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Quantification of Cardiac Arrest among Beauty Conscious Individuals

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ABSTRACT
Beauty conscious individuals go for diet control or medicines to take care of their beauty. They may switch from medication to diet control or from diet control to medication. These individuals may exit at any stage; like stop the treatment voluntarily or may exit due to cardiac arrest. Malnutrition due to diet control and side effects of medication leads to cardiac arrest. In this paper, we have designed the system of non-linear differential equations to study the phenomenon of cardiac arrest among beauty conscious individuals. The basic reproduction number and the stability are derived to check the endurance of the model. The numerical simulation is also done using validated data. It is observed that after adopting any of the route, beauty conscious individuals will suffer from cardic arrest in 24-years.
KEYWORDS: Beauty, Cardiac arrest, Mathematical model, Basic reproduction number, Stability

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I. INTRODUCTION

Beauty conscious individuals are those who possess the combination of all qualities that delight the sense and pleases the mind [3]. As glamour is taking control of our life, everyone is getting more conscious and sensitive regarding their look. It is human nature to keep one's self attractive. It can be in terms of complexion, uniformity and smoothness of skin, hair in terms of color, volume and styling, body ratios (Golden ratios), lips (thickness, color and shape), nose shape, measurements of bust, waist and hips, overall body weight, n-packs (6-Packs/ 8 packs) and biceps and triceps. By carrying out the program entitle "always look pretty", one has forgot their health complexities. As compared to our ancestors, how we treat body is a reason for more compromised health status. In addition to it, one also cares to keep their originality at the peak. It has been observed by many professors of biological sciences that as people are getting more beauty conscious, health and psychological hazards prevails among those individuals in the society.

Rapidly, people bring change in their lives just to look more attractive even by compromising with their health. Every individual is born genuine, never try to be duplicate of someone. One must stop changing themselves for the cause of having one in our own life. Beauty consciousness is not bad, but oversensitivity towards it is surely a disease. Khalil Jibran says in his "The Prophet" that beauty is not a need but an ecstasy [7].

Dillon *et al.*[2] has done their research entitled "A Mathematical Model of Depression in Young Women as a Function of the Pressure to be beautiful". Shah *et al.* has carried out their research entitled "Mathematical Analysis of a Motivated Stage Artist to be a Film Artist" [10].

Recent news of famous actress Sridevi's death [8] due to cardiac arrest triggered our interest in relationship between medication/ diet control and cardiac arrest. In this paper, a mathematical model for the beauty conscious individuals have been formulated in Section 2. Stability Analysis has been discussed in Section 3. Sensitivity of the model parameters have also been studied in Section 4. Numerical Simulation has been carried out in Section 5 to support the analytical results. Section 6 comprise of Conclusion.

II. MATHEMATICAL MODEL

A mathematical model for quantification of cardiac arrest among beauty conscious individuals is formulated as shown in figure 1.

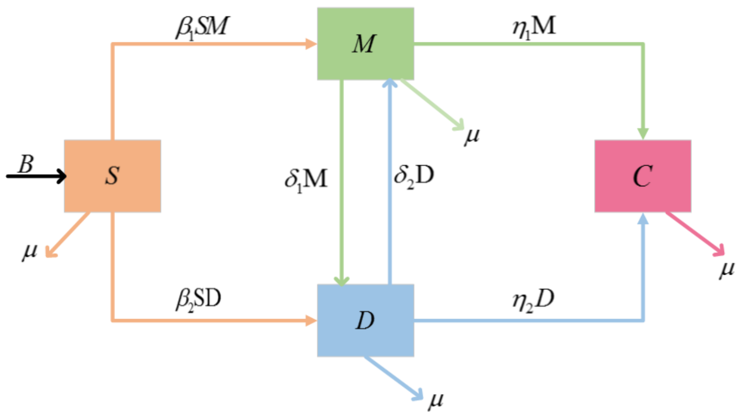

Figure 1: Transmission of beauty conscious individuals in various phases

Table 1: Description and values of parameters.

Notation	Description	Parametric Value
B	Recruitment rate of individuals as beauty conscious individual	0.05
β_1	Rate at which individuals choose medication option	0.30
β_2	Rate at which individuals choose diet control option	0.40
δ_1	Rate at which individuals under diet control switch to medication	0.60
δ_2	Rate at which individuals under medication switch to diet control	0.40
η_1	Rate at which individuals under medication end-up having cardiac arrest	0.20
η_2	Rate at which individuals under diet control end-up having cardiac arrest	0.60
μ	Rate at which individual leave the system	0.70

Figure 1 depicting the motion of beauty conscious individuals from one compartment to other can be described as the set of nonlinear ordinary differential equations.

$$\begin{aligned}
\frac{dS}{dt} &= B - \beta_1 SM - \beta_2 SD - \mu S \\
\frac{dM}{dt} &= \beta_1 SM - \eta_1 M - \delta_1 \mathrm{M} + \delta_2 D - \mu M \\
\frac{dD}{dt} &= \beta_2 SD - \eta_2 D + \delta_1 M - \delta_2 D - \mu D \\
\frac{dC}{dt} &= \eta_1 \mathrm{M} + \eta_2 D - \mu C
\end{aligned} \qquad (1)$$

Adding the above set of equation, we get

$$\frac{d}{dt}(S + M + D + C) = B - \mu(S + M + D + C) \geq 0$$

This gives, $\lim_{t\to\infty} \sup(S + M + D + C) \leq \frac{B}{\mu}$

Thus, the feasible region for (1) is,

$$\Lambda = \left\{ S + M + D + C \leq \frac{B}{\mu}; S > 0; M, D, C \geq 0 \right\}$$

Therefore, beauty conscious free equilibrium point of this model is $E_0 = \left(\frac{B}{\mu}, 0, 0, 0 \right)$.

Now, we need to calculate the basic reproduction number to know the migration of beauty conscious individuals in the system using the next generation matrix method given by Diekmann *et al*. [1]. The next generation matrix method is the spectral radius of matrix FV^{-1} where F and V are the Jacobian matrices of f and v evaluated with respect to each compartment at an equilibrium state [4].

Let $X=\left(M,D,C,S\right)$

Then, $\dfrac{dX}{dt}=f(X)-v(X)$

where $f(X)$ denotes the rate of new beauty conscious individuals and $v(X)$ denotes the rate of transmission of individuals from one compartment to other which is given by

$$f(X)=\begin{bmatrix}\beta_1 SM\\ \beta_2 SD\\ 0\\ 0\end{bmatrix} \text{ and } v(X)=\begin{bmatrix}\eta_1 M+\delta_1 M-\delta_2 D+\mu M\\ \eta_2 D-\delta_1 M+\delta_2 D+\mu D\\ -\eta_1 M-\eta_2 D+\mu C\\ -B+\beta_1 SM+\beta_2 SD+\mu S\end{bmatrix}$$

Now, the derivative of $\Im$ and v calculated at an equilibrium point E_0 gives matrices F and V of order 4×4 defined as

$$F=\left[\frac{\partial \Im_i(E_0)}{\partial X_j}\right] \quad \text{and} \quad V=\left[\frac{\partial v_i(E_0)}{\partial X_j}\right]; \text{ for } i,\ j=1,2,3,4$$

So, $$F(E_0)=\begin{bmatrix}\frac{\beta_1 B}{\mu} & 0 & 0 & 0\\ 0 & \frac{\beta_2 B}{\mu} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix} \text{ and } V(E_0)=\begin{bmatrix}\eta_1+\delta_1+\mu & -\delta_2 & 0 & 0\\ -\delta_1 & \eta_2+\delta_2+\mu & 0 & 0\\ -\eta_1 & -\eta_2 & \mu & 0\\ \frac{\beta_1 B}{\mu} & \frac{\beta_2 B}{\mu} & 0 & \mu\end{bmatrix}$$

where V is non-singular matrix. Thus, the basic reproduction number R_0 which is the spectral radius of matrix FV^{-1} is given as

$$R_0=\frac{B[\beta_1(\delta_2+\eta_2+\mu)+\beta_2(\delta_1+\eta_1+\mu)]}{\mu[(\eta_2+\mu)(\delta_1+\eta_1+\mu)+\delta_2(\eta_1+\mu)]}$$

Solving the set of equation (1) gives two equilibrium points namely:

(i) Beauty conscious free equilibrium point $E_0=\left(\dfrac{B}{\mu},0,0,0\right)$

(ii) Endemic equilibrium point $E^*=\left(S^*,M^*,D^*,C^*\right)$

$$S^*=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

where, $a=\beta_1\beta_2,\ b=-[\beta_1(\delta_2+\eta_2+\mu)+\beta_2(\delta_1+\eta_1+\mu)], c=[\delta_1(\eta_2+\mu)+(\eta_1+\mu)(\delta_2+\eta_2+\mu)]$

$$M^*=\frac{\delta_2(S^*\mu-B)}{[S^*\beta_1-(\eta_1+\mu+\delta_1)](\eta_2+\mu)-\delta_2(\eta_1+\mu)}$$

$$C^*=\frac{\begin{array}{c}S^*\beta_1\eta_2(B\beta_2\text{-}\delta_2\mu\text{-}\mu^2\text{-}\eta_2\mu)+S^*\beta_2\delta_2\eta_1\mu-B\beta_2(\eta_2\mu+\eta_1\eta_2+\delta_1\eta_2+\delta_2\eta_1)\\ +[\delta_1(\mu+\eta_2)+\delta_2(\mu+\eta_1)+(\eta_1+\mu)(\eta_2+\mu)]\eta_2\mu\end{array}}{\mu\beta_2[S^*\beta_1(\eta_2+\beta\mu)-\{\delta_1(\eta_2+\mu)+\delta_2(\eta_1+\mu)+(\eta_1+\mu)(\eta_2+\mu)\}]}$$

$$D^* = \frac{\begin{array}{l} S^*[\beta_1(\eta_1+\mu)(B\beta_2-\delta_2\mu)-\beta_1(\eta_2+\mu)(\delta_1+\eta_1+\mu)\mu] \\ +B[\beta_1\delta_1(\mu+\eta_2)-\beta_2\{\eta_1(2\mu+\eta_1)+\delta_1(\mu+\eta_1)+\mu^2\}]+[(\mu+2\eta_1)(\mu+\eta_2)]\mu^2 \\ +(\eta_2+\mu)\{\delta_1(\eta_1+\mu)\mu+\eta_1^2\mu\}+\delta_2(\eta_1+\mu)^2\mu \end{array}}{[\beta_1(\mu+\eta_2)-\beta_2(\eta_1+\mu)][(\delta_1+\eta_1+\mu)(\eta_2+\mu)+\delta_2(\eta_1+\mu)]}$$

III. STABILITY ANALYSIS

In this section, local and global stability of both the equilibrium points is to be analyzed.

3.1 Local Stability

Theorem 1: (Stability of E_0) If $\eta_1+\mu-\frac{\beta_1 B}{\mu}\ge 0$ and $\eta_2+\mu-\frac{\beta_2 B}{\mu}\ge 0$ then E_0 is locally asymptotically stable.

Proof: Jacobian matrix of the system evaluated at E_0 is

$$J(E_0)=\begin{bmatrix} -\mu & -\frac{\beta_1 B}{\mu} & -\frac{\beta_2 B}{\mu} & 0 \\ 0 & -c_1 & -\delta_2 & 0 \\ 0 & \delta_1 & -c_2 & 0 \\ 0 & \eta_1 & \eta_2 & -\mu \end{bmatrix}$$

where, $c_1=-\left(\frac{\beta_1 B}{\mu}-\eta_1-\delta_1-\mu\right)$ and $c_2=-\left(\frac{\beta_2 B}{\mu}-\eta_2-\delta_2-\mu\right)$.

The characteristic polynomial for the Jacobian matrix is

$$\lambda^4+a_1\lambda^3+a_2\lambda^2+a_3\lambda+a_4=0$$

where, $a_1=(2\mu+c_2+c_1)$, $a_2=(2c_2\mu+2c_1\mu+\mu^2+(c_2c_1-\delta_1\delta_2))$

$a_3=(2\mu(c_2c_1-\delta_1\delta_2)+c_2\mu^2)$, $a_4=\mu^2(\delta_1\delta_2+c_2c_1)$

If $\eta_1+\mu-\frac{\beta_1 B}{\mu}\ge 0$ and $\eta_2+\mu-\frac{\beta_2 B}{\mu}\ge 0$, then $c_1=-\frac{\beta_1 B}{\mu}+\eta_1+\mu+\delta_1\ge\delta_1>0$ and $c_2=-\frac{\beta_2 B}{\mu}+\eta_2+\mu+\delta_2\ge\delta_2>0$. So, $c_1c_2\ge\delta_1\delta_2; c_1\ge 0; c_2\ge 0.$

Thus, if $\eta_1+\mu-\frac{\beta_1 B}{\mu}\ge 0$ and $\eta_2+\mu-\frac{\beta_2 B}{\mu}\ge 0$ then $a_1,a_2,a_3,a_4>0$ and $a_1a_2a_3-a_3^2-a_1^2a_4>0$

Hence, by Routh-Hurwitz criteria [9], E_0 is locally asymptotically stable.

Theorem 2: (Stability of E^*) If $\mu+\eta_1-\beta_1S^*\ge 0$ and $\mu+\eta_2-\beta_2S^*\ge 0$ then E^* is locally asymptotically stable.

Proof: Jacobian Matrix of the system evaluated at E^* is

$$J(E^*)=\begin{bmatrix} -a_{11} & -\beta_1S^* & -\beta_2S^* & 0 \\ \beta_1M^* & -a_{22} & -\delta_2 & 0 \\ \beta_2D^* & \delta_1 & -a_{33} & 0 \\ 0 & \eta_1 & \eta_2 & -\mu \end{bmatrix}$$

where, $a_{11}=\mu+\beta_1M^*+\beta_2D^*$, $a_{22}=\mu+\eta_1-\beta_1S^*+\delta_1$, $a_{33}=\mu+\eta_2-\beta_2S^*+\delta_2$.

The characteristic polynomial for the Jacobian matrix is

$$\lambda^4+b_1\lambda^3+b_2\lambda^2+b_3\lambda+b_4=0$$

where, $b_1 = a_{11} + a_{22} + a_{33} + \mu$

$$b_2 = \mu a_{11} + \mu a_{22} + \mu a_{33} + \beta_2{}^2 D^* S + \beta_1{}^2 M^* S^* + a_{11}a_{22} + a_{11}a_{33} + (a_{22}a_{33} - \delta_1 d_2)$$

$$b_3 = \mu\beta_2{}^2 D^* S + \mu a_{33}a_{11} + \mu\beta_1^2 M^* S^* + \mu a_{22}a_{11} + \beta_1\beta_2 D^* S^* \delta_2 + \beta_1\beta_2 M^* S^* \delta_1 + \beta_2{}^2 D^* S^* a_{22} + \beta_1^2 M^* S^* a33 + \mu(a_{22}a_{33} - \delta_1\delta_2) + a_{11}(a_{22}a_{33} - \delta_1\delta_2)$$

$$b_4 = \mu(\beta_1^2\beta_2 D^* S^* \delta_2 + \beta_1^2\beta_2 M^* S^* \delta_1 + \beta_2{}^2 D^* S^* a_{22} + \beta_1^2 M^* S^* a_{33} + a_{11}(a_{22}a_{33} - \delta_1\delta_2))$$

If $\mu + \eta_1 - \beta_1 S^* \ge 0$ and $\mu + \eta_2 - \beta_2 S^* \ge 0$ then $a_{22} \ge \delta_1 > 0$ and $a_{33} \ge \delta_2 > 0$.

Therefore, $a_{22}a_{33} \ge \delta_1\delta_2$. Moreover, $a_{11} > 0$.

Thus, if $\mu + \eta_1 - \beta_1 S^* \ge 0$ and $\mu + \eta_2 - \beta_2 S^* \ge 0$ then $b_1, b_2, b_3, b_4 > 0$ and $b_1 b_2 b_3 - b_3^2 - b_1^2 b_4 > 0$.

Hence, by Routh-Hurwitz criteria [9], E^* is locally asymptotically stable.

3.2 Global Stability

Theorem 3: (Stability of E_0) Beauty Conscious free Equilibrium point E_0 is globally asymptotically stable.

Proof: Consider the Lyapunov function [5]:

$$L(t) = M(t) + D(t) + C(t)$$

$$\begin{aligned} \therefore L'(t) &= M'(t) + D'(t) + C'(t) \\ &= (\beta_1 S - \mu)M + (\beta_2 S - \mu)D - \mu C \\ &= -\mu\left(M + D + C\right) \\ &< 0 \end{aligned}$$

And, $L'(t) = 0$ if $M = 0, D = 0, C = 0$

By LaSalle's Invariance Principle solution [6], E_0 is globally asymptotically stable.

Theorem 4: (Stability of E^*) Endemic equilibrium point E^* is globally asymptotically stable.

Proof: Consider the Lyapunov function [5]:

$$L(t) = \frac{1}{2}\left[\left(S - S^*\right) + \left(M - M^*\right) + \left(D - D^*\right) + \left(C - C^*\right)\right]^2$$

$$\begin{aligned} \therefore L'(t) &= \left[\left(S - S^*\right) + \left(M - M^*\right) + \left(D - D^*\right) + \left(C - C^*\right)\right][S' + M' + D' + C'] \\ &= \left[\left(S - S^*\right) + \left(M - M^*\right) + \left(D - D^*\right) + \left(C - C^*\right)\right][B - \mu S - \mu M - \mu D - \mu C] \end{aligned}$$

Taking $B = \left(S^* + M^* + D^* + C^*\right)\mu$

$$L'\left(t\right) = -\mu\left[\left(S - S^*\right) + \left(M - M^*\right) + \left(D - D^*\right) + \left(C - C^*\right)\right]^2 \le 0$$

Hence, E^* is globally asymptotically stable.

IV. SENSITIVITY ANALYSIS

In this section, the sensitivity analysis for all parameters are discussed. The normalized sensitivity of the parameters is computed using the Christoffel formula

$$\Gamma_\alpha^{R_0} = \frac{\partial R_0}{\partial \alpha} \cdot \frac{\alpha}{R_0}$$

where $\Gamma_\alpha^{R_0}$ represents the change in threshold R_0 of the system with respect to the parameter α which inherits life-threatening diseases.

Table 1: Sensitivity Analysis

Parameters	Impact
β_1	+
β_2	+
δ_1	-
δ_2	-
η_1	-
η_2	-

From Table 2, it is observed that the beauty conscious individuals opt for medication and diet control. All other model parameters increase the rate of cardiac arrest among beauty conscious individuals.

V. NUMERICAL SIMULATION

In this section, transmission of individuals in compartments will be observed numerically.

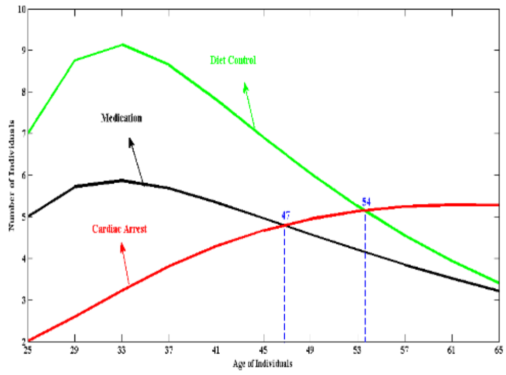

Figure 2: Age of Individuals v/s Number of Individuals in Compartments

Figure 2 shows that cardiac arrest occurs at the age of 47 years due to medication and at the age of 54 years due to diet control which has been observed recently for Bollywood actress Sridevi.

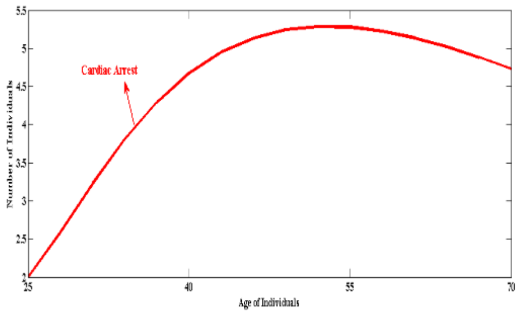

Figure 2: Age of Individuals v/s Number of Individuals Having Cardiac Arrest

Figure 3 shows that the risk of cardiac arrest is maximum till the age of 55 years.

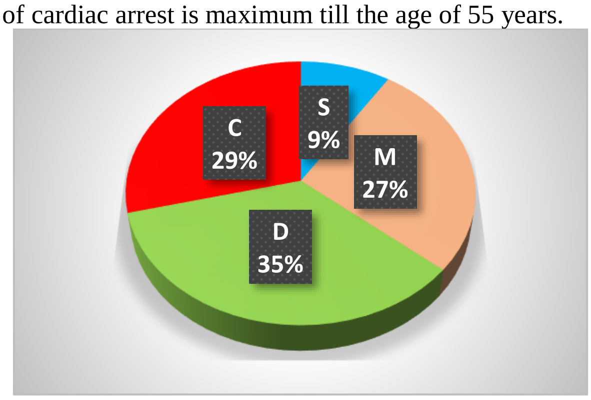

Figure 4: Percentage of Individuals in each Compartment

Figure 4 indicates that from 9% beauty conscious individuals 27% opts for medication whereas 35% opts for diet control due to which 29% of individuals suffers from cardiac arrest. Reasons could be side effects of medicines and malnutrition/ poor nutrition.

VI. CONCLUSION

In this paper, a mathematical model for impact of medication and diet control in beauty conscious individuals is formulated. Model suggest that such individuals may have cardiac arrest in late 40’s or in 50’s depending on the option selected. As suggested by sensitivity analysis, excessive medication and diet control impacts the health and increases risk of cardiac arrest. The model can be extended to study the impact of other factors like financial aspect of the susceptible individual, casting couch, health related problems etc.

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