probability $p(0 \leq p \leq 1)$ or departs from the system forever with probability $q\{= (1 - p)\}$. This mechanism is known as Bernoulli feedback. D'Avignon and Disney [7,8] have analysed queues with instantaneous feedback and single server queues with state dependent feedback. Disney and Dieter Konig [9] obtained stationary queue length and waiting time distributions in a single server feedback queues. Kalyanaraman and Renganathan [12] have studied a single server instantaneous Bernoulli feedback q with multiple vacation. Kalyanaraman and Sumathy [13] have considered a feedback queue with multiple servers and batch service. Santhakumaran and Thangaraj [18] have focused a single server queue with impatient and feedback customers. Santhakumaran and Shanmugasundaram [17] have discussed a preparatory work on arrival customers with a single server feedback queue. Gautam Choudhury and Maduchanda Paul [6] have obtained an M/G/1 queue with two phases of hetrogeneous services and Bernoulli feedback system, where the server provides first phase of regular service to all the customers. Thangaraj and Vanitha [22] have studied on the analysis of M/M/1 feedback queue with catastrophes using continued fraction approach.Shanmugasundaram and Chitra[20] have introduced the time dependent solution of feedback customer with two servers along with catastrophic effect.Chandrasekaran and Saravanarajan [5] has proposed a transient and reliability analysis of single server queue with feedback subject to catastrophes also discussed server failures and repairs. Kalidass and Kasturi [11] have studied an M/G/1 queueing system with two phases of hetrogeneous service and a finite number of immediate Bernoulli feedbacks. Thiagarajan and Srinivasan [23] have obtained various queueing models with controllable arrival rates having interdependent inter arrival and service times. Rani and Srinivasan [16] have discussed a single server interdependent queueing model with controllable arrival rates and feedback. Amina Angelika Bouchentouf and Lahcene Yahiaoui [4] have presented an analysis of a Markovian feedback queueing system with reneging and retention of reneged customers, multiple working vacations and Bernoulli Schedule vacation interruption, where customers' impatience is due to the servers' vacation. Anand Gnana Selvam et al., [19] analysed a study on queueing system with multiple arrivals and Bernoulli feedback subject to catastrophic events. Rajadurai and Chandrasekaran [15] have considered a single server feedback retrial queueing system with multiple working vacations and vacation interruption.Atencia, I.Fortes and Sanchez [1] have discussed a discrete-time Geo/G/1 retrial queue with general retrail times, Bernoulli feedback and the server subjected to starting failures. Ayyappan and Shyamala [3] have developed an $M^{[X]}$/G/1 queue with feedback, random server breakdowns and Bernoulli schedule server vacation with general distribution. Recently, Muthukumaran et.al., [14] analyzed a single server finite capacity Markovian queueing model with faster and slower rate of arrivals, where the service station is subject to failures due to catastrophic events at any time.

III. ASSUMPTIONS AND NOTATIONS OF THE MODEL

The queueing model considered in this paper is based on the following assumptions:

1) The arrivals to a state dependent queueing system occur one by one in accrodance with a Poisson stream withfaster $(\lambda_0 > 0)$ and slower rates $(\lambda_1 > 0)$ and $\lambda_0 > \lambda_1$. Whenever the queue size reaches a certain presicribed number r, the faster arrival rate reduces from λ_0 to slower arrival rate λ_1 and it continues with that rate.

2) There is a single-server who provides the service to all arriving customers. The service times areindependently, identically and exponentially distributed with parameter $\mu(> 0)$

3) The capacity of the system is finite, say N.

4) Customers are served according to first-come, first-served discipline.

5)The service of a customers is unsuccessful only when the queueing systems subjected to catastrophes andfeedback service.

6)Whenever the system fails with Poisson breakdown rate $\gamma(> 0)$, all the present customers are removed fromthe system and then server is sent immediately for repair state Q where the repair times are i.i.d randomvaraibles having exponential distribution function with mean repair rate $\beta(> 0)$. Service resumesimmediately after a repair process completed.

7) For certain reason, if a customer has feedback he joins system queue for receiving feedback service again and again until a successful service completed. If a customer does feedback, he joins the feedback streamwith

probability p. The feedback is assumed to occur instantaneously. If a customer does not feedback, hejoins the departure process forever with probability q with $p + q = 1$. It is assumed that there is no difference between the regular arrival and feedback arrival.

8) It is also assumed that various stochastic processes involved in the system are independent of each other.

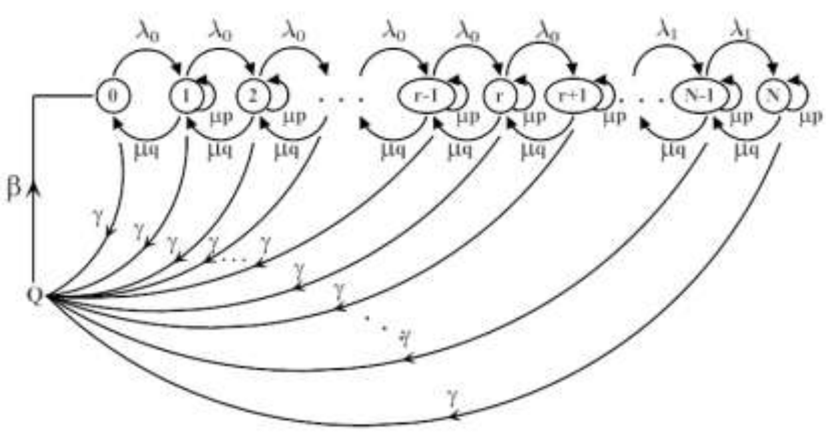

FIGURE 1. Transition Rate Diagram

IV. STEADY- STATE SOLUTION

In this section, we present the steady-state solution of the model. Let $P_{i,n}$ be the probability that there are 'n' customers in the system when the system has arrival rate λ_i ; $i = 0,1$.

Now in view of our queueing system subject to catastrophic effect and feedback mechanism, the govering equations for the faster arrival rate λ_0 and the slower arrival rate λ_1 are as follows.

$$(\lambda_0 + \gamma)P_{0,0} = \mu q P_{0,1} + \beta Q \qquad (4.1)$$

$$(\lambda_0 + \mu + \gamma)P_{0,m-1} = \lambda_0 P_{0,m-2} + \mu q P_{0,m} + \mu p P_{0,m-1};\ 2 \le m \le r \qquad (4.2)$$

$$(\lambda_0 + \mu + \gamma)P_{0,r} = \lambda_0 P_{0,r-1} + \mu q P_{1,r+1} + \mu p P_{0,r} \qquad (4.3)$$

$$(\lambda_1 + \mu + \gamma)P_{1,r+1} = \lambda_0 P_{0,r} + \mu q P_{1,r+2} + \mu p P_{1,r+1} \qquad (4.4)$$

$$(\lambda_1 + \mu + \gamma)P_{1,m} = \lambda_1 P_{1,m-1} + \mu q P_{1,m+1} + \mu p P_{1,m};\ \ r + 2 \le m \le N - 1 \qquad (4.5)$$

$$(\mu + \gamma)P_{1,N} = \lambda_1 P_{1,N-1} + \mu p P_{1,N} \qquad (4.6)$$

$$\beta Q = \gamma\left[\sum_{n=0}^{r} P_{0,n} + \sum_{n=r+1}^{N} P_{1,n}\right] \qquad (4.7)$$

Determination of Probability Generating Function (PGF) in both Faster and Slower Arrival Rates.

In this sub-section, we define the PGF for faster and slower rate of arrivals repectively.

$$h_0(z) = \sum_{n=0}^{r} P_{0,n} z^n$$

$$and h_1(z) = \sum_{n=r+1}^{N} P_{1,n} z^n$$

Multiplying by z^n for $n = 0,1,2,\cdots,r-1,r$ in respective steady - state equations (4.1) to (4.3) and then summing over n, we get

$$(\lambda_o + \mu + \gamma)h_0(z) - \mu P_{0,0} = [\mu q P_{0,1} + \cdots + \mu q P_{0,r} z^{r-1}]$$
$$+[\lambda_0 + P_{0,0} z + \cdots + \lambda_0 P_{0,r-1} z^r]$$
$$+\mu q P_{1,r+1} z^r + \mu p[P_{0,1} z + \cdots + P_{0,r} z^r] + \beta Q$$
$$= \frac{\mu q}{z}[P_{0,1} z + \cdots + P_{0,r} z^r]$$
$$+\lambda_0 z[P_{0,0} z + \cdots + P_{0,r-1} z^{r-1}]$$
$$+\mu q P_{1,r+1} z^r + \mu p[P_{0,1} z + \cdots + P_{0,r} z^r] + \beta Q$$
$$= \frac{\mu q}{z}[h_0(z) - P_{0,0}] + \lambda_0 z[h_0(z) - P_{0,r} z^r]$$
$$+\mu q P_{1,r+1} z^r + \mu p[h_0(z) - P_{0,0}] + \beta Q$$
$$(\lambda_o + \mu + \gamma) z h_0(z) - \mu z P_{0,0} = \mu q h_0(z) - \mu q P_{0,0} + \lambda_0 z^2 h_0(z) - \lambda_0 P_{0,r} z^{r+2}$$
$$+\mu q P_{1,r+1} z^{r+1} + \mu p z h_0(z) - \mu p z P_{0,0} + \beta Q z$$
$$[\lambda_0 z^2 - (\lambda_0 + \mu + \gamma)z + \mu q + \mu p z] h_0(z) = \mu p z P_{0,0} + \mu q P_{0,0} - \mu z P_{0,0} + \lambda_0 P_{0,r} z^{r+2}$$
$$-\mu q P_{1,r+1} z^{r+1} - \beta Q z$$
$$[\lambda_0 z^2 - (\lambda_0 + \mu - \mu p + \gamma)z + \mu q] h_0(z) = \mu[pz + q - z] P_{0,0} +$$
$$\lambda_0 P_{0,r} z^{r+2} - \mu q P_{1,r+1} z^{r+1} - \beta Q z$$
$$[\lambda_0 z^2 - (\lambda_0 + \mu q + \gamma)z + \mu q] h_0(z) = -\mu q(z-1) P_{0,0} + \lambda_0 P_{0,r} z^{r+2}$$
$$-\mu q P_{1,r+1} z^{r+1} - \beta Q z$$

$$Hence\, h_0(z) = \frac{-\mu q\,(z-1)P_{0,0}+\lambda_0 P_{0,r} z^{r+2}-\mu q\, P_{1,r+1} z^{r+1}-\beta Q z}{\lambda_0 z^2-(\lambda_0+\mu q+\gamma)z+\mu q} \quad (4.8)$$

The denominator of RHS of equation (4.8) put to zero

$$\lambda_0 z^2 - (\lambda_0 + \mu q + \gamma)z + \mu q = 0$$

has two positive roots

$$z_{11} = \frac{\omega_0 - \sqrt{\omega_0^2 - 4\lambda_0 \mu q}}{2\lambda_0} > 1$$

$$and\, z_{12} = \frac{\omega_0 + \sqrt{\omega_0^2 - 4\lambda_0 \mu q}}{2\lambda_0} < 1$$

$$where\, \omega_0 = \lambda_0 + \mu q + \gamma$$

As we know the fact that the numerator of (4.8) must vanish for $z = z_{11}$ and $z = z_{12}$. we get

$$-\mu q(z_{11} - 1)P_{0,0} + \lambda_0 P_{0,r} z_{11}^{r+2} - \mu q P_{1,r+1} z_{11}^{r+1} - \beta Q z_{11} = 0 \quad (4.9)$$

$$-\mu q(z_{12} - 1)P_{0,0} + \lambda_0 P_{0,r} z_{12}^{r+2} - \mu q P_{1,r+1} z_{12}^{r+1} - \beta Q z_{12} = 0 \quad (4.10)$$

Multiplying through steady - state equations (4.4) to (4.6) by appropriate powers of z^n for $n = r+1, r+2, \cdots, N-1, N$ and then summing over n and proceeding as in the earlier case of faster arrival rate λ_0, we get

$$(\lambda_1 + \mu + \gamma)h_1(z) - \lambda_1 P_{1,N} z^n = \lambda_0 P_{0,r} z^{r+1} + \mu q[P_{1,r+2} z^{r+2} + \cdots + P_{1,N} z^{N-1}]$$
$$+\lambda_1[P_{1,r+1} z^{r+2} + \cdots + P_{1,N-1} z^N]$$
$$+\mu p[P_{1,r+1} z^{r+1} + \cdots + P_{1,N} z^N]$$
$$= \lambda_0 P_{0,r} z^{r+1} + \frac{\mu q}{z}[P_{1,r+2} z^{r+2} + \cdots + P_{1,N} z^N]$$
$$+\lambda_1 z[P_{1,r+1} z^{r+1} + \cdots + P_{1,N-1} z^{N-1}]$$
$$+\mu p[P_{1,r+1} z^{r+1} + \cdots + P_{1,N} z^N]$$
$$(\lambda_1 + \mu + \gamma)h_1(z) - \lambda_1 P_{1,N} z^N = \lambda_0 P_{0,r} z^{r+1} + \frac{\mu q}{z}[h_1(z) - P_{1,r+1} z^{r+1}]$$
$$+\lambda_1 z[h_1(z) - P_{1,N} z^N] + \mu p h_1(z)$$
$$(\lambda_1 + \mu + \gamma) z h_1(z) - \lambda_1 P_{1,N} z^{N+1} = \lambda_0 P_{0,r} z^{r+2} + \mu q h_1(z) - \mu q P_{1,r+1} z^{r+1}$$
$$+\lambda_1 z^2 h_1(z) - \lambda_1 P_{1,N} z^{N+2} + \mu p z h_1(z)$$
$$[\lambda_1 z^2 - (\lambda_1 + \mu + \gamma)z + \mu q + \mu p z]h_1(z) = -\lambda_0 P_{0,r} z^{r+2} + \mu q P_{1,r+1} z^{r+1}$$
$$+\lambda_1 P_{1,N} z^{N+2} - \lambda_1 P_{1,N} z^{N+1}$$
$$[\lambda_1 z^2 - (\lambda_1 + \mu - \mu p + \gamma)z + \mu q]h_1(z) = -\lambda_0 P_{0,r} z^{r+2} + \mu q P_{1,r+1} z^{r+1}$$
$$+\lambda_1 (z-1) P_{1,N} z^{N+1}$$
$$[\lambda_1 z^2 - (\lambda_1 + \mu q + \gamma)z + \mu q]h_1(z) = -\lambda_0 P_{0,r} z^{r+2} + \mu q P_{1,r+1} z^{r+1}$$
$$+\lambda_1 (z-1) P_{1,N} z^{N+1}$$
$$Hence\, h_1(z) = \frac{-\lambda_0 P_{0,r} z^{r+2}+\mu q\, P_{1,r+1} z^{r+1}+\lambda_1(z-1)P_{1,N} z^{N+1}}{\lambda_1 z^2-(\lambda_1+\mu q+\gamma)z+\mu q} \quad (4.11)$$

Whenever the denominator of RHS of equation (4.11) has zeros, the two positive roots of the denominator are as given below.

$$z_{21} = \frac{\omega_1 - \sqrt{\omega_1^2 - 4\lambda_1 \mu q}}{2\lambda_1} > 1$$

$$and\, z_{22} = \frac{\omega_1 + \sqrt{\omega_1^2 - 4\lambda_1 \mu q}}{2\lambda_1} < 1$$

$$where\, \omega_1 = \lambda_1 + \mu q + \gamma$$

Putting $z = z_{21}$ and $z = z_{22}$ in the numerator of (4.11) and equated to zero as in the case of faster arrival rate, we have

$$-\lambda_0 P_{0,r} z_{21}^{r+2} + \mu q P_{1,r+1} z_{21}^{r+1} + \lambda_1 (z_{21} - 1) P_{1,N} z_{21}^{N+1} = 0 \quad (4.12)$$

$$-\lambda_0 P_{0,r} z_{22}^{r+2} + \mu q P_{1,r+1} z_{22}^{r+1} + \lambda_1 (z_{22} - 1) P_{1,N} z_{22}^{N+1} = 0 \quad (4.13)$$

The condition of normality $h_0(1) + h_1(1) + Q = 1$ yields,

$$\frac{1}{(-\gamma)}(\lambda_0 P_{0,r} - \mu q P_{1,r+1} - \beta Q) + \frac{1}{(-\gamma)}(-\lambda_0 P_{0,r} + \mu q P_{1,r+1}) + Q = 1$$
$$\frac{\beta Q}{\gamma} + Q = 1$$
$$\beta Q + \gamma Q = \gamma$$
$$(\beta + \gamma)Q = \gamma \quad (4.14)$$

Thus we get equations from (4.9),(4.10),(4.12),(4.13) and (4.14)

$$\mu q(z_{11} - 1)P_{0,0} - \lambda_0 P_{0,r} z_{11}^{r+2} + \mu q P_{1,r+1} z_{11}^{r+1} + \beta Q z_{11} = 0$$

$$\mu q(z_{12}-1)P_{0,0}-\lambda_0 P_{0,r} z_{12}^{r+2}+\mu q P_{1,r+1} z_{12}^{r+1}+\beta Q z_{12}=0$$
$$\lambda_0 P_{0,r} z_{21}^{r+2}-\mu q P_{1,r+1} z_{21}^{r+1}-\lambda_1 (z_{21}-1)P_{1,N} z_{21}^{N+1}=0$$
$$\lambda_0 P_{0,r} z_{22}^{r+2}-\mu q P_{1,r+1} z_{22}^{r+1}-\lambda_1 (z_{22}-1)P_{1,N} z_{22}^{N+1}=0$$
$$(\beta+\gamma)Q-\gamma=0$$

Solvig the above set of equations , we get

$$P_{0,0}=\frac{N_1}{D}$$
$$P_{0,r}=\frac{N_2}{D}$$
$$P_{1,r+1}=\frac{N_3}{D}$$
$$P_{1,N}=\frac{N_4}{D}$$
$$Q=\frac{N_5}{D}$$

$$where \quad D=-(\gamma+\beta)\lambda_0\lambda_1\mu^2 q^2\{A[z_{21}^{r+2}(1-z_{22})z_{22}^{N+1}-z_{22}^{r+2}(1-z_{21})z_{21}^{N+1}] \\ +B[z_{21}^{r+1}(1-z_{22})z_{22}^{N+1}-z_{22}^{r+1}(1-z_{21})z_{21}^{N+1}]\}$$
$$here \quad A=[-z_{11}^{r+1}(z_{21}-1)+z_{12}^{r+1}(z_{11}-1)]$$
$$B=[z_{11}^{r+2}(z_{12}-1)-z_{12}^{r+2}(z_{11}-1)]$$
$$N_1=\gamma\beta\lambda_0\lambda_1\mu q\{z_{12}[(1-z_{22})Z_{22}^{N+1}(-z_{11}^{r+1}z_{21}^{r+1}(z_{21}-z_{11})) \\ -(1-z_{21})z_{21}^{N+1}(-z_{11}^{r+1}z_{22}^{r+1}(z_{22}-z_{11}))] \\ -z_{11}[(1-z_{22})z_{22}^{N+1}(-z_{11}^{r+1}z_{21}^{r+1}(z_{21}-z_{12})) \\ -(1-z_{21})z_{21}^{N+1}(-z_{12}^{r+1}z_{22}^{r+1}(z_{22}-z_{12}))]\}$$
$$N_2=\lambda_1\mu^2 q^2\gamma\beta(z_{11}-z_{12})\{z_{21}^{r+1}(1-z_{22})z_{22}^{N+1}-z_{22}^{r+1}(1-z_{21})z_{21}^{N+1}\}$$
$$N_3=\lambda_0\lambda_1\mu q\gamma\beta(z_{11}-z_{12})\{z_{21}^{r+2}(1-z_{22})z_{22}^{N+1}-z_{22}^{r+2}(1-z_{21})z_{21}^{N+1}\}$$
$$N_4=\lambda_0\mu^2 q^2\gamma\beta(z_{11}-z_{12})(z_{21}-z_{22})z_{21}^{r+1}z_{22}^{r+1}$$
$$N_5=-\gamma\lambda_0\lambda_1\mu^2 q^2\{A[z_{21}^{r+2}(1-z_{22})z_{22}^{N+1}-z_{22}^{r+2}(1-z_{21})z_{21}^{N+1}] \\ +B[z_{21}^{r+1}(1-z_{22})z_{22}^{N+1}-z_{22}^{r+1}(1-z_{21})z_{21}^{N+1}]\}$$
$$where\ A\ \&\ B\ are\ same\ as\ in\ D.$$

V. SYSTEM PERFORMANCE MEASURES

In this section, we derive the various analytical expression for the system characteristics.

5.1 Proportion of time the server being in a state when the arrival rate is faster with catastrophes and Bernoulli feedbacks.

From equation (4.8), we get

$$h_0(1)=\frac{1}{\gamma}(\beta Q+\mu q P_{1,r+1}-\lambda_0 P_{0,r})$$

5.2 Proportion of time the server being in a state when the arrival rate is slower with catstrophes and Bernoulli feedbacks.

From equation (4.11), we get

$$h_1(1)=\frac{1}{\gamma}(\lambda_0 P_{0,r}-\mu q P_{1,r+1})$$

5.3 Expected number of customers in the system L_{s_0}, when the system is faster rate of arrivals with catastrophes and Bernoulli feedbacks.

$$L_{s_0}=h_{0'}(1)=\frac{-1}{\gamma^2}[\gamma\{(r+2)\lambda_0 P_{0,r}-(r+1)\mu q P_{1,r+1}-\mu q P_{0,0}-\beta Q\} \\ +\{(\lambda_0 P_{0,r}-\mu q P_{1,r+1}-\beta Q)(\lambda_0-\mu q-\gamma)\}]$$

5.4 Expected number of customers in the system L_{s_1} , when the sytem is in the slower rate of arrivals with catastrophes and Bernoulli feedbacks.

$$L_{s_1}=h_{1'}(1)=\frac{-1}{\gamma^2}[\gamma\{\lambda_1 P_{1,N}+(r+1)\mu q P_{1,r+1}-(r+2)\lambda_0 P_{0,r}\} \\ +\{(\mu q P_{1,r+1}-\lambda_0 P_{0,r})(\lambda_1-\mu q-\gamma)\}]$$

5.5 Expected number of customers in the system L_s is from (5.1) and (5..2), we have

$$L_s=L_{s_0}+L_{s_1}$$

5.6 Expected waiting time W_s of a customer in the system is

$$W_s=W_{s_0}+W_{s_1}$$
$$where W_{s_0}=\frac{L_{s_0}}{\lambda_0}\&W_{s_1}=\frac{L_{s_1}}{\lambda_1}$$

5.7 Probability that the server is in failure rate is

$Q = \frac{N_5}{D}$

where N_5 and D are given in section 4.
5.8 Probability that the server is in active is given by 1-Q, where Q is stated as above.
Special case:
This model includes the certain model as particular case. When q=1 and p=0, this model reduces to the effect of catastrophes on a single server Queueing system with finite capacity which was recently discussed by Muthukumaran et al.,[14]

VI. SENSITIVITY ANALYSIS

In this section, we present sensitivity analysis of the model. A system analyst often concerns with how the system performance measures can be affected by the changes of the input parameters in the investigated queueing service model. Sensitivity investigation on queueing model with various input parameters may provide some answers to this question.First we fix the following parameters as $(r, N, \lambda_1, \mu, \gamma, \beta) = (5,10,0.5,1.5,0.3,0.46)$ and consider the follwing figures.

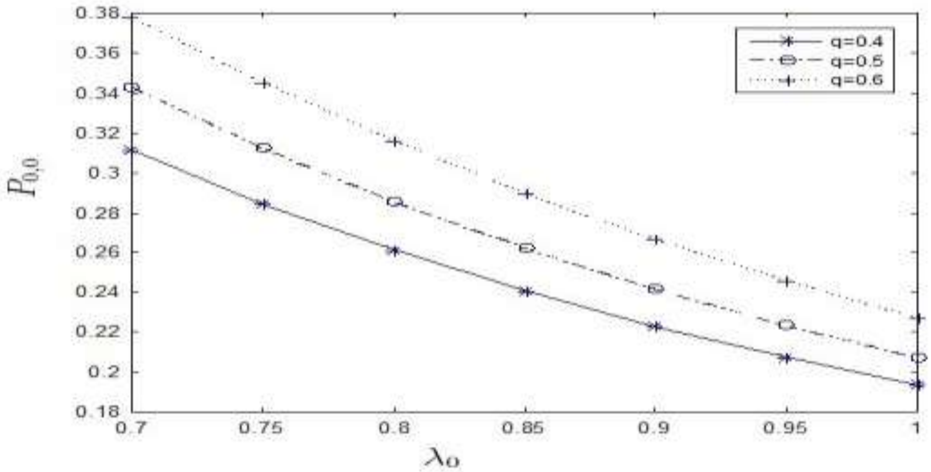

FIGURE 2. $P_{0,0}$ vs λ_0 for different values of μ

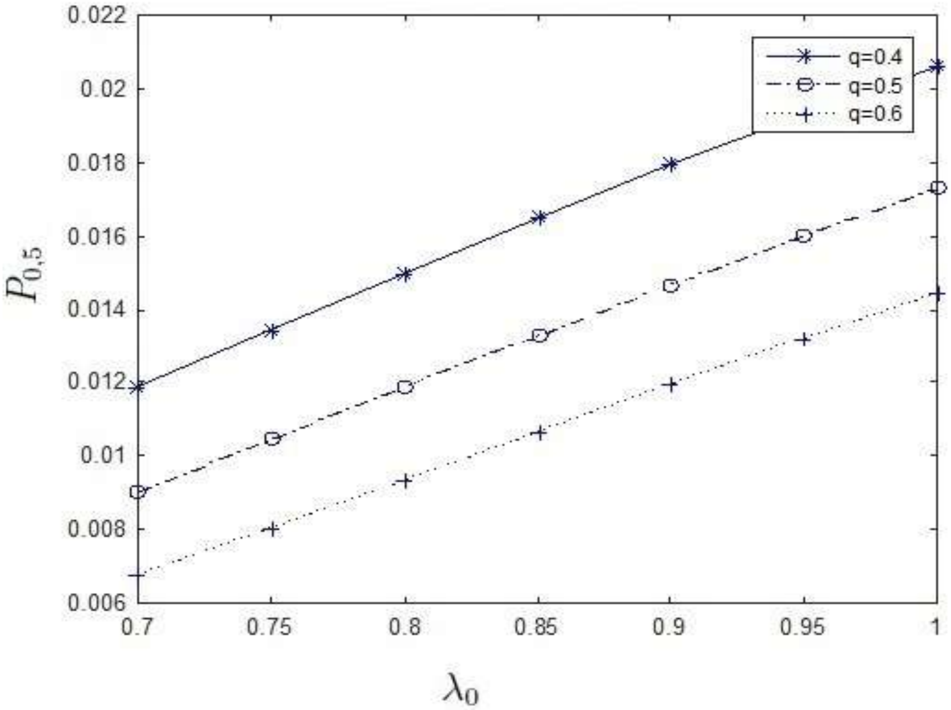

FIGURE 3. $P_{0,5}$ vs λ_0 for different values of μ

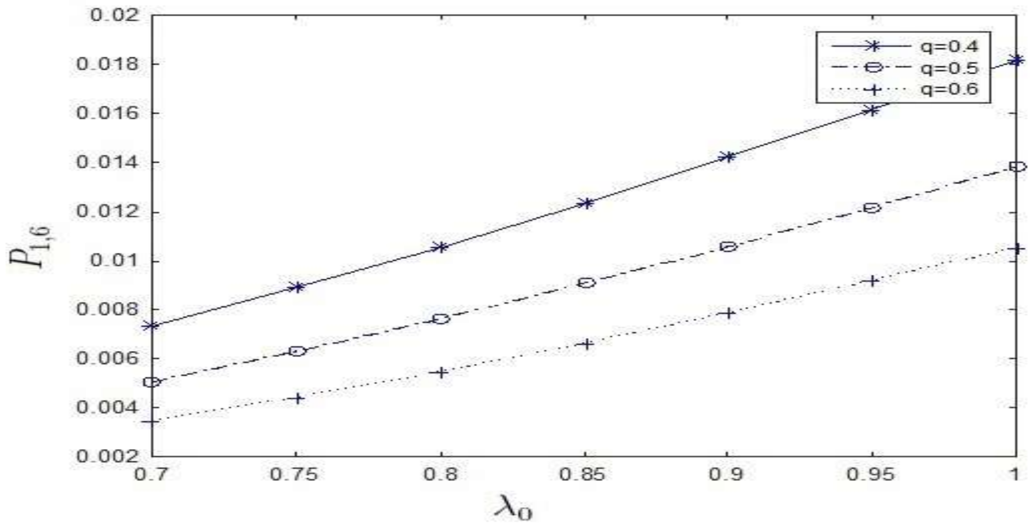

FIGURE 4. $P_{1,6}$ vs λ_0 for different values of μ

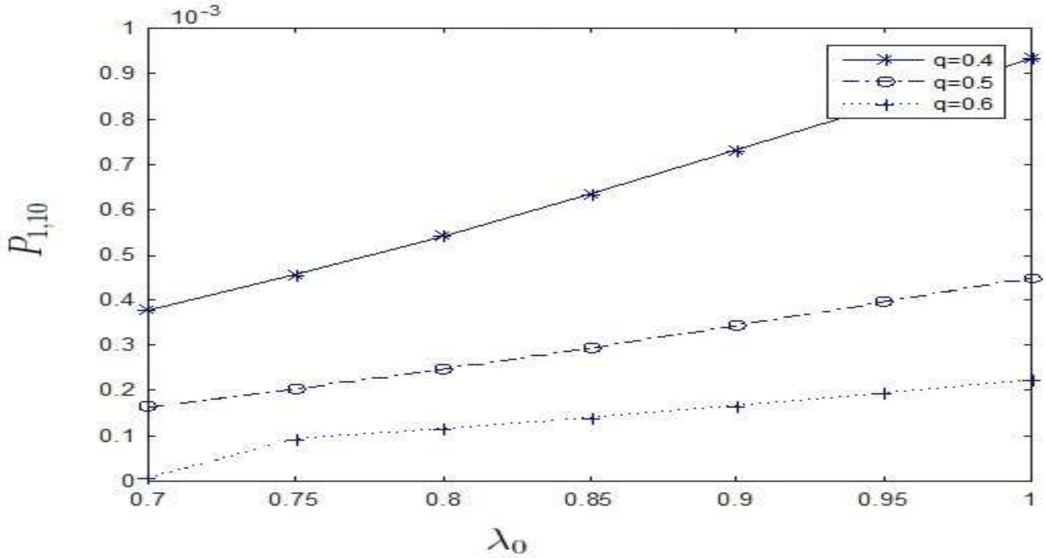

FIGURE 5. $P_{1,10}$ vs λ_0 for different values of μ

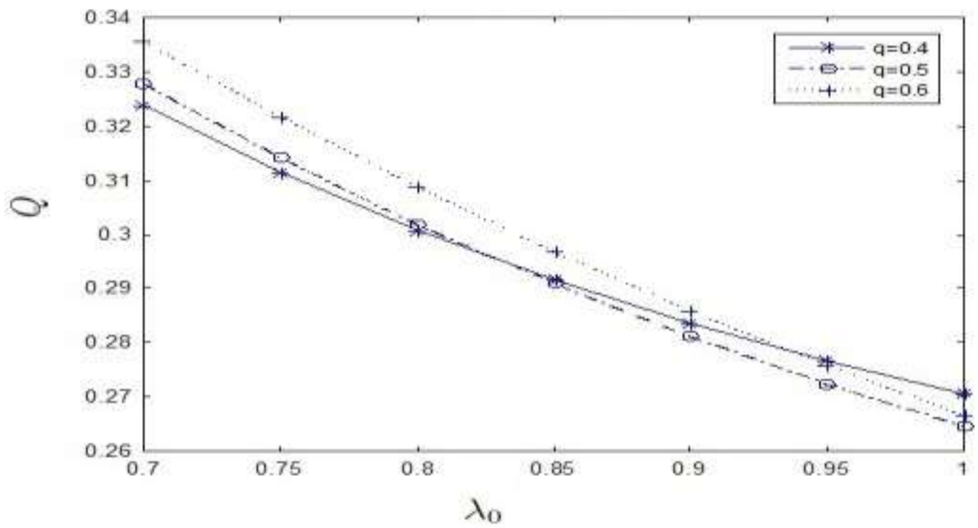

FIGURE 6. Q vs λ_0 for different values of μ

It is obsereved from the figures 2-6 that, when the customer does not feedback with the different probabilities $q = \{0.4, 0.5, 0.6\}$ theprobability values $P_{0,5}, P_{1,6}, P_{1,10}$ increases as λ_0 increases whereas $P_{0,0}$ and Q decreases as λ_0 increases while the other parameters are kept fixed.

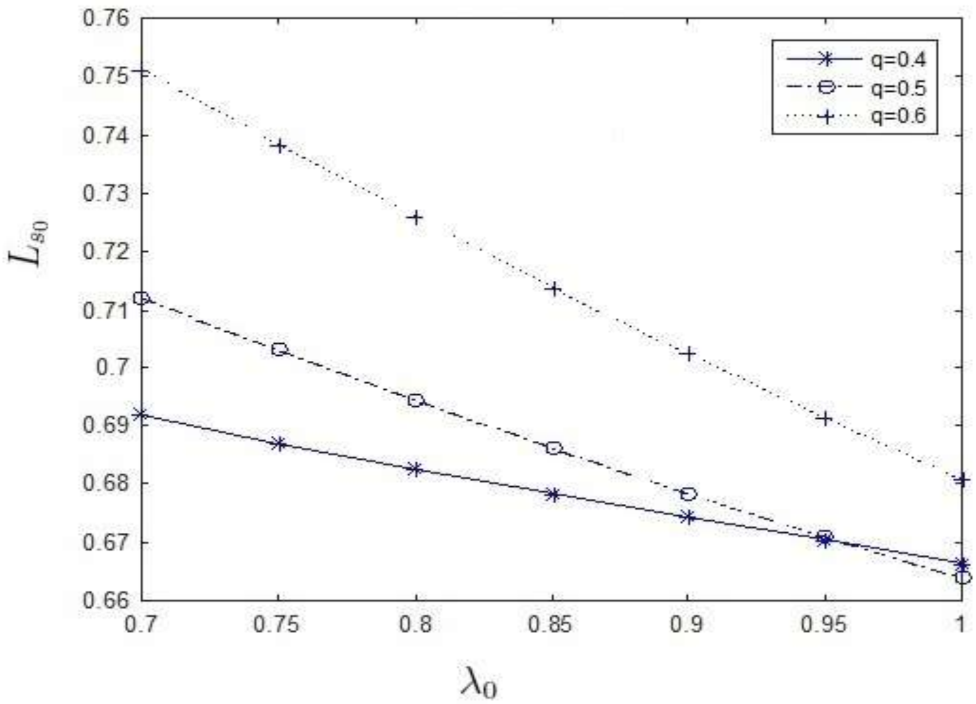

FIGURE 7. L_{s_0} vs λ_0 for different values of μ.

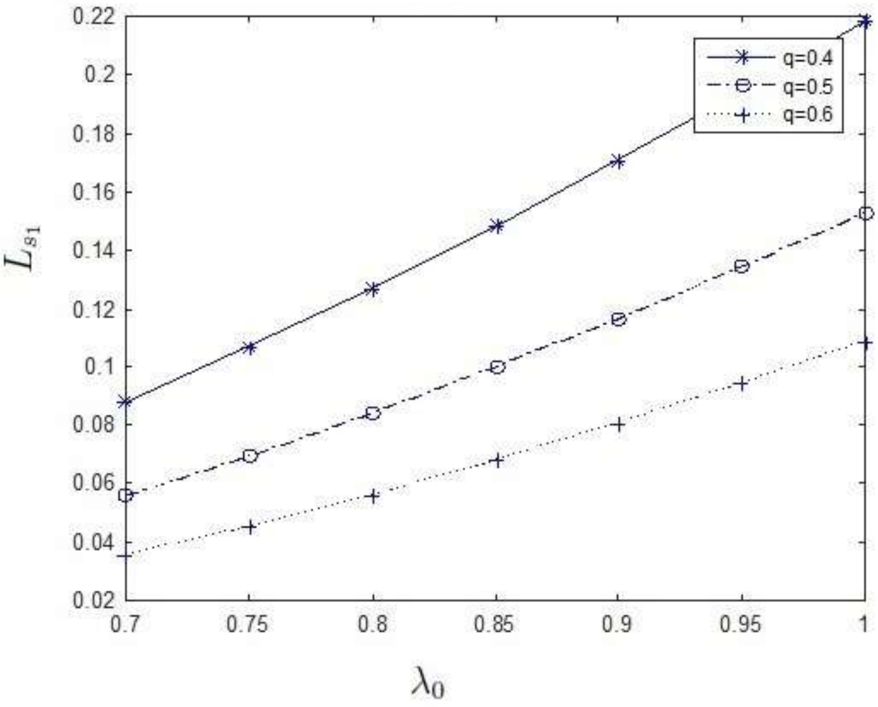

FIGURE 8. L_{s_1} vs λ_0 for different values of μ.

In the above figures 7 and 8 represents the variation in expected system size, L_{s_0} and L_{s_1} with respect to the faster arrival rate, λ_0. There is a proportionate decrease in L_{s_0} and increases in L_{s_1} for increasing values of λ_0 (the other parameters are kept fixed).

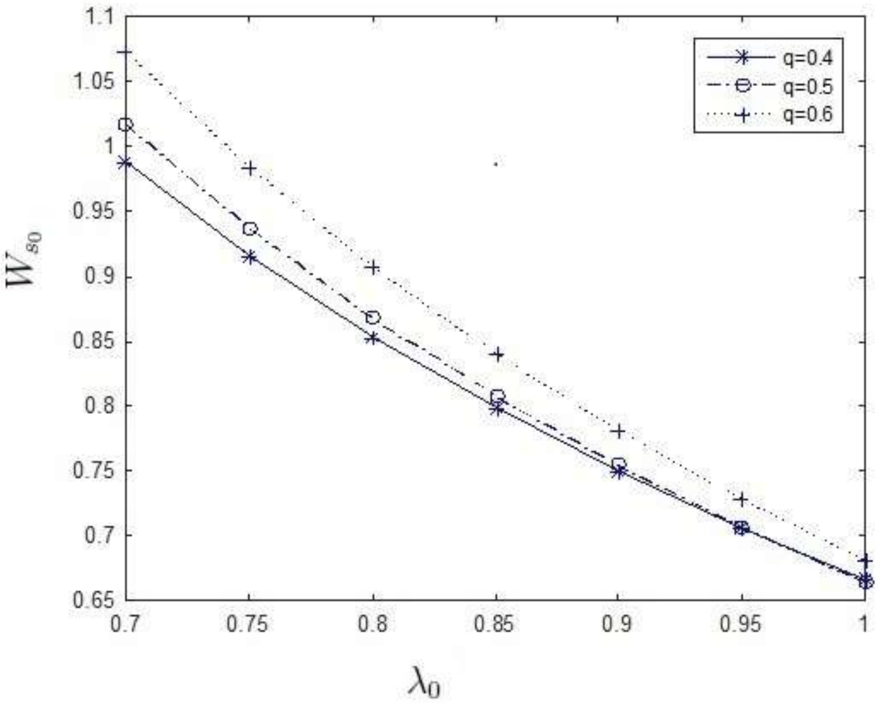

FIGURE 9. W_{s_0} vs λ_0 for different values of μ.

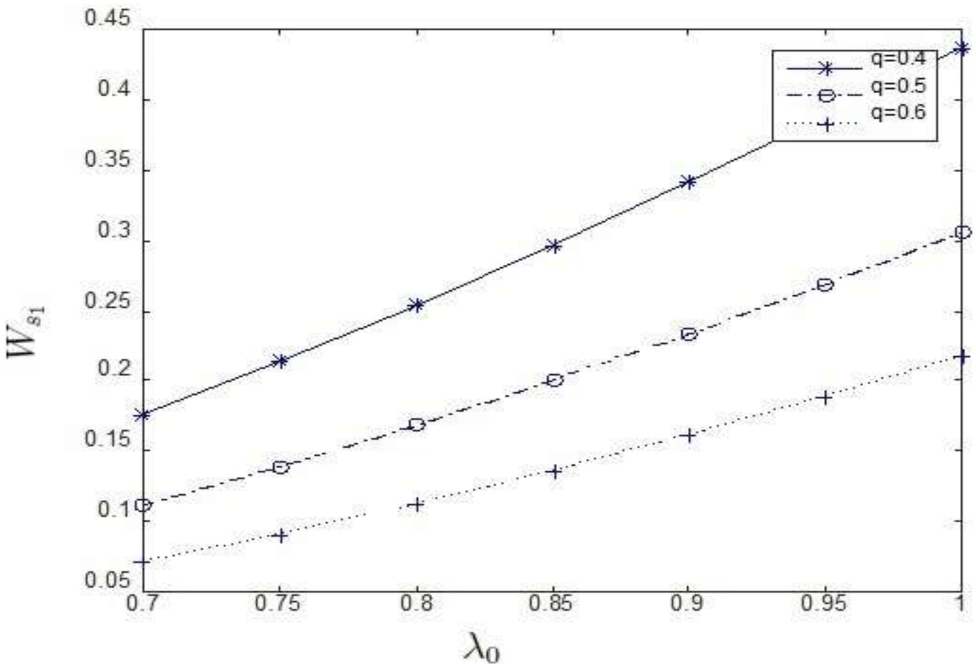

FIGURE 10. W_{s_1} vs λ_0 for different values of μ.

In the above figures 9 and 10, depicts the variation in expected waiting time, W_{s_0} and W_{s_1} with respect to the faster arrival rate, λ_0. When the faster arrival rate increases (the other parameters are kept fixed) the expected waiting time W_{s_0} decreases whereas W_{s_1} increases.

VII. CONCLUSION

Thus in this paper, we develop and incorporate the concept of Bernoulli feedbacks into an M/M/1/N queueing system having faster and slower arrival rates under catastrophic effects. The PGF technique is used in this paper efficiently to obtain the steady-state solution of the model. We have also derived various performance measures and moreover the particular case is discussed to highlight the confirmity of the corresponding result of a non-reliable server finite capacity Markovian queueing system which was recently studied by Muthukumaran et.al.,[14]. Further the validity of this model is justified with the help of the graphs.

Applications: The model studied in this paper can also be applied in various sector such as business sector, telecommunication sector and health care etc. for facilitating quality of service interms of reduced waiting time for many congestion situations encountered by the customers.

Future Research: The future scope of the idea of this model can be the application of fuzzy concept and finding out the crisp outputs and the investigation done in this paper can be extended by incorporating the cocepts of vacation, expected busy periods of the server, bulk arrival and / or bulk service.

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