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Application Ofcommuting Pair Ofmappings innon- Archimedean MengerSpace ofType C_g.

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ABSTRACT:*Italian mathematician Salvolose Sessa proposed the concept of weak commutativity of pair of mappings of a metric space, which lead to a new theory of weaker forms of commuting mappings. In this paper we introduce the concept of weakly commuting pair of mappings with respect to certain mappings in a non-Archimedean Menger probabilistic metric space of type Cg and we generalized some well-known results on non-Archimedean Menger probabilistic metric space of type Cg.*

KEYWORDS*: probabilistic metric spaces, contraction mapping, commuting mapping and fixed-point theorems*

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I. INTRODUCTION

In 1942, Menger [6] generalized the metric axioms by associating a distribution function with each pair of points of an abstract set X. (A distribution functions are a *mapping* $f: R \to R^+$ which is non-decreasing, left continuous, with $\inf f = 0$ and $\sup f = 1$). Thus, for any ordered pair of points p, q *of* X, we associate a distribution function denoted by$F_{p,q}$ and, for any positive number x, we interpret$F_{p,q}(x)$as the probability that the distance between p and q is less than x. This gives rise to a new theory of 'probabilistic metric spaces' which started developing rapidly after the publication of the paper of Schweizer and Sklar [11].

II. PROBABILISTIC METRIC SPACES

Definition 1.1. A mapping$f: R \to R^+$ is called a distribution function if it is non-decreasing, left continuous and $inf\ f(x) = 0, sup\ f(x) = 1.$

We shall denote by L the set of all distribution functions. The specific distribution functionH ∈ L is defined by

$$\left.\begin{aligned} H(x) &= 0, \quad x \le 0 \\ &= 1, \quad x > 0 \end{aligned}\right\}$$

Definition 1.2A probabilistic metric space (PM space) is an ordered pair (X, F), X is a nonempty set and $F: X \times X \to L$is mapping such that, by denoting $F(p, q)$ by$F_{p,q}$ for all $p, q\ in\ X$, we have

I. $F_{p,q}(x) = 1\ \forall x > 0\ iff\ p = q$
II. $F_{p,q}(0) = 0$
III. $F_{p,q} = F_{q,p}$
IV. $F_{p,q}(x) = 1,\ F_{q,r}(y) = 1 \Rightarrow F_{p,r}(x + y) = 1.$

We note that $F_{p,q}(x)$is value of the distribution function $F_{p,q} = F(p, q) \in L\ at\ x \in R.$

Definition 1.3.A mapping $t: [0,1] \times [0,1] \to [0,1]$ is called t −norm if it is non-decreasing (by non-decreasing, we mean $a \le c, b \le d \Rightarrow t(a,b) \le t(c,d)$, commutative, associative and $t(a,1) = a$ for all a in [0, 1], t(0,0) = 0.

Definition 1.4. A Menger PM space is a triple $(X, F;\ t)$ where (X, F) is a PM space and t is t −norm such that,
$F_{p,r}(x+y) \ge t\left(F_{p,q}(x), F_{q,r}(y)\right)\ \ \forall\ x, y \ge 0$.

NOTE [1]If $(X, F; t)$ is Menger Probabilistic metric space with$\sup t(x,x) = 1,\ 0 < x < 1$, then $(X, F;\ t)$ is a Hausdorff topological space in the topology T induced by the family of (ε, λ) neighborhoods$\{U_p(\varepsilon, \lambda): p \in X, \varepsilon > 0, \lambda > 0$where $Up\varepsilon, \lambda = x \in X: Fxp\varepsilon > 1 - \lambda$.

Definition 1.5 Suppose $\Omega = \{g: [0,1] \to [0,\infty)$ is continuous, strictly increasing such that $g(1) = 0,\ g(0) < \infty\}$ is a set of functions. A probabilistic metric space is said to be of typeC_g if $g \in \Omega$ such that,

$$g\left(F_{p,q}(x)\right) \le g\left(F_{p,r}(x)\right) + g\left(F_{r,q}(x)\right)\ \forall\ p,q,r \in X \text{ and } x \ge 0.$$

NOTE. Throughout this paper we consider (X, F, t) a complete non-Archimedean Menger probabilistic metric space of type C_g.

Through this mapping every metric space can be considered as probabilistic metric space. For topological details the measure of compactness, completion, product and quotient spaces, refer to [8].

In 1977, Fisher [5] proved an interesting result on common fixed-point theorem for a pair of self-mappings on a complete metric space satisfying a contractive inequality.

Theorem 1.1 [5]. Let Sand T be mappings of a complete metric space (X, d) into itself satisfying the inequality,

$$[d(Sp,Tq)]^2 \le \alpha d(p,Sp)d(q,Tq) + \beta d(p,Tq)d(q,Sp)\ \ \forall p,q \in X,$$

where $0 \le \alpha < 1$ and $\beta > 0$.Then Sand T have a common fixed point. Further, if $0 \le \alpha$, $\beta < 1$ then each of Sand T has a unique fixed common point.

In 1984, Rao and Rao [9] extended the above result of Fisher [5] for three mappings in the same setting of complete metric space.

Theorem 1.2 [9]. Let$S$,$T$and P be mappings from a complete metric space (X, d) to itself satisfying the inequality,

$$[d(SQp,TQq)]^2 \le \alpha[d(p,q)]^2 + \beta d(p,SQp)d(q,TQq) + \gamma d(p,TQq)d(q,SQp)\ \ \forall p,q \in X$$

where $\alpha,\beta,\gamma \ge 0$ with $\alpha+\beta<1$ and $\alpha+\gamma<1$ and $SQ = QS or\ TQ = QT$.

Then S, T *and* Qhave a unique common fixed point.

After this Chatterjee and Singh [3] extended the above results of Fisher [5] and Rao and Rao [9] for four mappings without changing the setting.

In 1987, the existence of common fixed-point theorems using the concept of weakly commuting pair of mappings with respect to certain mapping was introduced by Pathak [7].

Definition 1.6[7]. Let P, Sand Tbe mappings from a complete metric space (X, d) to itself. Then {S, T} is said to be weakly commuting pair of mappings with respect to mapping P if,

$$d(PSPx,TPx) \le d(SPPx,TPx)$$

and

$$d(SPx,PTPx) \le d(SPx,TPPx),$$

for all$x$ *in* X.

Now we define weakly commuting pair of mappings with respect to certain mapping in non- Archimedean Menger space of type C_g.

Definition 1.7 Let(X, F, t) be a non-Archimedean Menger space of type C_g. Suppose $S, T, P$are self-mappingson X. Then the pair (S, T) is said to be weakly commuting pair with respect to$P$if $\forall\ t \in X$,

$$g(F_{PSPx,TPx}(t)) \le g(F_{SPPx,TPx}(t))$$

and

$$g(F_{PTPx,SPx}(t)) \le g(F_{TPPx,SPx}(t)).$$

The above developments and results of Tripathi et al. [12], [13] motivated us to extend these results of common fixed points for three mappings, four mappings and more generally sequence of mappings in a complete non-Archimedean Menger space of type C_g.

Following Lemma and remark proved by Chang [25] has been used by us in proving our results.

Lemma 1.1 Let $\{p_n\}$be a sequence in X such that $\lim_{n\to\infty} F_{p_n,p_{n+1}}(x) = 1$ for all $x > 0$. If the sequence $\{p_n\}$ is not aCauchy sequence in X,then there exists $\varepsilon_0 > 0$, $t_0 > 0$ and two sequence $\{m_i\}$ and $\{n_i\}$ of positive integers such that,

I. $m_i > n_i + 1$ and $n_i \to \infty$ as $i \to \infty$,

II. $g(F_{p_{m_i},p_{n_i}}(t_0)) > g(1-\varepsilon_0)$ and $g(F_{p_{m_{i-1}},p_{n_i}}(t_0)) \le g(1-\varepsilon_0)$.

Remark 1.1. If sequence $\{p_n\}$ is not a Cauchy sequence in $X$and $\lim_{n\to\infty} g(F_{p_n,P_{n+1}}(x)) = 0$.

Then,

$$g(1-\varepsilon_0) < g(F_{p_{m_i},p_{n_i}}(t_0)) \le g(F_{p_{m_i},p_{m_{i-1}}}(t_0)) + g(F_{p_{m_{i-1}},p_{n_i}}(t_0))$$

and $\lim_{i\to\infty} g(F_{p_{m_i},p_{n_i}}(t_0)) = g(1-\varepsilon_0)$.

III. MAIN RESULTS

Theorem 2.1 Let (X, F, t) be a complete non- Archimedean Menger space. Suppose $S, T, P$are self-mappings on X satisfying,

(1). $$\begin{aligned}[g(F_{SPx,TPy}(t))]^2 \le\ & \alpha_1[g(F_{x,y}(t))]^2 + \alpha_2 g(F_{x,SPx}(t))g(F_{y,TPy}(t)) + \alpha_3 g(F_{x,TPy}(t))g(F_{y,SPx}(t)) \\ & + \alpha_4 g(F_{x,SPx}(t))g(F_{x,TPy}(t)) + \alpha_5 g(F_{y,TPy}(t))g(F_{y,SPx}(t)) \\ & + \alpha_6 g(F_{x,TPy}(t))g(F_{y,TPy}(t)) + \alpha_7 g(F_{SPx,TPy}(t))g(F_{x,y}(t)) \\ & + \alpha_8 g(F_{x,y}(t))g(F_{x,SPx}(t)) + \alpha_9 g(F_{x,y}(t))g(F_{x,TPy}(t)) \\ & + \alpha_{10} g(F_{x,y}(t))g(F_{y,SPx}(t)) + \alpha_{11} g(F_{x,y}(t))g(F_{y,TPy}(t))\end{aligned}$$

for $\alpha_i > 0, \quad i = 1, 2, \ldots 11$ such that

$\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4 + 2\alpha_5 + 2\alpha_6 + \alpha_7 + \alpha_8 + 2\alpha_9 + 2\alpha_{10} + \alpha_{11} < 1$.

(2). $SP = PS$ *or* $TP = PT$.

Then S, T, P have a unique common fixed point in X.

Proof. For $x_0 \in X$, we construct a sequence $\{x_n\}$ such that,

$x_{2n+1} = SPx_{2n}$ and $x_{2n} = TPx_{2n-1}$, $n = 1, 2\ldots$

Then from (1),

$$[g(F_{x_{2n+1},x_{2n}}(t))]^2 = [g(F_{SPx_{2n},TPx_{2n-1}}(t))]^2$$

i.e. $$\begin{aligned}[g(F_{x_{2n+1},x_{2n}}(t))]^2 \le\ & \alpha_1[g(F_{x_{2n},x_{2n-1}}(t))]^2 + \alpha_2 g(F_{x_{2n},x_{2n+1}}(t))g(F_{x_{2n-1},x_{2n}}(t)) + \alpha_3 g(F_{x_{2n},x_{2n}}(t))g(F_{x_{2n-1},x_{2n+1}}(t)) \\ & + \alpha_4 g(F_{x_{2n},x_{2n+1}}(t))g(F_{x_{2n},x_{2n}}(t)) + \alpha_5 g(F_{x_{2n-1},x_{2n}}(t))g(F_{x_{2n-1},x_{2n+1}}(t)) \\ & + \alpha_6 g(F_{x_{2n},x_{2n}}(t))g(F_{x_{2n-1},x_{2n}}(t)) + \alpha_7 g(F_{x_{2n+1},x_{2n}}(t))g(F_{x_{2n},x_{2n-1}}(t)) \\ & + \alpha_8 g(F_{x_{2n},x_{2n-1}}(t))g(F_{x_{2n},x_{2n+1}}(t)) + \alpha_9 g(F_{x_{2n},x_{2n-1}}(t))g(F_{x_{2n},x_{2n}}(t)) \\ & + \alpha_{10} g(F_{x_{2n},x_{2n-1}}(t))g(F_{x_{2n-1},x_{2n+1}}(t)) + \alpha_{11} g(F_{x_{2n},x_{2n-1}}(t))g(F_{x_{2n-1},x_{2n}}(t)),\end{aligned}$$

i.e. $$\begin{aligned}[g(F_{x_{2n+1},x_{2n}}(t))]^2 \le\ & (\alpha_1 + \alpha_{11})[g(F_{x_{2n},x_{2n-1}}(t))]^2 + (\alpha_2 + \alpha_7 + \alpha_8)g(F_{x_{2n},x_{2n+1}}(t))g(F_{x_{2n-1},x_{2n}}(t)) \\ & + (\alpha_5 + \alpha_{10})g(F_{x_{2n},x_{2n-1}}(t))g(F_{x_{2n-1},x_{2n+1}}(t))\end{aligned}$$

i.e. $$\begin{aligned}[g(F_{x_{2n+1},x_{2n}}(t))]^2 \le\ & (\alpha_1 + \alpha_{11})[g(F_{x_{2n},x_{2n-1}}(t))]^2 + (\alpha_2 + \alpha_7 + \alpha_8)g(F_{x_{2n},x_{2n+1}}(t))g(F_{x_{2n-1},x_{2n}}(t)) \\ & + (\alpha_5 + \alpha_{10})g(F_{x_{2n},x_{2n-1}}(t))[g(F_{x_{2n},x_{2n+1}}(t)) + g(F_{x_{2n},x_{2n-1}}(t))]\end{aligned}$$

or $$[g(F_{x_{2n+1},x_{2n}}(t))]^2 \le (\alpha_1 + \alpha_{11} + \alpha_5 + \alpha_{10})[g(F_{x_{2n},x_{2n-1}}(t))]^2 + (\alpha_5 + \alpha_{10} + \alpha_2 + \alpha_7 + \alpha_8)g(F_{x_{2n},x_{2n+1}}(t))g(F_{x_{2n-1},x_{2n}}(t))$$

or $$[g(F_{x_{2n+1},x_{2n}}(t))]^2 \le (\alpha_1 + \alpha_{11} + \alpha_5 + \alpha_{10})[g(F_{x_{2n},x_{2n-1}}(t))]^2 + (\alpha_5 + \alpha_{10} + \alpha_2 + \alpha_7 + \alpha_8)\frac{[\{g(F_{x_{2n},x_{2n+1}}(t))\}^2 + \{g(F_{x_{2n-1},x_{2n}}(t))\}^2]}{2}$$

i.e. $$[g(F_{x_{2n+1},x_{2n}}(t))]^2 \le \frac{\{(\alpha_1 + \alpha_{11} + \alpha_5 + \alpha_{10}) + \frac{1}{2}(\alpha_5 + \alpha_{10} + \alpha_2 + \alpha_7 + \alpha_8)\}}{1 - \frac{1}{2}(\alpha_5 + \alpha_{10} + \alpha_2 + \alpha_7 + \alpha_8)} g(F_{x_{2n-1},x_{2n}}(t))$$

i.e. $[g(F_{x_{2n+1},x_{2n}}(t))] \le k(F_{x_{2n-1},x_{2n}}(t))$,

where $$k^2 = \frac{(\alpha_1 + \alpha_{11} + \alpha_5 + \alpha_{10}) + \frac{1}{2}(\alpha_5 + \alpha_{10} + \alpha_2 + \alpha_7 + \alpha_8)}{1 - \frac{1}{2}(\alpha_5 + \alpha_{10} + \alpha_2 + \alpha_7 + \alpha_8)} < 1,$$

(because if $k^2 \ge 1$, then $\alpha_1 + \alpha_2 + 2\alpha_5 + \alpha_7 + \alpha_8 + 2\alpha_{10} + \alpha_{11} \ge 1$, which contradicts our assumption).

Hence Inductively,

$[g(F_{x_{2n+1},x_{2n}}(t))] \le k^n(F_{x_n,x_{n+1}}(t))$.

Now in limiting case as n, we have,

$\{g(F_{x_{2n+1},x_{2n}}(t)\} \to 0$.

i.e. $\lim_{n\to\infty} g(F_{x_n,x_{n+1}}(t)\} = 0$.

Next, we show that $\{x_n\}$ is a Cauchy sequence.

If $\{x_n\}$ is not a Cauchy sequence then by Lemma 1.1 and Remark 1.1 $\exists\ \varepsilon_0 > 0$, $t_0 > 0$ and sets of positive integers $\{m_i\},\{n_i\}$ such that,

$\lim_{i\to\infty} g(F_{x_{m_i},x_{n_i}}(t_0)) = g(1-\varepsilon_0)$,

$\lim_{i\to\infty} g(F_{x_{m_i-1},x_{n_i-1}}(t_0)) = g(1-\varepsilon_0)$,

$\lim_{i\to\infty} g(F_{x_{m_i+1},x_{n_i-1}}(t_0)) = g(1-\varepsilon_0)$

and $\lim_{i\to\infty} g(F_{x_{m_i},x_{n_i-1}}(t_0)) = g(1-\varepsilon_0)$.

Putting $x = x_{2m_i}$ and $y = x_{2n_i-1}$ in (1), we get,

$$\begin{aligned}[g(F_{SPx_{2m_i},TPx_{2n_i-1}}(t))]^2 \le\ & \alpha_1[g(F_{x_{2m_i},x_{2n_i-1}}(t))]^2 + \alpha_2 g(F_{x_{2m_i},x_{2m_i+1}}(t))g(F_{x_{2n_i-1},x_{2n_i}}(t)) + \alpha_3 g(F_{x_{2m_i},x_{2n_i}}(t))g(F_{x_{2n_i-1},x_{2m_i+1}}(t))\\ &+\alpha_4 g(F_{x_{2m_i},x_{2m_i+1}}(t))g(F_{x_{2m_i},x_{2n_i}}(t)) + \alpha_5 g(F_{x_{2n_i-1},x_{2n_i}}(t))g(F_{x_{2n_i-1},x_{2m_i+1}}(t))\\ &+\alpha_6 g(F_{x_{2m_i},x_{2n_i}}(t))g(F_{x_{2n_i-1},x_{2n_i}}(t)) + \alpha_7 g(F_{x_{2m_i+1},x_{2n_i}}(t))g(F_{x_{2m_i},x_{2n_i-1}}(t))\\ &+\alpha_8 g(F_{x_{2m_i},x_{2n_i-1}}(t))g(F_{x_{2m_i},x_{2m_i+1}}(t)) + \alpha_9 g(F_{x_{2m_i},x_{2n_i-1}}(t))g(F_{x_{2m_i},x_{2n_i}}(t))\\ &+\alpha_{10} g(F_{x_{2m_i},x_{2n_i-1}}(t))g(F_{x_{2n_i-1},x_{2m_i+1}}(t)) + \alpha_{11} g(F_{x_{2m_i},x_{2n_i-1}}(t))g(F_{x_{2n_i-1},x_{2n_i}}(t)).\end{aligned}$$

Taking limit as $n\to\infty$, we have,

$[g(1-\varepsilon_0)]^2 \le (\alpha_1+\alpha_3+\alpha_7+\alpha_9+\alpha_{10})[g(1-\varepsilon_0)]^2$

i.e. $[g(1-\varepsilon_0)]^2 < [g(1-\varepsilon_0)]^2$

(because $(\alpha_1+\alpha_3+\alpha_7+\alpha_9+\alpha_{10}) < 1$), which is not possible. Thus $\{x_n\}$ is a Cauchy sequence. Since (*X, F; t*) be a complete non- Archimedean Menger space so

$\{x_n\} \to z \in X$ hence from (1), we get,

$[g(F_{SPz,x_{2n}}(t))]^2 = [g(F_{SPz,TPx_{2n-1}}(t))]^2$,

$$\begin{aligned}\text{i.e. } [g(F_{SPz,x_{2n}}(t))]^2 \le\ & \alpha_1[g(F_{z,x_{2n-1}}(t))]^2 + \alpha_2 g(F_{z,SPz}(t))g(F_{x_{2n-1},x_{2n}}(t)) + \alpha_3 g(F_{z,x_{2n}}(t))g(F_{x_{2n-1},SPz}(t))\\ &+\alpha_4 g(F_{z,SPz}(t))g(F_{z,x_{2n}}(t)) + \alpha_5 g(F_{x_{2n-1},x_{2n}}(t))g(F_{x_{2n-1},SPz}(t))\\ &+\alpha_6 g(F_{z,x_{2n}}(t))g(F_{x_{2n-1},x_{2n}}(t)) + \alpha_7 g(F_{SPz,x_{2n}}(t))g(F_{z,x_{2n-1}}(t))\\ &+\alpha_8 g(F_{z,x_{2n-1}}(t))g(F_{z,SPz}(t)) + \alpha_9 g(F_{z,x_{2n-1}}(t))g(F_{z,x_{2n}}(t))\\ &+\alpha_{10} g(F_{z,x_{2n-1}}(t))g(F_{x_{2n-1},SPz}(t)) + \alpha_{11} g(F_{z,x_{2n-1}}(t))g(F_{x_{2n-1},x_{2n}}(t)).\end{aligned}$$

Making $n\to\infty$,we get,

$[g(F_{SPz,z}(x))]^2 \le 0$ i.e. $g(F_{SPz,z}(x)) = 0$ or $SPz = z.$

Similarly, by considering $[g(F_{x_{2n+1},TPz}(t))]^2$, we conclude from (1) that

$TPz = z$, i.e. $TPz = z = SPz$ (2.1)

Again, if SP = PS, then

$[g(F_{Pz,z}(t))]^2 = [g(F_{PSPz,TPz}(t))]^2 = [g(F_{SPPz,TPz}(t))]^2$ (2.2)

i.e.

$$\begin{aligned}[g(F_{Pz,z}(t))]^2 \le\ & \alpha_1[g(F_{Pz,z}(t))]^2 + \alpha_2 g(F_{Pz,Pz}(t))g(F_{z,z}(t)) + \alpha_3 g(F_{Pz,z}(t))g(F_{z,Pz}(t))\\ &+\alpha_4 g(F_{Pz,Pz}(t))g(F_{Pz,z}(t)) + \alpha_5 g(F_{z,z}(t))g(F_{z,Pz}(t))\\ &+\alpha_6 g(F_{Pz,z}(t))g(F_{z,z}(t)) + \alpha_7 g(F_{Pz,z}(t))g(F_{Pz,z}(t))\\ &+\alpha_8 g(F_{Pz,z}(t))g(F_{Pz,Pz}(t)) + \alpha_9 g(F_{Pz,z}(t))g(F_{Pz,z}(t))\\ &+\alpha_{10} g(F_{Pz,z}(t))g(F_{z,Pz}(t)) + \alpha_{11} g(F_{Pz,z}(t))g(F_{z,z}(t))\end{aligned}$$

i.e. $[g(F_{Pz,z}(t))]^2 \le (\alpha_1+\alpha_3+\alpha_7+\alpha_9+\alpha_{10})[g(F_{Pz,z}(t))]^2$,

i.e. $(1-(\alpha_1+\alpha_3+\alpha_7+\alpha_9+\alpha_{10}))[g(F_{Pz,z}(t))]^2 \le 0,$ hence $g(F_{Pz,z}(t)) = 0,$ thus $Pz = z.$

Hence by (2.2) $Tz = z = Sz.$ Similarly if $PT = TP$, then also $Pz = z = Sz = Tz$.

Therefore, *z* is a common fixed point of *S, T, P*.

For uniqueness suppose z and z' are common fixed points of S, T, P

.

Then from (1), we have,

$$[g(F_{z,z'}(t))]^2 = [g(F_{SPz,TPz'}(t))]^2 \le \alpha_1[g(F_{z,z'}(t))]^2 + \alpha_2 g(F_{z,z}(t))g(F_{z',z'}(t)) + \alpha_3 g(F_{z,z'}(t))g(F_{z',z}(t))$$
$$+\alpha_4 g(F_{z,z}(t))g(F_{z,z}(t)) + \alpha_5 g(F_{z',z'}(t))g(F_{z',z'}(t))$$
$$+\alpha_6 g(F_{z,z'}(t))g(F_{z',z'}(t)) + \alpha_7 g(F_{z,z'}(t))g(F_{z,z'}(t))$$
$$+\alpha_8 g(F_{z,z'}(t))g(F_{z,z}(t)) + \alpha_9 g(F_{z,z'}(t))g(F_{z,z'}(t))$$
$$+\alpha_{10} g(F_{z,z'}(t))g(F_{z',z}(t)) + \alpha_{11} g(F_{z,z'}(t))g(F_{z',z'}(t)),$$

i.e. $[g(F_{z,z'}(t))]^2 \le (\alpha_1 + \alpha_3 + \alpha_7 + \alpha_9 + \alpha_{10})[g(F_{z,z'}(t))]^2$

i.e. $(1-(\alpha_1 + \alpha_3 + \alpha_7 + \alpha_9 + \alpha_{10}))[g(F_{z,z'}(t))]^2 \le 0$, therefore $g(F_{z,z'}(t)) = 0$, hence $z = z'$.

This proves the uniqueness.

By using the definition of weakly commuting pair of mappings with respect to other mapping, we have proved the following theorem which is generalization of result of Sachdeva [10] in a metric space to more general setting of a non-Archimedean Menger space of type C_g .

Theorem 2.2. Let $(X, F;\ t)$ be a complete non- Archimedean Menger space. Suppose $S, T, P$are self-mappings on X satisfying the following conditions.

(1). $$[g(F_{SPx,TPy}(t))]^2 \le \alpha_1[g(F_{x,y}(t))]^2 + \alpha_2 g(F_{x,SPx}(t))g(F_{y,TPy}(t)) + \alpha_3 g(F_{x,TPy}(t))g(F_{y,SPx}(t))$$
$$+\alpha_4 g(F_{x,SPx}(t))g(F_{x,TPy}(t)) + \alpha_5 g(F_{y,TPy}(t))g(F_{y,SPx}(t))$$
$$+\alpha_6 g(F_{x,TPy}(t))g(F_{y,TPy}(t)) + \alpha_7 g(F_{SPx,TPy}(t))g(F_{x,y}(t))$$
$$+\alpha_8 g(F_{x,y}(t))g(F_{x,SPx}(t)) + \alpha_9 g(F_{x,y}(t))g(F_{x,TPy}(t))$$
$$+\alpha_{10} g(F_{x,y}(t))g(F_{y,SPx}(t)) + \alpha_{11} g(F_{x,y}(t))g(F_{y,TPy}(t)),$$

for all $\alpha_i > 0$, $i = 1, 2, \ldots 11$ such that

(2). $\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4 + 2\alpha_5 + 2\alpha_6 + \alpha_7 + \alpha_8 + 2\alpha_9 + 2\alpha_{10} + \alpha_{11} < 1$,

The pair (S, T) is weakly commuting with respect to$P$.

Then S, T, P have a unique common fixed point in X.

Proof. By proceeding as in the proof of above Theorem 2.1, we conclude that z is a common fixed point of SP $and\ TP$, $i.e.\ SPz\ =\ TPz\ =\ z$.

Now, using the condition (1) and definition of weakly commuting pair $(S, T\ )$ with respect to P, we have,

$[g(F_{Pz,z}(t))]^2 = [g(F_{PSPz,TPz}(t))]^2 \le [g(F_{SPPz,TPz}(t))]^2$.

Again, as in the proof of above theorem along with equation 2.2, zis unique common fixed point ofS, T and P.

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REFERENCES

[1]. **Bharucha-Reid A.T.** Fixed point theorems in probabilistic analysis,Bull. Amer. Math. Soc. 82 (1976), 641-657.
[2]. **Chang S. S., Cho Y. J., Kang S. M.** Probabilistic metric spaces and nonlinear operator theory, Sichuan Univ. Press, Chengdu, China, (1994).
[3]. **Chatterjee A. K. and Singh M. R.** Common fixed points for four maps on a metric space, Bull. Cal. Math. Soc. 81 (1989), 466A-466B.
[4]. **Egbert R. J.** Products and quotients of probabilistic metric spaces, Pacific J. Math. 24 (1958), 437-455.
[5]. **Fisher B.** Common fixed and constant mappings on metric space, Math. Sem. Notes, Kobe Univ. 5 (1977), 319-326.
[6]. Menger K. Statistical metrices, Proc. Nat. Acad. Sci. USA 28 (1942), 535-537.
[7]. Pathak H. K. A note on fixed point theorems of Rao and Rao, Bull. Cal. Math. Soc. 79 (1987), 267-273.
[8]. Qin Huang Xiao, Xi Z. C. and Jie L. X. Fixed point theorem in probabilistic inner product spaces and its applications. Appl. Math and computing 19 (2) (2005), 363 – 370.
[9]. Rao I. M. N. and Rao K. P. R. Common fixed point for three maps on a metric space, Bull. Cal. Math. Soc. 76(1984), 228-230.
[10]. Sachdeva N. Some general theorems on common fixed points, D. Phill. thesis Rani Durgavati University Jabalpur M. P. 1996.
[11]. Schweizer B. and Sklar A. Statistical metric spaces, Pacific J. Math. 10 (1960), 313-334.
[12]. **Tripathi P.K. and Gupta Manisha.** JungckType contractions and its application, Int. Journal of Math. Analysis, 4(37), (2010), 1837 – 1850.
[13]. **13. Tripathi P. K., Mishra S. N. and GuptaManisha.**Application of Contraction Mapping InMenger Spaces, Global Journal of Pure and Applied Mathematics 12(2) (2016), 1629-1634.

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