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Somecycle-supermagiclabeling of graphs

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ABSTRACT

A graph $G = (V,E)$ has an H-covering if every edge in $E(G)$belongs to a subgraph of G isomorphicto H.Suppose G admitsan H-covering. AnH-magic labeling is a total labeling f: $V(G)\cup E(G) \rightarrow \{1,2,3,\ldots, |V(G)|+|E(G)|\}$with the property that, for every subgraph H' of G isomorphic to H, $\sum_{v\in V(H')} f(v) + \sum_{e\in E(H')} f(e)$ is constant. Additionally, the labeling f is called H- supermagic labeling if $f(V(G)) = \{1, 2, 3, \ldots, |V(G)|\}$.In this paper, we first give some basic results on magic constant. Next, we prove cycle-supermagic labeling of generalized splitting graph. Finally,we prove the disconnected graph mC_n,is cycle – supermagicfor m≥2 and n≥ 3.

Keywords:H-supermagiclabeling,cycle-supermagiclabeling.
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I. INTRODUCTION

Weconsider finite and simple graphs. The vertex and edge sets of a graph G are denoted by $V(G)$ and $E(G)$, respectively. Let H be a graph. An edge- covering of G is a family of subgraphs$H_1,H_2,\ldots,H_k$ such that each edge of $E(G)$belongs to at least one of the subgraphH_i, $1 \le i \le k$. Then it is said that G admits an$(H_1,H_2,\ldots,H_k)$ –(edge) covering. If every H_i is isomorphic to a given graph H, then G admits an H-covering. Suppose G admits an H – covering. A total labeling f: $V(G)\cup E(G)\rightarrow\{1,2,3,\ldots, |V(G)|+|E(G)|\}$ is called an H-magic labeling of G if there exists a positive integer k(called the magic constant) such that for every subgraphH' of G isomorphic to H, $\sum_{v\in V(H')} f(v) + \sum_{e\in E(H')} f(e) = k$. A graph that admits such a labeling is called H-magic. An H-magic labeling f is called an H-supermagic labelingif $f(V(G)) = \{1, 2, 3,\ldots,|V(G)|\}$. A graph that admits an H-supermagic labeling is called an H-supermagic graph. The sum of all vertex and edge labels on H (under a labeling f)and is denoted by $\sum f(H)$.
Although magic labeling of graphs was introduced by Sedlacek [17], the concept of vertex magic total labeling (VMTL) first appeared in 2002 in[8]. In 2004, MacDougall et al. [9] introduced the notion of super vertex magic total labeling (SVMTL). In 1998, Enomoto et al. [1] introduced the concept of super edge- magic graphs. In 2005, Sugeng and Xie [19] constructed some super edge-magic total graphs. Most recently, Tao-ming Wang and Guang-Hui Zhang [21], generalized some results found in [10].
In 2007,Lladoand Moragas [7] studied some C_n –supermagic graphs. They proved that the wheel W_n, the windmill $W(r,k)$, the subdivided wheel $W_n(r,k)$, and the graph obtained by joining two end vertices of any number of internally disjoint paths of length $p \ge 2$ are C_h–supermagic for some h. Maryati et al.[14] studied some P_h– supermagic trees. They proved that shrubs, balanced subdivision of shrubs, and banana trees are P_h–supermagic for some h.

For $H \cong K_2$, an H- supermagic graph is also called a super edge- magic graph. The notion of a super-edge-magic graph was introduced by Enomoto et al. [1] as a particular type of edge-magic graph given by Kotzig and Rosa [4]. The usage of the word 'super' was introduced in [1]. There are many graphs that have been proved to be (super) edge-magic graphs; see for instance [15,16]. For further information about (super) edge – magic graphs, see [2]. The H- magic labeling of a plane graph was introduced by Lih [6].
MacDougall et al. [9] and Swaminathan and Jeyanthi [20] introduced different labelings with same name super vertex-magic total labeling. To avoid confusion Marimuthu and Balakrishnan [10] called a vertex magic total labeling is E-super if $f(E(G)) = \{1, 2, 3,\ldots, |E(G)|\}$. Note that the smallest labels are assigned to the edges. A graph G is called E-super vertex magic if it admits E-super vertex labeling. There are many graphs that have been proved to be E-super vertex magic; see for instance [11, 5,12]. A total H-magic labeling of $G$is an injection $f{:}V\cup E\rightarrow \{1, 2,\ldots,|V| + |E|\}$ such that for each subgraph$H' = (V' ,E')$ of Gisomorphic to H, we have $\sum_{v\in V'} f(v) + \sum_{e\in E'} f(e)$ is constant. Additionally, if$f(E) = \{1, 2, \ldots,|E|\}$ then $G$is called H-E-supermagic. For further information about H-E-super magic graphs, see [18].

In this paper, we first give some basic counting results on magic constant. Next, we prove the generalized spilitting graph $S_{n-1}[C_n]$ is C_{n-1}-supermagic for any integer $n\geq 5$.Finally, we prove the disconnected graph mC_n,is C_n- supermagicmagic for$m\geq 2$ and $n\geq 3$.

II. BASIC COUNTING ON MAGIC CONSTANT *k*.

Theorem 2.1 If G is a cycle -supermagic graph with magic constant k then $k>p+q$.

Proof. Suppose G iscycle supermagic with magic constant $k\leq p+q$.
Case 1: Assume $k<p+q$.

If $k<p+q$ then there exist an edge $e_i\in E(G)$ such that $f(e_i) = p + q$
Moreover there must be a cycle C_i which contains $e_i\in E(G)$ such that,

$$k = \sum_{u_i \in V(Ci)} f(u_i) + \sum_{e_j \in E(Ci)} f(e_j)$$

$$k = \sum_{u_i \in V(Ci)} f(u_i) + (p+q) + \sum_{e_j \varepsilon E(C_i)-\{p+q\}} f(e_j)$$

$$k-(p+q) = \sum_{u_i \in V(Ci)} f(u_i) + \sum_{e_j \varepsilon E(C_i)-\{p+q\}} f(e_j) < 0$$

which is a contradiction to the fact that $\sum_{u_i \in V(Ci)} f(u_i) + \sum_{e_j \varepsilon E(C_i)-\{p+q\}} f(e_j)$ is always positive integer.

Case 2: Assume k = p+q

If $k = p+q$ then there exist an edge$e_i\in E(G)$ such that $f(e_i) = p+q$
Moreover there must be a cycle C_i which contains $e_i\in E(G)$ such that,

$$k = \sum_{u_i \in V(Ci)} f(u_i) + \sum_{e_i \in E(C_i)} f(e_i)$$

$$k = \sum_{u_i \in V(C_i)} f(u_i) + (p+q) + \sum_{e_j \varepsilon E(C_i)-\{p+q\}} f(e_j)$$

$$k-(p+q) = \sum_{u_i \in V(Ci)} f(u_i) + \sum_{ej \varepsilon E(C_i)-\{p+q\}} f(e_j) = 0$$

which is a contradiction to the fact that $\sum_{u_i \in V(Ci)} f(u_i) + \sum_{ej \varepsilon E(C_i)-\{p+q\}} f(e_j)$ is always positive integer.□

Next, we prove that the magic constant for a cycle - supermagic graph is not unique.

Result 2.2. If G is a cycle - supermagic graph with magic constant k, then the magic constant $k$is not unique.

Example is given in the Figure 1 and Figure 2.

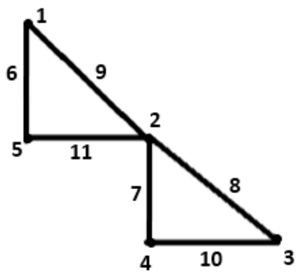

Figure 1: cycle – supermagic labeling with magic constant 34.

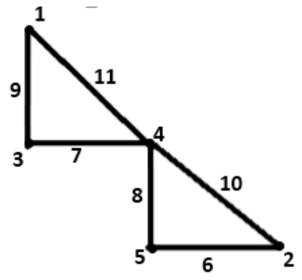

Figure 2: cycle – supermagic labeling with magic constant 35.

III. MAIN RESULTS

A generalizedspilittinggraph $S_{n-1}[C_n]$for $n\geq 5$is defined as a graph with
$V(S_{n-1}[C_n]) = \{v_1,v_2,\ldots,v_n\} \cup \{ u_i^1, u_i^2,\ldots, u_i^{n-4}: 1\leq i\leq n\}$ and
$E(S_{n-1}[C_n]) = \{v_iv_{i+1}: 1\leq i\leq n-1\} \cup \{ v_nv_1\} \cup \{ u_i^ju_i^{j+1}: 1\leq i\leq n, 1\leq j\leq n-5\}$
$\cup\{u_i^1v_{i-1}: 2\leq i\leq n\} \cup\{u_1^1v_n\}\cup\{u_i^2, v_{i+1}: 2\leq i\leq n-1\}\cup\{u_n^2 v_1\}$
The following theorem showsthat the generalized spilitting graph $S_{n-1}[C_n]$ is C_{n-1}-supermagic.

Theorem 3.1. The generalized spilitting graph $S_{n-1}[C_n]$is C_{n-1}-supermagicfor any integer $n\geq 5$.

Proof. Let $n\geq 5$ be any integer.
We define a total labeling g: $V(S_{n-1}[C_n])\cup E(S_{n-1}[C_n]) \rightarrow \{1,2,3,\ldots, (2n^2-5n)\}$ as given below.
Label the vertices of G in the following way:
For $1\leq i\leq n$, $g(x) = i$, if $x = v_i$.
Label the additional vertices of $S_{n-1}[C_n]$in the following way:

$$g(x)=\begin{cases}(i+1)\,n - j, & \text{if } x = u_i^j, \text{for } 1\leq j\leq n-1,\ 1\leq i\leq n-4, \text{ for odd } i,\\ ni+1+j, & \text{if } x = u_i^j, \text{for } 1\leq j\leq n-1,\ 1\leq i\leq n-4, \text{ for even } i.\end{cases}$$

$$g(x) = \begin{cases}(i+1)\,n, & \text{if } x = u_i^n, \text{ for } 1\leq i\leq n-4, \text{ for odd } i,\\ ni+1, & \text{if } x= u_i^n, \text{ for } 1\leq i\leq n-4, \text{ for even } i.\end{cases}$$

The edge labeling of $S_{n-1}[C_n]$ is defined by,

$$g(x) = \begin{cases} n^2 - 2n - i + 1, & \text{if } x= v_iv_{i+1}, \text{ for } 1\leq i\leq n-1,\\ n^2-3n+1, & \text{if } x=v_nv_1,\\ n^2 - 2n + j, & \text{if } x = u_1^j v_{j+1}, \text{for } 1\leq j\leq n-1,\\ n^2 - n, & \text{if } x = u_1^n v_1,\\ n^2-j+1, & \text{if } x = u_i^j v_{j-1}, \text{for } i = n-4 \text{ and } 2\leq j\leq n-1,\\ n^2, & \text{if } x = u_i^1 v_n, \text{for } i = n-4.\end{cases}$$

The edge labeling is given by,
For odd n and $1\leq j\leq n$,

$$g(x) = \begin{cases} n^2 + n\,(i-1) + j, & \text{if } x = u_i^j u_{i+1}^j, \text{for } 1\leq i\leq n-5, \text{ for odd } i,\\ (n+1)^2 + n\,(i-2) - j, & \text{if } x = u_i^j u_{i+1}^j, \text{for } 2\leq i\leq n-5, \text{ for even } i.\end{cases}$$

For even$n$,

$$g(x) = \begin{cases} n^2 + n\,(i-1) + j, & \text{if } x = u_i^j u_{i+1}^j, \text{for } 1\leq j\leq n \text{ and } 1\leq i\leq n-6, \text{ for odd } i,\\ (n+1)^2 + n\,(i-2) - j, & \text{if } x = u_i^j u_{i+1}^j, \text{for } 1\leq j\leq n \text{ and } 2\leq i\leq n-6, \text{ for even } i,\\ 2n^2 - 5n - j, & \text{if } x = u_i^j u_{i+1}^j, \text{ for } 1\leq j\leq n-1 \text{ and } i= n-5,\\ 2n^2-5n, & \text{if } x = u_i^n u_{i+1}^n, \text{ for } i = n-5.\end{cases}$$

Let $C_n^{(j)}, 1 \le j \le n$ be the subcycle of $S_{n-1}[C_n]$ with

$V(S_{n-1}[C_n])=\{v_1, v_2, \ldots, v_n\} \cup \{u_i^1, u_i^2, \ldots, u_i^{n-4} : 1 \le i \le n\}$ and
$E(S_{n-1}[C_n])= \{v_i v_{i+1} : 1 \le i \le n-1\} \cup \{v_n v_1\} \cup \{u_i^j u_i^{j+1} : 1 \le i \le n, 1 \le j \le n-5\}$
$\cup \{u_i^1 v_{i-1} : 2 \le i \le n\} \cup \{u_1^1 v_n\} \cup \{u_i^2, v_{i+1} : 2 \le i \le n-1\} \cup \{u_n^2 v_1\}$

It can be verified that for each $2 \le i \le n-1$ and $j = n$-4,
$\Sigma\, g(C_{n-1}^{(i)}) = g(v_i) + g(v_{i+1}) + g(v_{i-1}) + \sum_{j=1}^{n-4} g(u_j^i) + g(u_1^i v_{i+1}) + g(u_j^i v_{i-1}) + (\sum_{j=1}^{n-5} g(u_j^i u_{j+1}^i))$
$= 2n^3 - 9n^2 + 12n - 1$
$\Sigma\, g(C_{n-1}^{1}) = g(v_1) + g(v_2) + g(v_n) + \sum_{j=1}^{n-4} g(u_j^1) + g(u_1^1 v_2) + g(u_j^1 v_n) + (\sum_{j=1}^{n-5} g(u_j^1 u_{j+1}^1))$
$= 2n^3 - 9n^2 + 12n - 1$
$\Sigma\, g(C_{n-1}^{n}) = g(v_1) + g(v_n) + g(v_{n-1}) + \sum_{j=1}^{n-4} g(u_j^n) + g(u_1^n v_1) + g(u_j^n v_{n-1}) + (\sum_{j=1}^{n-5} g(u_j^n u_{j+1}^n))$
$= 2n^3 - 9n^2 + 12n - 1$
Hence$\Sigma\, g(C_{n-1}^{(i)}) = 2n^3 - 9n^2 + 12n - 1$for $1 \le i \le n$.
Hence the generalized spilitting graph $S_{n-1}[C_n]$ is C_{n-1}-supermagicfor any integer $n \ge 5$. □

An illustration is given in Figure 3 and Figure 4.

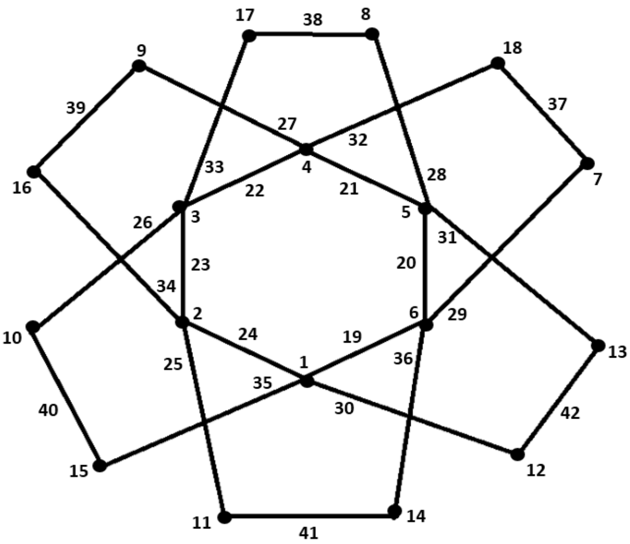

Figure 3: A $S_5[C_6]$ graph is C_5–supermagic.

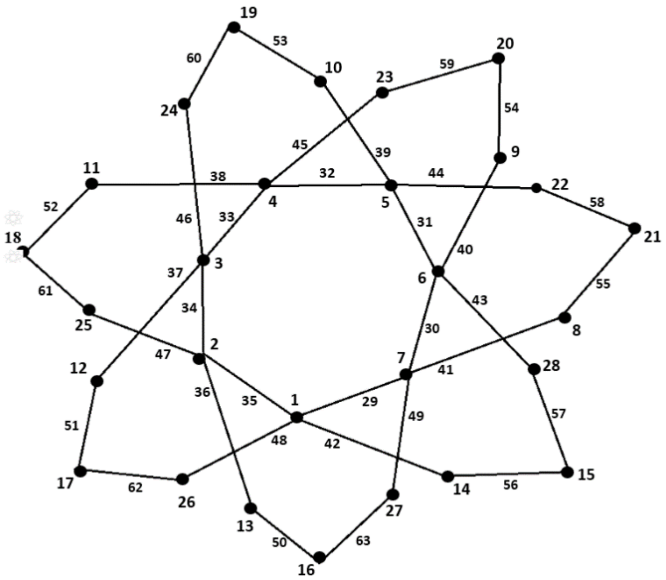

Figure 4: A $S_6[C_7]$ graph is C_6–supermagic.

In the next theorem we will deal with cycle-supermagiclabelingfor the disconnected graph mC_n,the disjoint union of m cycles of length $n$for $m\geq 2$and $n \geq 3$.

The vertex and edge set of mC_n is defined by,

$V(mC_n)= \{v_{ij}: 1 \leq i \leq m, 1 \leq j \leq n\}$and

$E(mC_n)= \{v_{i\,j}\, v_{i\,j+1}: 1 \leq i \leq m, 1 \leq j \leq n-1\} \cup \{v_{i\,n}\, v_{i\,1} : 1 \leq i \leq m\}$

The main result is the following.

Theorem 3.2. The graphmC_nisC_n-supermagicfor$m\geq 2$ and $n\geq 3$.

Proof. We define a total labeling as g: $V(mC_n)\cup E(mC_n) \longrightarrow \{1, 2, \ldots, 2mn\}$.
Label the vertices ofmC_nin the following way:

For $1 \leq i \leq m$,

$$g(x) = \begin{cases} i + (j-1)m, & \text{if } x = v_{i\,j}, \text{for } 1 \leq j \leq n, \text{ for } j \text{ is odd,} \\ mj - (i-1), & \text{if } x = v_{i\,j}, \text{for } 2 \leq j \leq n, \text{ for } j \text{ is even.} \end{cases}$$

The edge labeling of $mC_n$is defined by,
For odd n,for $1 \leq i \leq m$,

$$g(x) = \begin{cases} (n+j)m - (i-1), & \text{if } x = v_{i\,j}\, v_{i\,j+1}, \text{for } 1 \leq j \leq n-1, \text{ for } j \text{ is odd,} \\ (n+j-1)m + i, & \text{if } x = v_{i\,j}\, v_{i\,j+1}, \text{if } 2 \leq j \leq n-1, \text{ for } j \text{ is even.} \end{cases}$$

For even n, for $1 \leq i \leq m$,

$$g(x) = \begin{cases} (n+j)m + i - m, & \text{if } x = v_{i\,j}\, v_{i\,j+1}, \text{for } 1 \leq j \leq n-1, \text{ for } j \text{ is odd,} \\ (n+j)m - (i-1), & \text{if } x = v_{i\,j}\, v_{i\,j+1}, \text{for } 2 \leq j \leq n-2, \text{ for } j \text{ is even,} \\ 2nm - (i-1), & \text{if } x = v_{i\,n}\, v_{i\,1}. \end{cases}$$

For $1 \leq i \leq m$, $\sum g(C_n^{(i)}) = g(v_{ij}) + g(v_{i\,j}\, v_{i\,j+1}) + g(v_{i\,n}\, v_{i\,1}) = 2n^2m + n$.
Hence mC_n admits a C_n -supermagic labeling for $n\geq 3$. □

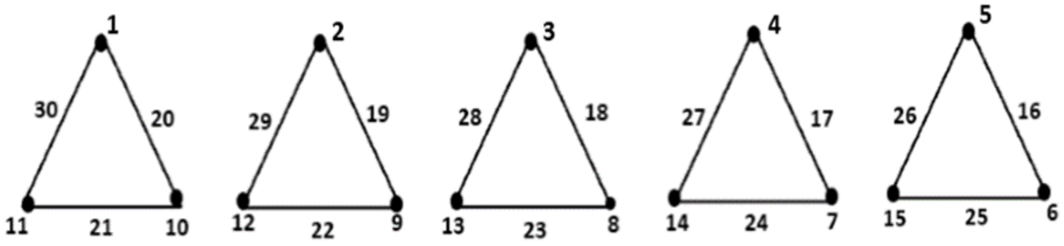

Figure 5:$5C_3$ is C_3 – supermagic with magic constant 93.

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