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Photogravitational elliptical magnetic binary problem

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ABSTRACT
This paper deals with the existence and linear stability of equilibrium points in the photogravitational elliptical magnetic binary problem when the bigger primary is a source of radiation . It is observed that there exists three collinear and two non-collinear equilibrium points
We have found that the radiation-pressure of the primary affects the position of the equilibrium points for any combination of the parameters considered here. Further we have observed that the one collinear equilibrium point L_1 is stable while the others collinear and non-collinear equilibrium points are unstable for various values of μ.
KEYWORDS equilibrium points, elliptical magnetic binary problem, stability.
AMS Subject Classification: *70F07.*

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I. INTRODUCTION

The elliptical restricted three-body problem describes the dynamical system more accurately on account that the realistic assumptions of the, motion of the primaries are subjected to move along the elliptical orbit. Several mathematician [1–7] have been discussed the different aspects of the elliptical restricted three-body problem.
A. Mavragnais [8-11] and Mohd. Arif [12-13] have studied the motion of a charge particle which is moving in the field of two rotating magnetic dipoles. Mohd. Arif [14] have discussed the motion of a charged particle when the dipoles are moving in the elliptic orbits.
Being motivated by the above discussion in this article we have discussed the motion of a charged particle when the dipoles (primaries) are moving in the elliptical orbit with the bigger primary is a source of radiation. The dimensionless variables are introduced by using the distance r between dipoles given by

$$r = \frac{a(1-e^2)}{1+e\,cos\gamma} \qquad (1.1)$$

Here a and e are the semi-major axis and the eccentricity of the elliptical orbit of the either dipole around other and γ is the true anomaly of one of the dipole of mass m_1. We have introduced a coordinate system which rotates with the variable angular velocity ω with

$$\frac{d\omega}{dt^*} = \frac{k\,(m_1+m_2)^{\frac{1}{2}}}{a^{3/2}\,(1-e^2)^{3/2}}\,(1+e\,cos\gamma)^2 \qquad (1.2)$$

Where t^* is the dimensional time and $k = k_1 + k_2$ where k_1 and k_2 are the product of the universal gravitational constants with the mass of dipoles. Equation (1.2) follows from the principle of the conservation of the angular momentum .

II. EQUATION OF MOTION

Consider the two dipoles (the primaries), of masses m_1 and m_2 , $m_1 > m_2$ with magnetic fields moving in a plane about their centre of mass O in Keplerian elliptical orbit having eccentricity e. We assume further the bigger primary is a source of radiation. A charged particle P of charge q and infinitesimal mass m is moving in the plane of motion of the primaries without influencing their motion. The question of the photogravitational elliptical magnetic-binaries problem is to describe the motion of this charged particle P. Then the equation of motion of charged particle P in a dimensionless, pulsating rotating, co-ordinate system are follows,

$$\xi'' - \eta' f = U_\xi \qquad (2.1)$$
$$\eta'' + \xi' f = U_\eta \qquad (2.2)$$

Where

$f = 2 - 2\zeta\left(\frac{q_1}{r_1^3} + \frac{\lambda}{r_2^3}\right)$, $U_\xi = \frac{\partial U}{\partial \xi}$ and $U_\eta = \frac{\partial U}{\partial \eta}$

$$U = (\xi^2 + \eta^2)\left\{\frac{1}{2(1+e\cos\gamma)} + \zeta\left(\frac{q_1}{r_1^3} + \frac{\lambda}{r_2^3}\right)\right\} - \xi\,\zeta\left\{\frac{\mu\, q_1}{r_1^3} - \frac{\lambda(1-\mu)}{r_2^3}\right\} - \frac{(1-\mu)}{(1+e\cos\gamma)}\left\{\frac{q_1}{r_1} + \frac{\lambda}{r_2}\right\} + \frac{2\,q\,\xi\,\eta\,e\sin\gamma}{m\,c\,a^{3/2}\,(1-e^2)^{3/2}} \tag{2.3}$$

$\zeta = \frac{q\sqrt{a(1-e^2)}}{m\,c\,(1+e\cos\gamma)}$, $r_1^2 = (\xi - \mu)^2 + \eta^2$, $r_2^2 = (\xi + 1 - \mu)^2 + \eta^2$, $\lambda = \frac{M_2}{M_1}$ (M_1, M_2 are the magnetic moments of the primaries which lies perpendicular to the plane of the motion),
$m_2 = \mu =$ mass parameter, hence $m_1 = 1 - \mu$, a is the semi-major axis of the orbit,
$c =$ velocity of light.
Let the radiation repulsive force F_p exerted on a charged particle P define as

$$F_p = F_g(1 - q_1)$$

Here F_g is a gravitational force and $q_1 = 1 - \frac{F_p}{F_g} = 1 - \frac{5.6 \times 10^{-3}}{a_{1\,\delta}}\, x_1$, $a_1 =$ radius of the charged particle with density δ and x_1 is a radiation- pressure efficiency factor.
The assumption $q_1 =$ constant is equivalent to neglecting fluctuations in the beam of solar radiation and effect of planet's shadow.
Now introduce the averaged potential function of the problem with respect to true anomaly as:

$$U^* = \frac{1}{2\pi}\int_0^{2\pi} U\, d\gamma, \tag{2.4}$$

Then

$$U^* = (\xi^2 + \eta^2)\left\{\frac{1}{2\sqrt{(1-e^2)}} + a^{\frac{1}{2}}\left(\frac{q_1}{r_1^3} + \frac{\lambda}{r_2^3}\right)\right\} - \xi\, a^{\frac{1}{2}}\left\{\frac{\mu\, q_1}{r_1^3} - \frac{\lambda(1-\mu)}{r_2^3}\right\} - \frac{(1-\mu)}{\sqrt{(1-e^2)}}\left\{\frac{q_1}{r_1} + \frac{1}{r_2}\right\} \tag{2.5}$$

is the modified potential function, for numerical calculation we have taken a particular case $q = mc$.

III. EQUILIBRIUM POINTS

For the locations of the equilibrium points, we solve the following equations

$$\frac{\partial U^*}{\partial \xi} = 0 \text{ and } \frac{\partial U^*}{\partial \eta} = 0 \tag{3.1}$$

We group the solution of equation (3.1) into two kinds; those $\eta = 0$ the collinear equilibrium points and those with $\eta \neq 0$ non-collinear equilibrium points.
We investigate the existence and location of the collinear equilibrium points into the following three intervals.
$C_1 = \{\xi : \xi > \mu\}$, $C_2 = \{\xi : -(\mu - 1) < \xi \leq \mu\}$ and $C_3 = \{\xi : \xi \leq -(\mu - 1)\}$.
If $\xi \in C_1$ the substitution $r_1 = \xi - \mu = \tau$ and $r_2 = \xi + 1 - \mu = \tau + 1$ in (3.1) we have 9 degree equation

$$2\left[(\tau+1)^4\tau^4(\tau+\mu) - a^{\frac{1}{2}}\sqrt{1-e^2}\{3(\tau+1)^4\tau(\tau+\mu)q_1 + 3(\tau+1)\tau^4(\tau+\mu)\lambda - q_1(\tau+1)^4\tau(\tau+\mu) - (\tau+1)\tau^3(\tau+\mu)\lambda - (\tau+1)^4\tau^2 q_1 + \lambda(\tau+1)^2\tau^4\} - q_1(\tau+1)^4\tau^2(1-\mu) - (\tau+1)^2\tau^4\mu q_1\right] + 2(1-\mu)q_1(\tau+1)^4\tau^2 + 2(1-\mu)(\tau+1)^2\tau^4 = 0 \tag{3.2}$$

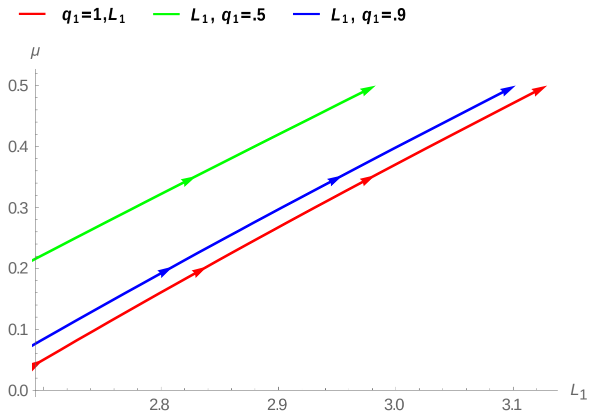
If $\xi \in C_2$ the substitution $r_1 = \xi - \mu = 1 - \tau$ and $r_2 = \xi + 1 - \mu = \tau$ in (3.1) again given 9 degree equation

$$2\left[(1-\tau)^4\tau^4(1-\tau+\mu) - a^{\frac{1}{2}}\sqrt{1-e^2}\{3(1-\tau)\tau^4(1-\tau+\mu)q_1 + 3(1-\tau)^4\tau(1-\tau+\mu)\lambda - (1-\tau)\tau^4(1-\tau+\mu)q_1 - (1-\tau)^4\tau(1-\tau+\mu)\lambda - (1-\tau)^2\tau^4 q_1 + \lambda(1-\tau)^4\tau^2\} - (1-\tau)^2\tau^4 4(1-\mu)q_1 - (1-\tau)^4\tau^2\mu\, q_1\right] + 2(1-\mu)q_1(1-\tau)^2\tau^4 + 2(1-\mu)(1-\tau)^4\tau^2 = 0 \tag{3.3}$$

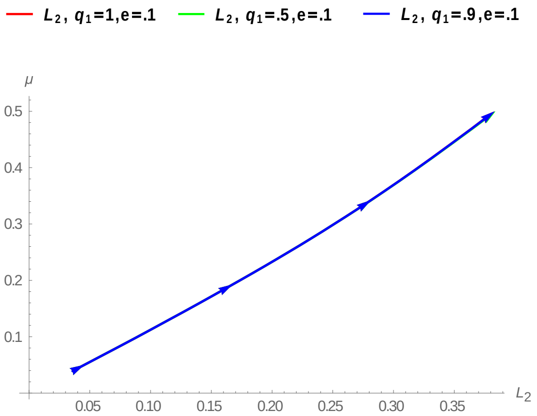
And when $\xi \in C_3$ the substitution $r_1 = \mu - \xi = 1 + \tau$ and $r_2 = -(\xi + 1 - \mu) = \tau$ in (3.1) we have again 9 degree equation

$2\left[(1+\tau)^4\tau^4(-1-\tau+\mu) - a^{\frac{1}{2}}\sqrt{1-e^2}\{3(1+\tau)\tau^4(-1-\tau+\mu)q_1 + 3(1+\tau)^4\tau(-1-\tau+\mu)\lambda -\right.$
1+ττ4−1−τ+μq1−1+τ4τ−1−τ+μ λ−q11+τ2τ4+λ 1+τ4τ2−1+τ2τ41−μq1−1+τ4τ2μ
$\left. q_1\right] + 2(1-\mu)q_1(\tau+1)^6 + 2(1-\mu)(\tau+1)^4\tau^2 = 0$ (3.4)

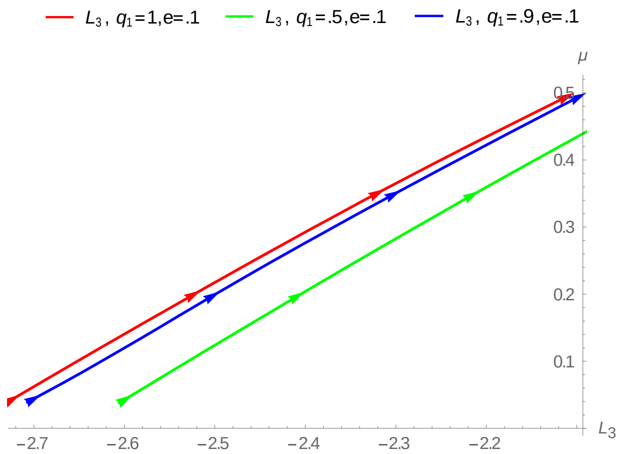
We solve the equations (3.2), (3.3) and (3.4) numerically by use the Mathematica-11. We have observed that the each equation have one real root for various values of μ and q_1 and these roots are denoted by L_1, L_2 and L_3 respectively. The variation in the values of L_i $(i = 1,2,3)$ for various values of μ and q_1 are shown in the figures (1,2,3). We have seen that the radiation- pressure of the primary affects the position of the equilibrium points. The point L_1 go away from the center of mass and this deviation decreases as μ increases fig(1).



Fig(1), Position of equilibrium point L_1

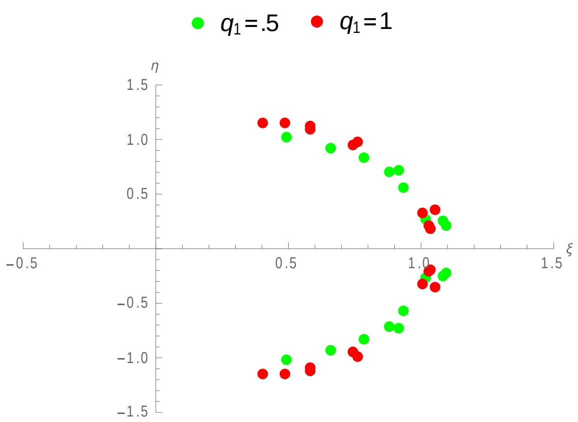


Fig(2), Position of equilibrium point L_2



Fig(3), Position of equilibrium point L_3

We have also observed that this effect is insignificant of the position of L_2 Fig(2). We further observed that the point L_3 move towards the center of mass for different values of � and q_1 and this deviation decreases as � increases Fig(3).



Fig(4), Position of equilibrium points L_4, L_5

The non-collinear equilibrium points denoted by L_4 and L_5 are the solution of the equation (3.1) when $\eta \neq 0$. In fig (4) we give the position of the points L_4 and L_5 for various values of � and q_1 and this figure shows that both L_4 and L_5 shifted towards the primaries as the factor q_1 included.

IV. STABILITY OF EQUILIBRIUM POINTS

Let (ξ_0 , η_0) be the coordinate of any one of the equilibrium point and let α, β denote small displacement from the equilibrium point. Therefore we have

$$\alpha = \xi - \xi_0 ,$$
$$\beta = \eta - \eta_0,$$

Put this value of ξ and η in equation (2.1) and (2.2), we have the variation equation as:

$$\alpha'' - \beta' f_0 = \alpha\left(U^*_{\xi\xi}\right)^0 + \beta\left(U^*_{\xi\eta}\right)^0 \quad (4.1)$$

$$\beta'' + \alpha' f_0 = \alpha\left(U^*_{\xi\eta}\right)^0 + \beta\left(U^*_{\eta\eta}\right)^0 \quad (4.2)$$

Retaining only linear terms in α and β. Here superscript indicates that these partial derivative of U^* are to be evaluated at the equilibrium point (ξ_0 , η_0) . So the characteristic equation at the equilibrium points is

$$\lambda_1{}^4 + \lambda_1{}^2\left\{f_0{}^2 - \left(U^*_{\xi\xi}\right)^0 - \left(U^*_{\eta\eta}\right)^0\right\} + \left(U^*_{\xi\xi}\right)^0\left(U^*_{\eta\eta}\right)^0 - \left(U^*_{\xi\eta}\right)^{0^2} = 0 \quad (4.3)$$

The equilibrium point (ξ_0 , η_0) is said to be stable if all the four characteristic roots of equation (4.3) are either negative real numbers or pure imaginary.

The four characteristic roots are given in Tables 1–4 below, for values of $q_1 = .5$, $\lambda = 1, a = 1.2$, and various values of the parameters □. The table 1 shows that for each particular set of values of the parameters all the roots are complex roots this mean that the point L_1 is stable while the tables 2, 3 and 4 shows that the at least one of the roots is a positive real number. This means that the points L_2, L_3 and $L_{4,5}$ are unstable.

□	L_1	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.05	2.5300	$1.2030 \pm 0.78672i$	$-1.2030 \pm 0.78672i$
.1	2.5785	$1.2117 \pm 0.78001i$	$-1.2117 \pm 0.78001i$
.15	2.6275	$1.2202 \pm 0.77319i$	$-1.2202 \pm 0.77319i$
.20	2.6770	$1.2286 \pm 0.76629i$	$-1.2286 \pm 0.76629i$
.25	2.7274	$1.2368 \pm 0.75938\ i$	$-1.2368 \pm 0.75938i$
.30	2.7774	$1.2449 \pm 0.75221i$	$-1.2449 \pm 0.7522i$
.35	2.8282	$1.2528 \pm .0.7450i$	$-1.2528 \pm .0.7450i$
.40	2.8794	$1.2605 \pm 0.7378i$	$-1.2605 \pm 0.7378i$
.45	2.9312	$1.2681 \pm 0.7305i$	$-1.2681 \pm 0.7305i$
.5	2.9829	$1.2755 \pm 0.7231i$	$-1.2755 \pm 0.7231i$

Table (1)

□	L_2	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.05	0.0454	± 1093.8	$\pm 1.0307 \times 10^7 i$
.1	0.0899	± 460.86	$\pm 9777215i$
.15	0.1333	± 256.84	$\pm 216991i$
.20	0.1750	± 156.34	$\pm 64955i$
.25	0.2150	± 101.69	$\pm 23778i$
.30	0.2529	± 68.630	$\pm 9803.7i$
.35	0.2888	± 48.046	$\pm 4490.7i$
.40	0.3223	± 34.404	$\pm 2204.2i$
.45	0.3224	± 16.352	$\pm 505.94i$
.5	0.3839	± 19.247	$\pm 665.88i$

Table (2)

□	L_3	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.05	-2.5908	± 3.0250	± 0.82474
.1	-2.5298	± 3.0020	± 0.82467
.15	-2.4681	± 2.9789	± 0.82422
.20	-2.4057	± 2.9558	± 0.82348
.25	-2.3425	± 2.9324	± 0.82213
.30	-2.2784	± 2.9090	$\pm .820311$
.35	-2.2134	± 2.8853	± 0.81796
.40	-2.1472	± 2.8615	± 0.81479
.45	-2.0799	± 2.8374	± 0.81092
.5	-2.0111	± 2.8130	± 0.80594

Table (3)

□	$(L_{4,5})_\xi$	$(L_{4,5})_\eta$	$(\lambda_1)_{1,2}$
.05	1.03317	± 0.19004	± 1.8037
.1	1.03045	± 0.21247	± 1.9309
.15	1.01684	± 0.2712	± 2.0117

.20	1.00555	$\pm$.32230	$\pm$2.0649
.25	0.93450	$\pm$0.56448	$\pm$2.0013
.30	0.881122	$\pm$0.70707	$\pm$1.8868
.35	0.665256	$\pm$0.92419	$\pm$3.1574
.40	0.786232	$\pm$0.83168	$\pm$3.2464

Table (4)

V. CONCLUSION

In this paper the equation of motion (2.1), (2.2) of a charged particle have been derived under the assumption that the bigger primary is the source of radiation . These equations are affected by radiation pressure force, magmatic moment, semi major axis, and eccentricity of the orbits of the primaries. Then the positions of non-collinear and collinear equilibrium points are given in (3.1), (3.2), (3.3) and (3.4) respectively. We have seen that the radiation- pressure of the primary affects the position of the equilibrium points. The point L_1 go away from the center of mass and this deviation decreases as μ increases fig(1). We have also observed that this effect is insignificant of the position of L_2 Fig(2). We further observed that the point L_3 move towards the center of mass for different values of ? and q_1 and this deviation decreases as ? increases Fig(3). Afterward the stability of the equilibrium points has been investigated, and tables 1,2,3 and 4 shows that the point L_1 is stable where the L_2, L_3 and $L_{4,5}$ are unstable.

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