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Mathematical modeling on conservation and production of animal species

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ABSTRACT: *In a multispecies network, the interaction of biotic and abiotic systems in both plant and animal kingdom play a key role for their stability. Most of the warm blood animals including human beings depend upon both plants and animals to meet out carbohydrates, minerals, proteins and other nutrients for their physiological and developmental performances. Thus the demand and conservation rate of both animals and plants species is one of the challenging research area in the present scenario. Human beings mostly living in cold zones, slaughter animals for food purposes to compete with the adverse cold conditions. Thus, it is imperative to study the conservation and production of animal species with optimal slaughter rates. In this direction a mathematical model has been formulated to understand the behaviour and stability of animal species under slaughtering conditions. Slaughter and Conservation rates of animal populations has been established by*

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I. INTRODUCTION

One of the important phenomena in real life is to maintain the stability in the existing system. The survival and well-being of a nation depend on sustainable development. It is a process of social and economic betterment that satisfies the needs and values of all interest groups without foreclosing future options. To this end, we must ensure that the demand on the environment from which we derive our sustenance, does not exceed its carrying capacity for the present as well as future generations.

The human beings together with other animal species living in polar and cold regions are mainly dependent on animal protein. Thus, it is important to study the production and conservation of animal species in these regions during the process of slaughtering. The live-stocks meant for slaughtering to meet out the demands of Predator population can continuously decline, so it is imperative to follow a method by virtue of which, we can easily get the maximum production from the process and ensure that the prey populations do not die out. In this direction, a mathematical model describing the conservation and optimal slaughtering for maximum profit in animal populations has been presented.

Population is an important resource for development, yet it is a major source of environmental degradation when it exceeds the threshold limits of the support systems. Unless the relationship between the multiplying population and life support systems can be stabilized, development programmes, however, innovative, are not likely to yield the desired results. It is possible to expand the “carrying capacity' through technological advances and spatial distribution as discussed by Moyo [1994].

The scenario for the coming years is alarming and we are likely to face food crisis unless we are in a position to increase crop and animal productivity on a continuing basis, since the only option open to us for increasing production is productivity improvement. Also, access to food will have to be ensured through opportunities for productive employment.

The population comprising of omnivores depend on plants as well as on animals. The animals residing at cold places can get rid from the consequences of the cold by taking meat of slaughtered animal species. The nutritive support for human beings in terms of animals is the best example to compete with the cold and other diseases. The slaughtering of animal populations is mainly dependent on the production, habitat and ratio between their survival and production. As we know there is a continuous slaughtering of animal population taking place throughout the year, but on some occasions there is a rapid decline of animals for stock and for the conservation. Thus, it is our moral responsibility to maintain the stability in conservation of animal species for their survival as well as for the human beings of future generations. In order to address such issues, we have developed a mathematical model based on the studies of Rasool N. et al (2012) for analysing the effect of slaughtering effect on three categories of animal populations. We have classified the animal species into pre-reproductive, reproductive and post-reproductive groups.

II. MATERIALS AND METHODS

Convention of biological diversity research carried out by Brownlee [1977] is the one of the developments in this type of research. Hammond [1994] gives an insight to study the modelling slaughtering for conservation and production. The other researchers including Newman [1998] and Buckland et al, [2004] for state-space modeling of animal movement and mortality with application to salmon and for the dynamics of wild animal populations respectively. Miller et al [2002] extended this work by considering density dependent matrix model for grey wolf population projection. The recent work done in this direction by Rasool N. et al [2012] has incorporated the slaughtering of the animal population for conservation purposes. The study involving slaughtering process and production has not being carried out by any researcher so far, so the main focus of this study will be conservation of animal species in which we shall make use of linear algebra models and estimate the behaviour of three categories in animal population by using eigenvalue approach. The stability analysis for the three groups has been extensively studied out by means of various mathematical conditions.

III. MATHEMATICAL MODEL

Let $x_1(t)$, $x_2(t)$ and $x_3(t)$ be the populations of pre-reproductive, reproductive and post-reproductive cattle population at time t. Let the respective birth, death and slaughtering rates in the three groups are (0, b_2, 0), (d_1, d_2, d_3) and (s_1, s_2, s_3) and let m_1, m_2 denote the rates at which the animals of the first and second category transfer into the second and third groups respectively on maturity and survival. Under these conditions we get the following system of differential equations for our model

$$\begin{aligned}\frac{dx_1}{dt} &= b_2 x_2 - (d_1 + m_1 + s_1)x_1 \\ \frac{dx_2}{dt} &= m_1 x_1 - (d_2 + m_2 + s_2)x_2 \\ \frac{dx_3}{dt} &= m_2 x_2 - (d_3 + s_3)x_3\end{aligned} \qquad \text{... (1)}$$

which can also be written in the matrix form as

$$\frac{dX}{dt} = MX \qquad \text{... (2)}$$

where

$$X = [x_1(t), x_2(t), x_3(t)]' \;;$$

$$M = \begin{bmatrix} -(d_1 + m_1 + s_1) & b_2 & 0 \\ m_1 & -(d_2 + m_2 + s_2) & 0 \\ 0 & m_2 & -(d_3 + s_3) \end{bmatrix} \qquad \text{... (3)}$$

Let λ_1, λ_2 and λ_3 are the characteristic values of the matrix associated with the system of equations (1). The characteristic equation associated with the matrix in (3) is given by

$$/^{3} + /\left\{\left(d_1 + m_1 + s_1 + d_2 + m_2 + s_2 + d_3 + s_3\right)\right\} + \left[\left\{\left(d_1 + m_1 + s_1\right)\times\left(d_2 + m_2 + s_2\right) - b_2 m_1\right\} + \left(d_1 + m_1 + s_1\right)\left(d_3 + s_3\right) + \left(d_2 + m_2 + s_2\right)\left(d_3 + s_3\right)\right] = 0$$

$$\Rightarrow \quad \left[\lambda^2 + \lambda(d_1 + m_1 + s_1 + d_2 + m_2 + s_2) + (d_1 + m_1 + s_1) \times (d_2 + m_2 + s_2) - b_2 m_1\right]\left[\lambda + d_3 + s_3\right] = 0 \qquad \text{... (4)}$$

So that

$$\lambda_1, \lambda_2 = -\frac{1}{2}\left[d_1 + m_1 + s_1 + d_2 + m_2 + s_2\right] \pm \frac{1}{2}\left[\left(d_1 + m_1 + s_1 + d_2 + m_2 + s_2\right)^2 - 4\left(d_1 + m_1 + s_1\right)\left(d_2 + m_2 + s_2\right) + 4b_2 m_1\right]^{1/2}$$

$$= -\frac{1}{2}\left[d_1 + m_1 + s_1 + d_2 + m_2 + s_2\right] \pm \frac{1}{2}\left[\left(d_1 + m_1 + s_1 - d_2 - m_2 - s_2\right)^2 + 4b_2 m_1\right]^{1/2} \qquad \ldots(5)$$

$$\lambda_3 = -\left(d_3 + s_3\right) \qquad \ldots (6)$$

So that all the eigenvalues are real and in general distinct, λ_2 and λ_3 are negative and λ_1 will be negative and positive according as

$$b_2 m_1 < \left(d_1 + m_1 + s_1\right)\left(d_{21} + m_2 + s_2\right)$$

or

$$b_2 m_1 > \left(d_1 + m_1 + s_1\right)\left(d_{21} + m_2 + s_2\right) \qquad \ldots(7)$$

In general (2) can be written as

$$\frac{dX}{dt} = YDY^{-1}X(t) \qquad \ldots (8)$$

where D is the diagonal matrix of the eigenvalues of M, Y is the matrix whose columns are the right eigenvectors of M. The solution of the differential equation given in (8) is

$$X(t) = Y \exp(Dt) Y^{-1} X(0) \qquad \ldots (9)$$

which gives

$$x_1(t) = \frac{1}{m_1(\lambda_1 - \lambda_2)}\left\{e^{\lambda_1 t}\left(d_2 + m_2 + s_2 + \lambda_1\right)\left[m_1 x_1(0) - \left(d_2 + m_2 + s_2 + \lambda_2\right)x_2(0)\right] + e^{\lambda_2 t}\left(d_2 + m_2 + s_2 + \lambda_2\right)\left[-m_1 x_1(0) + \left(d_2 + m_2 + s_2 + \lambda_1\right)x_2(0)\right]\right\} \qquad \ldots (10)$$

$$x_2(t) = \frac{1}{m_1(\lambda_1 - \lambda_2)}\left\{e^{\lambda_1 t} m_1\left[m_1 x_1(0) - \left(d_2 + m_2 + s_2 + \lambda_2\right)x_2(0)\right] + e^{\lambda_2 t} m_2\left[-m_1 x_1(0) + \left(d_2 + m_2 + s_2 + \lambda_1\right)x_1(0)\right]\right\} \qquad \ldots (11)$$

$$\begin{aligned} x_3(t) = \frac{1}{m_1(\lambda_1 - \lambda_2)}\Big\{ & e^{\lambda_1 t} m_1 m_2\left[m_1 x_1(0) - \left(d_2 + m_2 + s_2 + \lambda_2\right)x_2(0)\right]\left[d_3 + s_3 + \lambda_1\right]^{-1} \\ & + e^{\lambda_2 t} m_1 m_2\left[-m_1 x_1(0) + \left(d_2 + m_2 + s_2 + \lambda_1\right)x_2(0)\right]\left[d_3 + s_3 + \lambda_1\right]^{-1} \\ & + e^{\lambda_3 t}\left(\lambda_1 - \lambda_2\right)\left[m_1^{\,2} m_2 x_1(0) - m_1 m_2\left(d_3 + s_3 - d_1 - m_1 - s_1\right)x_2(0) \right. \\ & \left. + m_1\left(d_3 + s_3 + \lambda_1\right)\left(d_3 + s_3 + \lambda_2\right)x_2(0)\right]\left[\left(d_3 + s_3 + \lambda_1\right)\left(d_3 + s_3 + \lambda_2\right)\right]^{-1}\Big\} \end{aligned} \qquad \ldots (12)$$

Now λ_1, λ_2 are the roots of

$$f(\lambda) \equiv \left(\lambda + d_1 + m_1 + s_1\right)\left(\lambda + d_2 + m_2 + s_2\right) - b_2 m_1 = 0 \qquad \ldots (13)$$

So that

$$f(-\infty) > 0, \quad f\left(-d_1 - m_1 - s_1\right) < 0, \quad f\left(-d_2 - m_2 - s_2\right) < 0, \quad f(\infty) > 0 \qquad \ldots (14)$$

As such λ_1 and λ_2 are respectively greater than and less than both $-\left(d_1 + m_1 + s_1\right)$ and $-\left(d_2 + m_2 + s_2\right)$, so that

$$d_1 + m_1 + s_1 + \lambda_2 < 0, \quad d_2 + m_2 + s_2 + \lambda_2 < 0$$

$$d_1 + m_1 + s_1 + \lambda_1 > 0, \quad d_2 + m_2 + s_2 + \lambda_1 > 0 \qquad \ldots (15)$$

Also $\lambda_1 > \lambda_2$ and we assume $\lambda_2 > \lambda_3$. In this case terms containing $e^{\lambda_1 t}$ dominate in (10), (11) and (12) and since using (15)

$$m_1 x_1(0) - (d_2 + m_2 + s_2 + \lambda_2) \neq 0 \qquad \text{... (16)}$$

we get

$$\underset{t\to\infty}{Lt}\, x_1(t) : x_2(t) : x_3(t) = (d_2 + m_2 + s_2 + \lambda_1)(d_3 + s_2 + \lambda_1) : m_1(d_3 + s_2 + \lambda_1) : m_1 m_2 \qquad \text{... (17)}$$

The ratios $x_1(t) : x_2(t) : x_3(t)$ determines the reproductive structure of the population at time t and (17) gives the ultimate reproductive structure when slaughtering rates are s_1, s_2, s_3.

(i) $\quad b_2 m_1 < (d_1 + m_1)(d_2 + m_2)$ ... (18)

then $\lambda_1, \lambda_2, \lambda_3$ are negative even when there is no slaughtering and animal populations of all three groups will eventually die out.

(ii) $\quad b_2 m_1 > (d_1 + m_1)(d_2 + m_2)$ ... (19)

then in the absence of slaughtering $\lambda_1 > 0$ and as such all group populations will increase in the absence of slaughtering.

(iii) $\quad b_2 m_1 \geq (d_1 + m_1 + s_1)(d_2 + m_2 + s_2)$ ... (20)

then we can undertake slaughtering at rates s_1, s_2 without dooming the animal population to extinction.
If (20) is strict inequality, the three group populations will grow in spite of slaughtering, but if

(iv) $\quad b_2 m_1 = (d_1 + m_1 + s_1)(d_2 + m_2 + s_2)$ ...(21)

$\lambda_1 = 0$ and the population will tend to constant values as $t \to \infty$. Equation (21) gives in some sense the permissible limits for slaughtering in the first two groups. There is no such limit in the slaughtering of the third group except that

$$s_3 \geq 0 \qquad \text{...(22)}$$

The slaughtering of animals mentioned above is permissible on the basis of the following:

Slaughtering of animals can be done at any rate subject to the populations not dying out, i.e., $\lambda_1 \geq 0$ or subject to (20) being satisfied. The minimum birth rate with permissible slaughtering at rates s_1, s_2 without extinction of populations of animals is given by (21).

Now s_3 occurs only in (12) so that the populations of the first and second groups are not affected by the slaughtering rate of the third population. This is otherwise obvious. However, the ultimate ratios of the three populations as given by (17) are influenced by s_3 and as s_3 increase the population of the pre-reproductive and reproductive group's increases relative to that of the post-reproductive group, though the ratio of the populations of the first two groups does not change. We can therefore give s_3 any value greater than zero. We shall however permit s_1, s_2 only such values as satisfy (20).

If $s_1 = s_2 = s$ i.e. on slaughtering the same proportion of the first two groups, then (21) gives

$$b_2 m_1 = (d_1 + m_1 + s)(d_2 + m_2 + s) \qquad \text{... (23)}$$

or

$$s = -\frac{1}{2}(d_1 + m_1 + d_2 + m_2) + \frac{1}{2}\left[(d_1 + m_1 - d_2 - m_2)^2 + 4 b_2 m_1\right]^{1/2} \qquad \text{... (24)}$$

Slaughter only the first group, gives

$$b_2 m_1 = (d_1 + m_1 + s_1)(d_2 + m_2) \qquad \text{... (25)}$$

or

$$h_1 = \frac{b_2 m_1}{d_2 + m_2} - (d_1 + m_1) \qquad \text{... (26)}$$

Now $s_1 > s$ if

$$\left[\frac{b_2 m_1}{d_2 + m_2} - \frac{(d_1 + m_1)}{2} + \frac{(d_2 + m_2)}{2}\right]^2 > \frac{1}{4}\left[(d_1 + m_1 - d_2 - m_2)^2 + 4 b_2 m_1\right]$$

or

$$b_2 m_1 > (d_1 + m_1)(d_2 + m_2) \qquad \text{... (27)}$$

which is same as (14) and is supposed to be satisfied.

Thus if slaughtering is done in such a way that the animal populations neither grow nor die out and s denotes the common proportions of the first two groups if both groups are slaughtered at same rate and if s_1 denotes the proportion when only the first group slaughtered, $s_1 > s$. Similarly if s_2 is the corresponding proportion of the second group when this alone is slaughtered, hence the above argument gives

$$s_2 > s \qquad \text{…(28)}$$

It is an important and worthwhile to mention that this work is effective in the sense that it can be helpful to monitor not only in one population but it will be helpful for the fish, water bodies, living stocks, plantation etc for the benefit of the human beings.

IV. PRODUCTION OF ANIMAL SPECIES IN OPTIMAL SLAUGHTERING

If p_1, p_2, p_3 are the profits per unit of the three type of animals and if μ is the instantaneous discount rate, then the present value of the profits is given by

$$P = \int_0^\infty e^{-\mu t}\left[p_1 s_1 x_1(t) + p_2 s_2 x_2(t) + p_3 s_3 x_3(t)\right]dt \qquad \text{… (29)}$$

Substituting the values of x_1 (t), x_2(t) and x_3(t) from the equations (10), (11) and (12) and integrating, we get

$$P = \frac{1}{m_1(\lambda_1 - \lambda_2)}\{A \times B + C \times D + E + F \times G\} \qquad \text{… (30)}$$

where

$$A = [m_1 x_1(0) - (d_2 + m_2 + s_2 + \lambda_2)x_2(0)][\mu - \lambda_1]^{-1}$$
$$B = \left[p_1 s_1(d_2 + m_2 + s_2 + \lambda_1) + p_2 s_2 m_1 + p_3 s_3 m_1 m_2 (d_3 + s_3 + \lambda_1)^{-1}\right]$$
$$C = [-m_1 x_1(0) - (d_2 + m_2 + s_2 + \lambda_1)x_2(0)][\mu - \lambda_2]^{-1}$$
$$D = \left[p_1 s_1(d_2 + m_2 + s_2 + \lambda_2) + p_2 s_2 m_1 + p_3 s_3 m_1 m_2 (d_3 + s_3 + \lambda_2)^{-1}\right]$$
$$E = p_3 s_3 (\lambda_1 - \lambda_2)(\mu - \lambda_3)^{-1}\left[m_1^{\,2} m_2 x_1(0) - m_1 m_2 (d_3 + s_3 - d_1 - m_1 - s_1)x_2(0)\right.$$
$$F = m_1(d_3 + s_3 + \lambda_1)(d_3 + s_3 + \lambda_2)x_3(0)]$$
$$G = [(d_3 + s_3 + \lambda_1)(d_3 + s_3 + \lambda_2)]^{-1}$$

Now P is to be maximized as a function s_1, s_2, s_3 subject to (5), (6), (20) and

$$s_1,\ s_2,\ s_3 > 0 \qquad \text{… (31)}$$

We can consider the simpler problem of maximizing P subject to (21) so that

$$\lambda_1 = 0,\ \ \lambda_2 = -[d_1 + m_1 + s_1 + d_2 + m_2 + s_2],$$
$$b_2 m_1 = (d_1 + m_1 + s_1)(d_2 + m_2 + s_2) \qquad \text{…(32)}$$

and P reduces to a function of two variables s_1, s_2 to be maximized subject to (26).

If P_1, P_2, P_3 are the selling prices of animals per unit of the three types $c_1(x_1,x_2,x_3), c_2(x_1,x_2,x_3), c_3(x_1,x_2,x_3)$ are the respective costs of these types in this production, then P becomes

$$P = \int_0^\infty e^{-\mu t}\{[P_1 - c_1(x_1,x_2,x_3)]s_1 x_1(t) + [P_2 - c_2(x_1,x_2,x_3)]s_2 x_2(t) + [P_3 - c_3(x_1,x_2,x_3)]s_3 x_3(t)\}dt \quad \ldots(33)$$

Since $x_1(t)$, $x_2(t)$ and $x_3(t)$ are known functions of t, we can always integrate and express P as a function of $s_1,\ s_2,\ s_3$ which has to be maximized subject to appropriate constraints. The integrations are particularly simple if $c_1,\ c_2,\ c_3$ are constants or linear functions of $x_1,\ x_2,\ x_3$.

V. DISCUSSION AND CONCLUSION

The need to conserve genetic variation in domestic animals has been recognized for many years. But, the rapid decline of this diversity under changing environmental and economic climate makes conservation needs more urgent than ever before.

Slaughter is the term used to describe the killing and butchering of animals, usually for food. Commonly it refers to killing and butchering of domestic livestock (tame animals).

The animals most commonly slaughtered for food are cattle (for beef and veal), water buffalo, sheep (for lamb and mutton), goats, pigs (for pork), horses (for horse meat), and fowl, largely chickens, turkeys, and ducks and increasingly fish from the aquaculture industry (fish farming).

One of the important aspects of this model is to study the behaviour of all three groups viz pre-reproductive, reproductive and post-reproductive groups. The two applications associated with the slaughtering of animal species leading to conservation and proper production has been studied to ensure the optimal rate in both cases. Thus, it is worthwhile to mention here that the theoretical model considered here can be helpful for the procedure applied for slaughtering of animals to get maximum production together with the conservation of animals for future generations. The linear algebra is considered to be one of the important application oriented course in mathematical studies. This model is mainly based on linear algebra and the matrix representation of the equations was established for all three groups after formulation. The eigenvalue were taken as a tool to determine the stability of the system after finding the characteristic equation.

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