Dynamics and Trajectory Control of Two Degree of Freedom Planar Robot Using Multibond Graph Approach

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ABSTRACT

Modeling of dynamics and trajectory control of different types of robots is important as these are widely used in industries, space exploration, surgical and other ultra-precision applications. The advantages of using robots are many like increase in productivity, high repeatability, and accuracy and to minimize risk to human life as robots can work in hazardous environment. Different approaches have been followed to model the dynamics of different configurations of robots. Bond graph approach and joint space scheme are two examples of the approaches which are followed. A bond graph model is developed to solve the dynamics of a two degree of freedom planar robot. Trajectory control is achieved using PID controller at each joint. The planar robot consists of two links without gripper. All links are assumed as rigid cylindrical links. Joints are flexible, and are modeled using spring damper systems to constraint the undesired translational and rotational movement of the links. A small gap is assumed between two successive links in which the spring damper systems are inserted. The first link is considered to be heavy then second link. The model determines the forces and torques at both links. System equations are derived from the bond graph model. These system equations are first order states differential equations. Cubic equations are generated for a cubic trajectory generation including an intermediate point (called via point). MATLAB code is generated algorithmically from the bond graph. System equations are solved using one of the ordinary differential equation solvers available in MATLAB. The model is validated through simulation.

**Keywords:** Bond Graph, Controller, Dynamics, Kinematics, Modeling, Simulation, Trajectory.

I. INTRODUCTION

The designing and development of robotics start programmable robot is designed by George Devol in 1945. He coins the term Universal Automation. Then the era of robotics begins with a great approach. Through it all, research in many areas of robotics have made it possible to produce varieties of robots. However, the designing of robots can be done with kinematics, dynamics and trajectory control successfully. Here robotic manipulator is being studied. A robot manipulator is a serial chain of rigid limbs designed to perform a task with its end effector. Different approaches have been followed to model the dynamics and trajectory control of different configurations of robots. An approach to solve the dynamics using Newton–Euler formulation is said to be a “force balance” approach to dynamics. Takehiko Kawase and Hiroaki Yoshimura [3] described a bond graph method of modeling multi-body dynamics was demonstrated that structural understanding and representation in bondgraph theory was quite powerful, for the modeling of suchlarge scale systems, and that thenon-energetic multiport of junction structure, which was a multiport expression of the systemStructure.D.W. Roberts, DJ. Ballance and PJ. Gawthrop [4] described the use of a bond-graph model-based nonlinear observer to estimate the velocities in the control of an experimental two-link manipulator.R. M. Berger, H. A. EIMaraghy and W.H. ElMaraghy [5] demonstrated the application of bond graph modeling of robotic manipulator by progressively developing a distributed mass model of a two-link robot. Dean Karnopp [6] used bond graph approach to describe the mechanical systems using the example of a multi-link inverted pendulum which can explain the form of multibody system equations that had been often studied. Recep BurkanandIbrahim Uzmay [7] presented a new robust control law for robot manipulators subjected to uncertainties to find tracking errors. Anand Vaz and Shinichi Hirai [8] A hand prosthesis system was modeled using bond graph approach with a concept of word bond graph was applied to represent component subsystem as objects. Such Word Bond Graph Objects...
(WBGO) is compact representations of subsystems, within the overall system, and has a well-defined structure. Anand Vaz and Shinichi Hirai [9] presented a model for a joint which can capture most of the behavior of the bone joint system. The model was applied to a ball and socket joint system with soft cartilage. Pushpraj Mani Pathak and Amalendu Mukherjee et. al. [10] presented a scheme for robust trajectory control of free-floating space robots that was based on the overwhelming robust trajectory control of a ground robot on a flexible foundation and robust foundation disturbance compensation presented elsewhere. Chieh-Li Chen, Tung-Chin Wu and Chao-Chung Peng [11] presented a common idea concerning trajectory control of robot manipulators was to tackle the motion of the end-effector. Acc. to traditional trajectory designs, a prescribed profile in a work space was first decomposed into independent joint positions such that the success in a contoured task lies with good tracking capability of individual joints. Joel Perez P., et.al [12] presented the application of adaptive neural networks to robot manipulator control. The main methodologies, on which the approach was based, were recurrent neural networks and Lyapunov functions methodology and Proportional-Integral-Derivative (PID) control for nonlinear systems. Amit Kumar, Pushparaj Mani Pathak and N. Sukavanam [13] demonstrated model based control schemes uses inverse dynamics of the robot arm to produce the main torque component necessary for trajectory tracking and presented a control scheme for trajectory control of the tip of a two arm rigid-flexible space robot, with the help of a virtual space vehicle. Anand Vaz, Anil Kumar Narwal, K. D. Gupta [14] presented the evaluation of dynamics of soft contact rolling by taking an example of a circular disc rolling over a layer of silicon rubber with the help of Multibond graph approach. The disc was moved with controlled force using proportional and derivative controller. Anand Vaz, Anil Kumar Narwal, K. D. Gupta [15] presented the soft contact interaction between a non-circular rigid body and a soft material using Multibond graph approach. The model was simulated for a rectangular block and an elliptical disc in contact with the soft material. The model determines contact area and distribution of forces over the contact nodes in static and moving contacts. The model was verified through simulation. An approach to plan the trajectory of two degree of freedom of planar robot is Joint-space scheme. It is generation of path in which the path shapes (in space and in time) are described in terms of functions of joint angles. Each path point is usually specified in terms of a desired position and orientation of the tool frame, (T), relative to the station frame, (S). Each of these via points is "converted" into a set of desired joint angles by application of the inverse kinematics. Joint-space schemes achieve the desired position and orientation at the via points. In the present work, Multibond Graph approach and Joint space scheme is used to model the Dynamics and Trajectory Control of Two Degree of Freedom Planar Robot. System equations are derived from the model. MATLAB codes are generated from the model directly. Proportional Integral Derivative (PID) controller is used for Dynamics and Trajectory Control of Two Degree of Freedom planar Robot.

II. TRAJECTORY GENERATION OF TWO DEGREE OF FREEDOM PLANAR ROBOT

In trajectory planning of path of two degree of freedom planar robot the path is followed by end effector when end effector moves from A to B. There is obstacle between initial and final position. While tracking path from point A to B, Some intermediate point named "vella point" / "via point" are to be considered. Motion between two points can be point to point motion or continuous path motion. Task specification can be done in world coordinates system.

![Fig.1 Basic structure for world coordinates and joint space realtion](image_url)
The following equation are obtained there:

\[
\phi = \theta_1 + \theta_2
\]

\[
X_b = l_1 \cos \theta_1 + l_2 \cos \phi \\
Y_b = l_1 \sin \theta_1 + l_2 \sin \phi
\]

Let us consider given equations:

\[
\theta_1, \phi, X_a, Y_b \quad \text{(Linear algebraic equation)}
\]

To correlate them with time and velocity, the above equations are differentiated, and written in matrix form.

\[
\begin{align*}
X_b &= l_1 \dot{\theta}_1 \sin \theta_1 - l_2 \sin \phi \\
Y_b &= l_1 \cos \theta_1 + l_2 \cos \phi
\end{align*}
\]

\[
\begin{bmatrix}
X_b \\
Y_b
\end{bmatrix} =
\begin{bmatrix}
-l_1 \sin \theta_1 & -l_2 \sin \phi \\
l_1 \cos \theta_1 & l_2 \cos \phi
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\phi}
\end{bmatrix}
\]

In general form

\[
\begin{bmatrix}
X \\
\theta
\end{bmatrix} = [C] \begin{bmatrix}
\dot{\theta}
\end{bmatrix}
\]

This is the generalized form to obtain solution.

The dynamics of two degree of freedom planar robot can be evaluated using matlab code which are obtained from bondgraph model. To control the trajectory of two degree of freedom planar robot consider that both link cover the specific path with a point in between called via point i.e. point j as shown in Fig. 2.

Fig.2 basic model for twodegree of freedom planar robot

According to the problem a initial position(i), final position(k), via point(j), angle of link1( \( \theta_1 \) ) w.r.t. inertial reference frame, angle of link2 ( \( \theta_2 \) )w.r.t. link1 frame position are predefined. The cubic equation with respect to time ( \( \tau \) ) for \( \theta_1 \) and \( \theta_2 \) are generated separately. The equations are as follows:

For link1:

\[
\begin{align*}
\theta_1 ij(\tau) &= a_0 + a_1 \tau + a_2 \tau^2 + a_3 \tau^3 \\
\theta_1 ik(\tau) &= b_0 + b_1 \tau + b_2 \tau^2 + b_3 \tau^3
\end{align*}
\]

Where \( a_0, a_1, a_2, a_3, b_0, b_1, b_2, b_3 \) are the co-efficients of cubic equation.

There are 8 co-efficients, so to find the optimal solution 8 equations will be generated. With in boundary limit we can differentiate above equation to obtain optimal solution

At time ( \( \tau = 0 \) )

\[
\begin{align*}
\theta_1 ij(0) &= \pi / 6 = a_0 \\
\dot{\theta}_1 ij(0) &= 0 = a_1
\end{align*}
\]
At time $(\tau) = \tau / 2$

\[ \theta_{ij}(\tau / 2) = 0.2606 \pi = a_o + a_1* \tau / 2 + a_2* \tau / 2^2 + a_3* \tau / 2^3 \]  

\[ \theta_{jk}(\tau / 2) = 0.2606 \pi = b_o + b_1* \tau / 2 + b_2* \tau / 2^2 + b_3* \tau / 2^3 \]  

At time $(\tau) = \tau$

\[ \theta_{ij}(\tau) = \frac{5}{6} \pi = b_o + b_1* \tau + b_2* \tau^2 + b_3* \tau^3 \]  

\[ \theta_{jk}(\tau) = 0 = b_1 + 2*b_2* \tau + 3*b_3* \tau^2 \]  

The above 6 equations can be generated easily and for another two equations we have to drive velocity and acceleration equation of $\theta_{ij}$ at via point separately for ij and jk position.

For velocity equation

\[ \dot{\theta}_{ij}(\tau / 2) = a_1*1/2 + a_2* \tau / 2 + 3*a_3* \tau^2 / 8 \]

\[ \dot{\theta}_{jk}(\tau / 2) = b_1*1/2 + b_2* \tau / 2 + 3*b_3* \tau^2 / 8 \]

But the position of via point is same for both of velocities

\[ \dot{\theta}_{ij}(\tau / 2) = \dot{\theta}_{jk}(\tau / 2) \]

Therefore

\[ a_1*1/2 + a_2* \tau / 2 + 3*a_3* \tau^2 / 8 - b_1*1/2 - b_2* \tau / 2 - 3*b_3* \tau^2 / 8 = 0 \]  

For acceleration equation

\[ \dot{\theta}_{ij}(\tau / 2) = a_o*1/2 + 3*a_3* \tau \]

\[ \dot{\theta}_{jk}(\tau / 2) = b_o*1/2 + 3*b_3* \tau \]

But the position of via point is same for both of acceleration

\[ \dot{\theta}_{ij}(\tau / 2) = \dot{\theta}_{jk}(\tau / 2) \]

\[ a_o*1/2 + 3*a_3* \tau - b_o*1/2 - 3*b_3* \tau = 0 \]

with the help of these 8 equation we can build up a matrix for optimal solution

\[
A = \begin{bmatrix}
  a_o \\
  a_1 \\
  a_2 \\
  a_3 \\
  b_o \\
  b_1 \\
  b_2 \\
  b_3
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
  1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
  1 & \tau / 2 & (\tau / 2)^2 & (\tau / 2)^3 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 1 & \tau / 2 & (\tau / 2)^2 & (\tau / 2)^3 \\
  0 & 0 & 0 & 0 & 1 & \tau & \tau^2 & \tau^3 \\
  0 & 0 & 0 & 0 & 1 & 2\tau & 3\tau^2 & \tau^4 \\
  0 & 1 / 2 & \tau / 2 & 3/8 + \tau^2 & 0 & 1 / 2 & \tau / 2 & 3/8 + \tau^2 \\
  0 & 0 & 1 / 2 & 3/4 + \tau & 0 & 0 & 1 / 2 & 3/4 + \tau
\end{bmatrix}
\]
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\[
C = \begin{bmatrix}
\theta_{ij}(0) \\
\dot{\theta}_{ij}(0) \\
\theta_{ij}(\tau/2) \\
\dot{\theta}_{jk}(\tau/2) \\
\theta_{jk}(\tau) \\
\dot{\theta}_{jk}(\tau) \\
0 \\
0
\end{bmatrix}
\]

\[A = B^{-1} \cdot C\]

This matrix solution will help us to control the trajectory of two degree of freedom planar robot.

For link2:

\[
\theta_{ij}(\tau) = c_0 + c_1 \tau + c_2 \tau^2 + c_3 \tau^3
\]

\[
\theta_{jk}(\tau) = d_0 + d_1 \tau + d_2 \tau^2 + d_3 \tau^3
\]

The above 8 coefficients are calculated using further 8 boundary conditions. With in boundary limit we can differentiate above equation to obtain optimal solution

At time \((\tau) = 0\)

\[
\theta_{ij}(0) = \frac{\pi}{6} = c_0 \tag{9}
\]

\[
\theta_{ij}(0) = 0 = c_1 \tag{10}
\]

At time \((\tau) = \tau/2\)

\[
\theta_{ij}(\tau/2) = 5384 \pi = c_0 + c_1 \tau/2 + c_2 (\tau/2)^2 + c_3 (\tau/2)^3 \tag{11}
\]

\[
\theta_{jk}(\tau/2) = 5384 \pi = d_0 + d_1 \tau/2 + d_2 (\tau/2)^2 + d_3 (\tau/2)^3 \tag{12}
\]

At time \((\tau) = \tau\)

\[
\theta_{ij}(\tau) = -\frac{\pi}{6} = d_0 + d_1 \tau + d_2 \tau^2 + d_3 \tau^3 \tag{13}
\]

\[
\theta_{jk}(\tau) = -\frac{\pi}{6} = d_0 + d_1 \tau + d_2 \tau^2 + d_3 \tau^3 \tag{14}
\]

The above 6 equation can be generated easily and for another two equation we have to drive velocity and acceleration equation for \(\theta_j\) at via point separately for \(ij\) and \(jk\) position.

For velocity equation

\[
\dot{\theta}_{ij}(\tau/2) = c_1 /2 + c_2 + 3/4 c_3 \tau^2 /8 \tag{15}
\]

\[
\dot{\theta}_{jk}(\tau/2) = d_1 /2 + d_2 + 3/4 d_3 \tau^2 /8 \tag{16}
\]

But the position of via point is same for both of velocities

\[
\dot{\theta}_{ij}(\tau/2) = \dot{\theta}_{jk}(\tau/2)
\]

Therefore

\[
c_1 /2 + c_2 + 3/8 c_3 \tau - d_1 /2 - d_2 + 3/8 d_3 \tau = 0 \tag{15}
\]

For acceleration equation

\[
\ddot{\theta}_{ij}(\tau/2) = c_1 /2 + 3/4 c_3 \tau \tag{17}
\]

\[
\ddot{\theta}_{jk}(\tau/2) = d_1 /2 + 3/4 d_3 \tau
\]
But the position of via point is same for both of acaccelation
\[ \dot{\theta}_{ij}(\tau /2) = \dot{\theta}_{jk}(\tau /2) \]
\[ c_2 \cdot 1/2 + 3/4 \cdot c_3 \cdot \tau - d_1 \cdot 1/2 - 3/4 \cdot d_3 \cdot \tau = 0 \]  
\[ (16) \]
with the help of these 8 equation we can build up a matrix for optimal solution
\[ D = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 & d_0 & d_1 & d_2 & d_3 \end{bmatrix}, \]
\[ E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & (\tau/2)^2 & (\tau/2)^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \tau & \tau^2 & \tau^3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2\tau & 3\tau^2 \\ 0 & 1/2 & \tau/2 & 3/8\tau^2 & 0 & 1/2 & \tau/2 & 3/8\tau^2 \\ 0 & 0 & 1/2 & 3/4\tau & 0 & 0 & 1/2 & 3/4\tau \end{bmatrix} \]
\[ F = \begin{bmatrix} \theta_{ij}(0) \\ \dot{\theta}_{ij}(0) \\ \ddot{\theta}_{ij}(\tau /2) \\ \theta_{jk}(\tau /2) \\ \dot{\theta}_{jk}(\tau) \\ \dot{\theta}_{jk}(\tau) \\ 0 \\ 0 \end{bmatrix} \]
\[ D = E^{-1} \cdot F \]
By this methodology the desired trajectory will be tracked with in limits.

### III. BONDGRAPH MODELING

The complete bond graph model of dynamics and trajectory control of two degree of freedom planar robot is explained. It is obtained by integrating the concepts of rigid body dynamics and PID controller as explained earlier in the section. The bond graph is as follows:
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IV. SIMULATION RESULT

Simulation of dynamics and trajectory control of two degree of freedom planar robot model is obtained by solving system states equations using ODE 45 solver available in MATLAB. ODE 45 solver is based upon Runga Kutta numerical method to solve ordinary differential equation. During simulation desired angles for the desired trajectory are calculated at different instances, and achieved using PID controller at both the joints as shown in Fig. 3. The parameters for different links and other elements are given in Table 1 and Table 2 as given below.

![Image of bond graph model](image_url)

**Fig.3** Bondgraph model

<table>
<thead>
<tr>
<th>Table.1 Links parameters used in simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PARAMETERS</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>LINK1 Mass</td>
</tr>
<tr>
<td>LINK1 Height</td>
</tr>
<tr>
<td>LINK2 Radius</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table.2 Joints parameters used in simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PARAMETERS</td>
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<tr>
<td>---------------------------------</td>
</tr>
<tr>
<td>Spring Damper System for Translation Stiffness constant (in x &amp; y direction) in N/m</td>
</tr>
<tr>
<td>Spring Damper System for Rotation Stiffness constant (in x direction) in N/m</td>
</tr>
<tr>
<td>Spring Damper System for Translation Stiffness constant (in y direction) in N/m</td>
</tr>
<tr>
<td>Spring Damper System for Rotation Stiffness constant (in z direction) in N/m</td>
</tr>
<tr>
<td>Spring Damper System for Translation Damping coefficient (in x &amp; y direction) in N-s/m</td>
</tr>
<tr>
<td>Spring Damper System for Rotation Damping coefficient (in x direction) in N-s/m</td>
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<td>Spring Damper System for Rotation Damping coefficient (in x &amp; y direction) in N-s/m</td>
</tr>
</tbody>
</table>

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**Link1:** The set rotation of first link is from 30° to 150°. Fig 4 represents the rotation of link1 with respect to time.

![Fig 4 Rotation of link 1 w.r.t. time.](image)

The desired path is to be followed in 10 sec. After the initial transient translational momentum, all the translational momentum components become zero after 10 sec as shown in Fig 6. The variation of angle $\theta_1$ and $\theta_2$ with respect to time during trajectory control is shown in Fig. 7.

![Fig 6 Translational momentum of link1](image)

**5. Trajectory Orientation control of Two Degree of Freedom planar robot**

<table>
<thead>
<tr>
<th>Table 3 Initial and final joint angles</th>
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<tbody>
<tr>
<td>Initial Angle</td>
</tr>
<tr>
<td>$\theta_1$</td>
</tr>
<tr>
<td>$\theta_2$</td>
</tr>
</tbody>
</table>

Table 3 shows initial and final values of $\theta_1$ and $\theta_2$ as calculated geometrically from the initial and final configuration of the robot. Third column gives the actual values of final angles achieved in simulation. The actual trajectory followed by the end effector is shown in Fig. 8. Initial, intermediate at via point and final position of the robot is shown in Fig. 9, Fig. 10 and Fig. 11 respectively.
V. CONCLUSION

Model determines control trajectory for two degree of freedom planar robot. Bond graph model for a rigid link is developed, and it is used as an object to develop the bond graph model for the system. Causality based representation of bond facilitates systematic derivation of system states equations from the bond graph model. PID controller is used to control the orientation of different links. Controlled torque is generated by PID controller on the basis error between the desired angular position and actual position of a link. The approach is algorithmic and computationally simple. First order state differential equations can be integrated easily using a number of numerical methods as compared to higher order differential equations that are given by Newtonian and Lagrangian Approaches. Joint space scheme is used to control the trajectory of two degree of freedom planar robot. The solution of generated cubic equation is easily available with some calculations. The future scope of the present work is to control trajectory of two degree of freedom planar robot with a straight line.
trajectory by developing a general algorithm. The trajectory control of robot manipulator with higher degree of freedom can also be achieved.

REFERENCES


Sandeep chhillar.“ Dynamics and Trajectory Control of Two Degree of Freedom Planar Robot Using Multibond Graph Approach.”. International Journal of Computational Engineering Research (IJCER), vol. 8, no. 7, 2018, pp. 25-34.