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The Upper Total Edge Monophonic Number Of A Graph

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ABSTRACT

A set M of vertices of a connected graph G is a monophonic set if every vertex of G lies on an x-y monophonic path for some elements x and y in M. The minimum cardinality of a monophonic set of G is the monophonic number of G,and is denoted by m(G). A monophonic set of cardinality m(G) is called a m-set of G. Any monophonic set of order m(G)is a minimum monophonic set of G. An edge monophonic set M in a connected graph G is called a minimal edge monophonic set if no proper subset of M is a edge monophonic set of G. The total edge monophonic set M of a graph G is a edge monophonic set M such that the subgraph induced by M has no isolated vertices, and is denoted by $em_t(G)$. The upper total edge monophonic set of a graph G is a minimal total edge monophonic set M such that the subgraph induced by M has no isolated vertices. The upper total edge monophonic number is the maximum cardinality of a minimal total edge monophonic set of G, and is denoted by $em_t^+(G)$. The upper total edge monophonic number of some connected graphs are realized. It is proved that for any integers, a, b and c such that $2\leq a\leq b<c$, there exist a connected graph G with $em(G)=a, em^+(G)=b$ and $em_t^+(G)=c$.

KEYWORDS: Monophonic set, monophonic number, edge monophonic set, edge monophonic number, total edge monophonic number, upper total monophonic number, upper total edgemonophonic number.
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I. INTRODUCTION

By a graph $G = (V, E)$ we mean a simple graph of order at least two. The order and size of G are denoted by p and q, respectively. For basic graph theoretic terminology,we refer to Harary [2]. The neighborhood of a vertex v is the set $N(v)$ consisting of all vertices u which are adjacent with v. The closed neighborhood of a vertex v is the set$N[v] =N(v) \cup \{v\}$.A vertex v is an extreme vertex if the sub graph induced by its neighbors is complete. A vertex v is a semi-extreme vertex of G if the sub graph induced by its neighbors has a full degree vertex in $N(v)$. In particular, every extreme vertex is a semi - extreme vertex and a semi - extreme vertex need not be an extreme vertex.

For any two vertices x and y in a connected graph G, the distance $d(x, y)$ is the length of a shortest x-y path in G. An x-y path of length $d(x, y)$ is called an x-y geodesic. A vertex v is said to lie on an x-y geodesic P if v is a vertex of P including the vertices x and y.The geodetic number of a graph was introduced in [4].The eccentricity $e(v)$ of a vertex v in G is the maximum distance from v and a vertex of G. The minimum eccentricity among the vertices of G is the radius, rad (G) or $r(G)$ and the maximum eccentricity is its diameter, diam$G$ of G.

A chord of a path $u_1, u_2, \ldots, u_k$ in G is an edge u_iu_j with $j\geq i + 2$. Anu-v path P is called a monophonic path if it is a chordless path. A set M of vertices is a monophonic set if every vertex of G lies on a monophonic path joining some pair of vertices in M,and the minimum cardinality of a monophonic set of G is the monophonic number of G, and is denoted by m(G). The monophonic number of a graph G was studied in [9]. A monophonic set M in a connected graph G is called a minimal monophonic set if no proper subset of M is a monophonic set of *G*.The upper monophonic number $m^+(G)$ of *G* is the maximum cardinality of a minimal monophonic set of *G*. The upper monophonic number of a graph *G* was studied in [8]. A set *M* of vertices of a graph *G* is an edge monophonic set if every edge of *G* lies on a$x - y$ monophonic path for some elements *x* and *y* in *M*. The minimum cardinality of an edge monophonic set of *G* is the edge monophonic number of *G*, denoted by $em(G)$. The edge monophonic number of a graph was introduced and studied in [6]. A total edge monophonic set of a graph *G* is a edge monophonic set *M* such that the subgraph induced by *M* has no isolated vertices. The minimum cardinality of a total edge monophonic set of *G* is the total edge monophonic number, denoted by $em_t(G)$. The total edge monophonic number of a graph G was studied in [1]. An edge monophonic set *M* in a connected graph *G* is called a minimal edge monophonic set if no proper subset of *M* is an edge monophonic set of *G*. The upper edge monophonic number $em^+(G)$ of *G* is the maximum cardinality of a minimal edge monophonic set of *G*. The upper edge monophonic number of a graph *G* was studied in [7]. The upper total edge monophonic set of a graph *G* is a minimal total edge monophonic set *M* such that the subgraph induced by *M* has no isolated vertices. The upper total edge monophonic number is the maximum cardinality of a minimal total edge monophonic set of *G*, and is denoted by $em_t^+(G)$.

The following Theorems will be used in the sequel.
Theorem 1.1[6]: Each simplical vertex of G belongs to every edge monophonic set of G.
Theorem 1.2[7]: No cut vertex of G belongs to any minimal edge monophonic set of G.
Theorem1.3[9]: Each extreme vertex of a connected graph *G* belongs to every monophonic set of *G*.
Theorem 1.4[9] **:** Let *G* be a connected graph with diameter *d*. Then $m(G) \leq p-d+1$.
Theorem 1.5[8] :Let *G* be a connected graph with cut vertices and *M* be a minimal monophonic set of *G*. If *v* is a cut vertex of *G*, then every component of $G-v$ contains an element of *M*.
Theorem 1.6 [1] : Let *G* be a connected graph with cut vertices and *M* be total edge monophonic set of *G*. If *v* is a cut vertex of *G*, then every component of $G-v$ contains an element of *M*.
Throughout this paper *G* denotes a connected graph with atleast two vertices.

The Upper Total Edge Monophonic Number of a Graph

Definition 2.1:
The total edge monophonic set *M* in a connected graph *G* is called a minimal total edge monophonic set if no proper subset of *M* is a total edge monophonic set of *G*. The upper total edge monophonic number $em_t^+(G)$ is the maximum cardinality of a minimal total edge monophonic set of *G*.

Example 2.2: For the graph *G* given in figure 2.1, $M_1 = \{v_1,v_3\}$ is the only minimum edge monophonic set of *G*, so that $m(G) =2$. $M_2 = \{v_1,v_3,v_4\}$ is the minimum total edge monophonic set of *G*, so that $em_t(G) = 3$. The set $M_3 = \{v_2,v_4,v_5\}$ is the only minimal edge monophonic set of *G*, so that $em^+(G) = 3$. $M_4 = \{v_2,v_3,v_4,v_5\}$ is the total edge monophonic set of *G*and it is clear that no proper subset of $M_4$is an total edge monophonic set of G, and so$M_4$is a minimal total edge monophonic set of *G* so that $em_t^+(G) \geq 4$. It is easily verified that no five elements set of *G* is the total edge monophonic set of *G*. Hence it follows that $em_t^+(G) = 4$

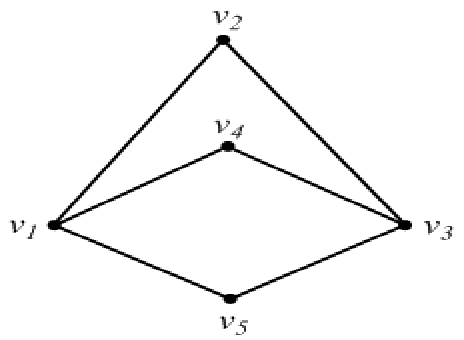

Figure: 2.1

Remark 2.3: Every minimum total edge monophonic set of *G* is a minimal total edge monophonic set of *G* and the converse is not true. For the graph *G* given in Figure 2.1, $M_4= \{v_2,v_3,v_4,v_5\}$ is a minimal total edge monophonic set but not a minimum total edge monophonic set of G.

Theorem 2.4: For any connected graph $G, 2 \leq em_t(G) \leq em^+_t(G) \leq p$

Proof : Any total edge monophonic set needs atleast 2 vertices and so $em_t(G) \geq 2$. Since every minimal total edge monophonic set is the total edge monophonic set ,$em_t(G) \leq em_t^+(G)$. Also since $V(G)$ is the total edge monophonic set of *G*, it is clear that $em_t^+(G) \leq p$. Thus $2 \leq em_t(G) \leq em_t^+(G) \leq p$.

Theorem 2.5: For any connected graph G, $em_t(G) = p$ if and only if $em^{+}_{t}(G) = p$

Proof: If $em_t(G) = p$, then $M = V(G)$ is the unique minimal total edge monophonic set of G. Since no proper subset of M is the total edge monophonic set, it is clear that M is the unique minimal total edge monophonic set of G and so $em^{+}_{t}(G) = p$. The converse part follows from Theorem 2.4.

Theorem 2.6 : For the complete graph $K_P(p \geq 2)$, $em_t^{+}(K_p) = em^{+}(K_p) = p$.

Proof : Since every vertex of the complete graph Kp $(p \geq 2)$ is an extreme vertex, the vertex set of Kp is the unique monophonic set and the minimal total edge monophonic set contains all the vertices.Thus$em^{+}(Kp) = em_t^{+}(Kp) = p$.

Theorem 2.7: Let Gbe a connected graph of order p with $em_t(G) = p$-1.Then $em_t^{+}(G)=p$-1.

Proof: Since $em_t(G) = p-1$, it follows from Theorem 2.4,$em_t^{+}(G) = p$ or $p-1$. If $em_t^{+}(G)$ =p, then by Theorem 2.5,$em_t(G)= p$,which is a contradiction. Hence $em_t^{+}(G) = p-1$.

Theorem 2.8 : For a connected graph G of order p, the following are equivalent:

i. $em_t^{+}(G) = p$

ii. $em(G) = p$

iii. $G = K_p$

Proof :(i)=>(ii). Let $em^{+}_{t}(G) = p$. Then $M = V(G)$ is the unique minimal total edge monophonic set of G. Since no proper subset of M is a edge monophonic set, it is clear that M is the unique minimum totaledge monopnic set of G and so $em(G) = p$.

(ii)=>(iii). Let $em(G) = p$. If $G \neq K_p$, then by theorem 1.2, $m(G) \leq p$-1, which is a condradiction. Therfore$G = K_{p.}$

(iii) $\Rightarrow$ (i). Let G = K_p. Then by Theorem 2.5,$em_t^{+}(G) = p$.

Theorem 2.9 : Let G be a connected graph with cut vertices and M be a minimal monophonic set of G. If v is a cut vertex of G, Then every component of G-v contains some vertices of M.

Proof: Since every minimal total edge monophonic set is also a total edge monophonic set, the result follows from Theorem 1.5 and Theorem 1.6.

Theorem 2.10: For any connected graph G, no cut vertex of G belongs to any minimal total edge monophonic set of G.

Proof : Let M be a minimal total edge monophonic set of G and $v \in M$ be any vertex. We claim that v is not a cut vertex of G. Suppose that v is a cut vertex of G. Let $G_1, G_2, ..., G_r (r \geq 2)$ be the components of G-v. By theorem 2.9, each component Gi, $(1 \leq i \leq r)$ contains an element of M. Let $M_1 = M - \{v\}$. Let uw be an edge of G which lies on a monophonic path P joining a pair of vertices u and v of M. Assume without loss of generality that $u \in G_1$. Since v is adjacent to atleast one vertex of each $G_i(1 \leq i \leq r)$, assume that v is adjacent to z in G_k, $k \neq 1$. Since M is an edge monophonic set, vz lies on a monophonic path Q joining v and a vertex w of M such that w must necessarily belongs to G_k. Thus $w \neq v$. Now, since v is a cut vertex of G, $P \cup Q$ is a path joining u and w in M and thus the edgeuv lies on this monophonic path joining two vertices of M_1. Hence it follows that every edge of G lies on a monophonic path joining two vertices of M_1, which shows thatM_1 is a edge monophonic set of G.Since $M_1 \subsetneq M$, this contradicts the fact that M is a minimal total edge monophonic set of G. Hence $v \notin M$,so that no cut vertex of G belongs to any minimal total edge monophonic set of G.

Theorem 2.11: For any Tree T with k vertices, $em_t^{+}(T) = k$.

Proof: By Theorem 1.3, any monophonic set contains all the end vertices of T. Hence it follows that, the set of all end vertices of T is the unique minimal edge monophonic set of T, so that $em^{+}_{t}(T) = k$.

Theorem 2.12: For a cycle $G = C_p(p \geq 4)$, $m^{+}_{t}(G) = 3$.

Proof : First suppose that $G = C_3$. It is a complete graph, by Theorem 2.5, we have $em^{+}_{t}(G) = 3$. For any cycle ,suppose that $em^{+}_{t}(G) > 3$, then there exist a minimal total edge monophonic set M_1 such that $|M_1| \geq 3$. Now it is clear that edge monophonic set $M \subsetneq M_1$, which is a contradiction to M_1 is a minimal total edge monophonic set of G.Therfore$em^{+}_{t}(G) = 3$.

Theorem 2.13: For the complete bipartite graph $G = K_{m,n.}$

(i) $em_t^{+}(G) = 2$ if $m = n = 1$

(ii) $em^{+}_{t}(G) = n+1$ if $m = 1$, $n \geq 2$

(iii) $em_t^{+}(G) = max\{m,n\}+1$, if $m,n \geq 2$.

Proof: (i) and (ii) follows from Theorem 2.10. (iii) Let $m,n \geq 2$. Assume without loss of generality that $m \leq n$. First assume that $m<n$. Let $X = \{x_1, x_2, \ldots, x_m\}$ and $Y = \{y_1, y_2, .., y_n\}$ be a bipartion of G. Let $M = Y$. We prove that M is a minimal total edge monophonic set of G. Any edge$y_i x_j (1 \leq i \leq n, 1 \leq j \leq m)$ lies on a monophonic path y_i, x_j, y_k for $k \neq$ i so that M is a edge monophonic set of G. Let $M' \subsetneq M$ Then there exists a vertex $y_j \in M$ such that $y_j \notin M'$.Then the edge$y_j x_i$ $(1 \leq j \leq m, 1 \leq i \leq n)$ does not lie on a monophonic path joining a pair of vertices of M'. Thus M' isnot aedgemonophonic set of G. This shows that M is a minimal edge monophonic set of G. Hence $em_t^{+}(G) \geq n$. Let M_1 be a minimal edge monophonic set of G such that $|M_1| \geq n+1$. Since the vertex $x_i y_j$ $(1 \leq i \leq m$ and $1 \leq j \leq n)$ lies on a monophonic path $x_i x_k y_j$for any $k \neq i$, it follows that X is an edge monophonic set of G. Hence M_1 cannot contain X.Similarly ,since Y is a minimal edge monophonic set of G, M_1 cannot contain Y also. Hence $M_1 \subsetneq X' \cup Y'$,where$X' \subsetneq X$ and $Y' \subsetneq Y$. Hence there exist a vertex $x_i \in X (1 \leq i \leq m)$ and a vertex $y_j \in$ Y

($1 \leq i \leq n$) such that $x_iy_j \notin M_1$. Hence the edge x_iy_j does not lie on a monophonic path joining a pair of vertices of M_1. It follows that M_1 is not a edge monophonic set of G, which is a contradiction. Thus M is a minimal total edge monophonic set of G. Hence $em_t^+(G) = \max\{m, n\}+1$.

Realization Results:

Theorem 3.1:
For every positive integers a,b and c where $2 \leq a \leq b < c$, there exists a connected graph G with $em(G) = a$, $em(G) = b$ and $em^+_t(G) = c$.

Proof:
Let $V(K_2) = \{u,x\}$ and $V[K_{b-a+1}] = \{v_1,v_2,\ldots,v_{b-a+1}\}$. Let $H = K_{b-a+1}+K_2$. Let G be the graph in figure 3.1 obtained from H by adding $a-1$ new vertices $x_1,x_2,\ldots,x_{a-1}$ and joining each vertex x_i($1 \leq i \leq a-1$) with x. Subdivide the edge xx_i, where $1 \leq i \leq c-b-1$, calling the new vertices $y_1,y_2,\ldots,y_{c-b-1}$, where x_i is adjacent to y_i and y_i is adjacent to x for all $i \in \{1,2,\ldots,c-b-1\}$. The graph G is shown in figure 3.1.

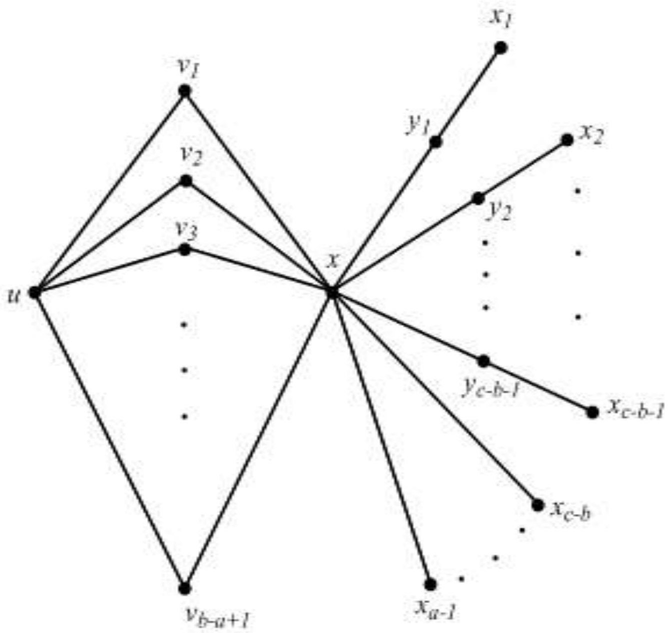

Figure 3.1

Let $M = \{x_1,x_2,\ldots x_{a-1}\}$ be the set of all end vertices of G. Clearly M is a subset of every edge monophonic set of G. Let $M_1 = M \cup \{u\}$. Then M_1 is an edge monophonic set of G, so that $em(G) = a$. Now $T = M \cup \{v_1,v_2,\ldots,v_{b-a+1}\}$ is an edge monophonic set of G. We show that T is a minimal edge monophonic set of G. Clearly, no proper subset of T is an edge monophonic set of G. Hence T is a minimal edge monophonic set of G, so that $em^+(G) = a-1+b-a+1 = b$. Also $M_2 = T \cup \{y_1,y_2,\ldots,y_{c-b-1}, x\}$ is a minimal total edge monophonic set of G, $em_t^+(G) = c$.

Theorem 3.2: For positive integers r_m,d_m and $l \geq 5$ with $r_m < d_m \leq 2r_m$, there exists a connected graph G with $\mathrm{rad}_m(G) = r_m$, $\mathrm{diam}_m(G) = dm$ and $em_t^+(G) = l$.

Let $r_m = 1$. Let $r_m \geq 2$. Let $C_{r_m+2}: v_1,v_2,\ldots,v_{r_m+2}$ be a cycle of length r_m+2 and let $P_{d_m-r_m}: u_0,u_1,u_2,\ldots,u_{d_m-r_m}$ be a path of length d_m-r_m+1. Let H be a graph obtained from C_{r_m+2} and by identifying v_1 in C_{r_m+2} and u_0 in $P_{d_m-r_m+1}$. Now add $l-5$ new vertices $w_1,w_2,\ldots,w_{l-5}$ to H and join each w_i($1 \leq i \leq l-5$) to the vertex $u_{d_m-r_m-1}$ and obtain the graph G of Figure 3.2.

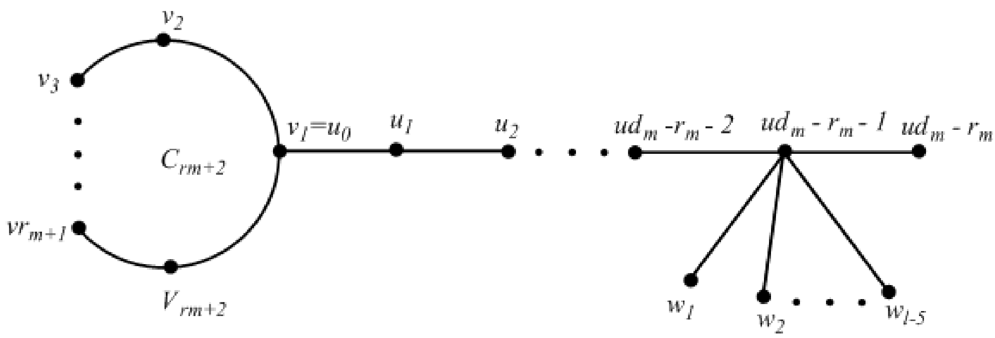

Figure 3.2

Then $\mathrm{rad}_m(G) = r_m$, $\mathrm{diam}_m(G) = d$. *Let* $M = \{w_1, w_2, \ldots, w_{l-5}, u_{d_m - r_m}\}$ be the set of all end vertices of G . It is clear that M is not an edge monophonic set of G and so $em(G) \geq l$. The set $M \cup \{x\}$, where $x \in \{v_3, v_4, \ldots, v_{r_m+1}\}$ is an edge monophonic set of G . We show that $em^+_t(G) = l$. Now $M_1 = M \cup \{v_2, v_{r_m+2}\}$ is a minimal edge monophonic set of G and so $em^+(G) \geq l$. Suppose that $em^+(G) \geq l+1$. Then there exists a minimal edge monophonic set T such that $|T| \geq l+2$. Hence there exists $y \in T$ such that $y \notin M_1$. By Theorem 1.1, $M \subset T$.If $y \in \{v_3, v_4, \ldots, v_{r_m+1}\}$, then $M \cup \{y\}$ is an edge monophonic set of G, which is a contradiction to T is a minimal edge monophonic set of G. If $y \notin \{v_3, v_4, \ldots, v_{r_m+1}\}$,then by corollary 1.1, $y \notin M$. Therfore$y = u_i (0 \leq i \leq d_m - r_m - 1)$. By Theorem 1.2,$y \notin T$, which is a contradiction . Thus M_1 is a minimal edge monophonic set of G . Now$M_2 = M_1 \cup \{u_{d_m - r_m - 1}, v_1\}$ is a minimal total edge monophonic set of G. It is clear that no proper subset of M_2 is a total edgemonophonic set of G, so that $em_t^+(G) = l$.

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Let $r_m \geq 2$. Let $C_{rm+2:}\ v_1, v_2, \ldots, v_{rm+2}$ be a cycle of length r_{m+2} and let $P_{dm-rm+1}$: $u_0, u_1, u_2, \ldots, u_{dm-rm}$ be a path of length $d_m - r_m + 1$. Let H be a graph obtained from C_{rm+2} and$P_{dm-rm+1}$ by v_1 in C_{rm+2} and u_0 in $P_{dm-rm+1.}$ Now add l-5 new vertices $w_1, w_2, \ldots, w_{l-5}$ to H and join each $w_i (1 \leq i \leq l-5)$ to the vertex $u_{dm-rm-1}$ and obtain the grThenrad$_m(G) = r_m$, $\mathrm{diam}_m(G) = dw_1, w_2, \ldots, w_{l-5}, u_{dm} - r_m\}$ be the set of all end vertices of G .It is clear that M is not an edge monophonic set of G and so $em(G) \geq l$. The set $M \cup \{x\}$, where $x \in \{v_3, v_4, \ldots, v_{rm+1}\}$ is an edge monophonic set of G . We show that $em^+_t(G) = l$. Now $M_1 = M \cup \{v_2, v_{rm+2}\}$ is a minimal edge monophonic set of G and so $em^+(G) \geq l$. Suppose that $em^+(G) \geq l+1$. Then there exists a minimal edge monophonic set T such that $|T| \geq l+2$. Hence there exists $y \in T$ such that $y \notin M_1$. By Theorem 1.1, $M \subset T$.If $y \in \{v_3, v_4, \ldots, v_{rm+1}\}$, then $M \cup \{y\}$ is an edge monophonic set of G, which is a contradiction to T is a minimal edge monophonic set of G. If $y \notin \{v_3, v_4, \ldots, v_{rm+1}\}$,then by corollary 1.1, $y \notin M$. Therfore$y = u_i (0 \leq i \leq d_m - r_m - 1)$. By Theorem 1.2, $y \notin T$,which is a contradiction . Thus M_1 is a minimal edge monophonic set of G . Now $M_2 = M_1 \cup \{u_{dm\text{-rm-}1}, v_1\}$ is a minimal total edge monophonic set of G. It ``````` of M_2 is a total edge monophonresult follows from Theorem 2.9. Let

Then $\mathrm{rad}_m(G) = r_m$, $\mathrm{diam}_m(G) = dw_1, w_2, \ldots, w_{l-5}, u_{dm} - r_m\}$ be the set of allis clear that M is not an edge monophonic set of G and so $em(G) \geq l$. The set $M \cup \{x\}$, where $x \in \{v_3, v_4, \ldots, v_{rm+1}\}$ is an edge monophonic set of G . We show that $em^+_t(G) = l$. Now $M_1 = M \cup \{v_2, v_{rm+2}\}$ is a minimal edge monophonic set of G and so $em^+(G) \geq l$. Suppose that $em^+(G) \geq l+1$. Then there exists a minimal edge monophonic set T such that $|T| \geq l+2$. Hence there exists $y \in T$ such that $y \notin M_1$. By Theorem 1.1, $M \subset T$.If $y \in \{v_3, v_4, \ldots, v_{rm+1}\}$, then $M \cup \{y\}$ is an edge monophonic set of G, which is a contradiction to T is a minimal edge monophonic set of G. If $y \notin \{v_3, v_4, \ldots, v_{rm+1}\}$,then by corollary 1.1, $y \notin M$. Therfore$y = u_i (0 \leq i \leq d_m - r_m - 1)$. By Theorem 1.2, $y \notin T$,which is a contradiction . Thus M_1 is a minimal edge monophonic set of G . Now $M_2 = M_1 \cup \{u_{dm\text{-rm-}1}, v_1\}$ is a minimal total edge monophonic set of G. It ``````` of M_2 is a total edge monophonic set of G, so **References**

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Let $r_m \geq 2$. Let $C_{rm+2:}\ v_1, v_2, \ldots, v_{rm+2}$ be a cycle of length r_{m+2} and let $P_{dm-rm+1}$: $u_0, u_1, u_2, \ldots, u_{dm-rm}$ be a path of length d_m-r_m+1. Let H be a graph obtained from C_{rm+2} and$P_{dm-rm+1}$ by v_1 in C_{rm+2} and u_0 in $P_{dm-rm+1.}$ Now add l-5 new vertices $w_1, w_2, \ldots, w_{l-5}$ to H and join each $w_i(1 \leq i \leq l$-$5)$ to the vertex $u_{dm-rm-1}$ and obtain the graph G of Figure 3.2.
Then $\mathrm{rad}_m(G) = r_m$, $\mathrm{diam}_m(G) = dw_1, w_2, \ldots, w_{l-5}, u_{dm}$-$r_m\}$ be the set of all end vertices of G . It is clear that M is not an edge monophonic set of G and so $em(G) \geq l$. The set $M \cup \{x\}$, where $x \in \{v_3, v_4, \ldots, v_{rm+1}\}$ is an edge monophonic set of G . We show that $em^+_t(G) = l$. Now $M_1 = M \cup \{v_2, v_{rm+2}\}$ is a minimal edge monophonic set of G and so $em^+(G) \geq l$. Suppose that $em^+(G) \geq l+1$. Then there exists a minimal edge monophonic set T such that $|T| \geq l+2$. Hence there exists $y \in T$ such that $y \notin M_1$. By Theorem 1.1, $M \subset T$.If $y \in \{v_3, v_4, \ldots, v_{rm+1}\}$, then $M \cup \{y\}$ is an edge monophonic set of G, which is a contradiction to T is a minimal edge monophonic set of G. If $y \notin \{v_3, v_4, \ldots, v_{rm+1}\}$,then by corollary 1.1, $y \notin M$. Therfore$y = u_i(0 \leq i \leq d_m$-$r_m$-$1)$. By Theorem1.2, $y \notin T$,which is a contradiction . Thus M_1 is a minimal edge monophonic set of G .Now $M_2 = M_1 \cup \{u_{dm-rm-1}, v_1\}$ is a minimal total edge monophonic set of G. It ``````` of M_2 is a total edge monophonic set of G, so

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