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Numerical Solution of Stagnation Point Flow Of Nano Fluid Due To an Inclined Stretching Sheet.

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ABSTRACT

This paper deals with numerical solution of stagnation point flow of nanofluid due to an inclined stretching sheet. The governing partial differential equations are transformed in to a system of coupled ordinary differential equations using similarity transformations. The resulted ordinary differential equations were solved using the fourth order Runge-Kutta method with shooting technique. The various parameters in the system such as the Grashof number Gr, the solutal Grashof number Gc, the angle of inclination γ, the Prandtle number Pr, the Lewis number Le, the Brownian motion parameter Nb, the thermophoresis parameter Nt, the skin friction Cf,the local Sherwood number Sh, the local Nusselt number Nu were determined for different cases and analyzed. On the analysis it was found that the roles of the parameters γ, Gr, Gc and Pr on the dynamics of the cooling liquid due to an inclined stretching sheet were qualitatively similar except for the Pr that has an opposite effect to that of the above parameters on the flow and The temperature thermal boundary layer thickness appreciably increased in line with the Brownian motion and thermophoresis effects strengthened. Apart from that the volume fraction of the non-particles was increased while the thermophoresis effect intensified.

Key words: Inclined stretching sheet, Stagnation point flow, Nanofluid, Runge-Kutta method, Brownian motion parameter, Nusselt Number Skin friction.

Nomenclature

a	Velocity of the stretching surface
b	Free Stream velocity
T	Temperature of the fluid in the boundary layer
C	Concentration of the fluid in the boundary layer
T_w	Stretching sheet temperature
C_w	Stretching sheet concentration
T_∞	Ambient fluid temperature
C_∞	Ambient fluid concentration
u	Velocity component along the x- axis
v	Velocity component along the y- axis
u_w	Velocity component at the wall
v_w	Velocity component at the wall
υ	Kinematic viscosity
τ	The effective heat capacity of the Nano particle to the heat capacity of the fluid
$(\rho_c)_p$	Effective heat capacity of Nano particle material
$(\rho_c)_f$	Effective heat capacity of the Fluid
α	Thermal diffusivity
D_B	Brownian diffusion coefficient

D_T	Thermophoresis diffusion coefficient
k	Thermal conductivity
λ	The velocity ratio parameter
γ	The angle of inclination
Gr	Local Grashof number
Gc	Local Solutal Grashof number
Pr	Prandtl number
Nb	Brownian motion parameter
Nt	Thermophoresis parameter
Le	Lewis number
Cf_x	Local skin friction
Nu_x	Local Nusselt number
Sh_x	Local Sherwood number

I. INTRODUCTION

The study of stagnation point flow of nanofluid over stretching sheet has various applications in manufacturing industries and technological process. Such as the polymer processing of chemical engineering plant metallurgy for the metal processing of glass fiber, paper production, plastic sheets, liquid films in the condensation process filament extrusion from a dye, artificial fibers, hot rolling in metal and polymer processing industries and others. Based on the importance and applicability of the flow many researchers have been involving in the investigation of the behaviors of the flow.

Hazem [1] has analyzed the effect of the strength of the uniform magnetic field, the surface stretching velocity and the heat generation/absorption coefficient on both the flow and heat transfer. Sin Wei Wong et al. [2] have investigated the behaviors of the flow and characteristics of heat transfer and witnessed that dual solution exists for the shrinking case where as the solution is unique for the stretching case. Norfifah and Anuar [3] the variation of the skin friction coefficient and the heat transfer rate at the surface with the governing parameters have been studied. Suali e al. [4] have investigated The unsteady two dimensional stagnation point ad heat transfer over a stretching/shrinking sheet with prescribed surface and the effect of the governing parameters have been examined. Muhammad et al. [5] the stagnation point flow over stretching have been investigated and the features of the flow and heat transfer characteristics for different parameters with various values have been considered. Aliereza et al. [6] have Studied that the steady two dimensional MHD stagnation point flow due to permeable stretching sheet with chemical reaction and have examined the effect of the different parameters involved in the three profiles Krishnendu [7] has examined the role of non uniform heat flux on the heat transfer in boundary layer stagnation point flow over shrinking sheet and has concluded that the direct and inverse variations of heat flux along the sheet have completely different effects on the temperature distribution. Aurangzaid and Sharidan [8] the unsteady MHD flow, heat and mass transfer in micro polar fluid near the forward stagnation point having thermophoresis has been investigated with suction or injection effects. Krishnendu [9] has presented the features of the flow and characteristics of heat transfer of MHD stagnation point flow of electrically conducting casson fluid and heat transfer towards a stretching sheet. Meraj et al. [10Have studied that stagnation point flow of nanofluid past an exponentially stretching sheet the different parameters have been examined and results have been presented. Krishnendu [11] has witnessed that the range of velocity ratio parameter for which similarity solution exists is unaltered for any change in casson parameter though the skin friction changes with casson parameter and concluded that the possibility of similarity solution for casson fluid flow is the same as that of the Newtonian fluid flow. Sin Wei Wong et al. [12] have investigated the flow and heat transfer characteristics of stagnation point flow of an incompressible viscous fluid towards a vertical stretching sheet and for different values of the governing parameters have been studied and analyzed. Sarajimma and Vasundhara [13] the flow ,heat and mass transfer characteristics of MHD casson fluid in parallel plate channel with stretching walls subject to a uniform transverse magnetic field have been studied and the different parameters have been examined. Emmanuel et al. [14The flow over a vertical porous surface of casson fluid having chemical reaction along with the existence of magnetic field has been investigated and they have examined and witnessed the effects of the governing parameters involved. Hayat et al. [15] have reported that the flow and heat transfer of the third grade fluid due to exponentially stretching surface along with the effects of the inclined magnetic field. NurFatihah et al. [16] The effect of slip on the flow of stagnation point due to permeable shrinking sheet has been studied and the effect of the different parameters have been checked and reported. Subhas and Jayashree [17] have reported the heat transfer from a warm laminar liquid to a melting stretching sheet of the steady two dimensional stagnation point flow and concluded the effect of the different parameters included. Rashidi et al.[18] The stagnation point flow in porous medium over a permeable stretching surface along with the heat generation/absorption having convective boundary condition has been examined and the results has been reported. Santosh [19] The flow of steady two dimensional and laminar due to a stretching sheet having heat generation in electrically conducting incompressible viscous fluid with porous medium has been studied. Syahira et al.[20] have examined the MHD stagnation point flow of nanofluid due to a permeable stretching/shrinking sheet and have reported the effect of the governing parameters. El-Sayed et al. [21] The steady two dimensional boundary layer MHD stagnation point flow along with heat transfer of an incompressible micro polar fluid due to a stretching sheet with the existence of radiation, heat generation and absorption has been investigated. Dulapal et al.[22] have studied the bouncy-driven radiative non-isothermal heat transfer in nanofluid stagnation point flow due to stretching/shrinking sheet with a porous medium and effects of thermal radiation and internal heat generation/absorption along with suction/injection at the boundary layer are also analyzed. Khairy and Anuar [23] The stagnation point flow and heat transfer over a stretching vertical sheet along with the effect of partial slip and buoyancy parameters on velocity, temperature, skin friction coefficient and local Nusselt number has been investigated and the results have been reported. Mageswari and Nirmala [24] have investigated the stagnation point flow due to a stretching sheet with Newtonian heating and shown the power of the Laplace Adomain Decomposition method in approximating the solution of non linear differential equations. Majeed et al.[25] the unsteady two dimensional mixed convection stagnation point flow due to a vertical stretching cylinder with sinusoidal wall temperature has been studied and the governing parameters has been examined. Monica et al. [26] have investigated the stagnation point flow of Williamson fluid due to a non linear stretching sheet with thermal radiation and reported the effects of the parameters involved. FerozAhmed et al. [27] the effect of velocity slip on the stagnation point flow due to porous stretching sheet has been examined and the influence of the parameters included has been presented. Madasi et al. [28] the effect of slip boundary condition and chemical reaction on a steady MHD stagnation point flow of a nanofluid over a non linear permeable stretching sheet has been studied and the results of the effects of the governing parameters has been reported. Sirinivasulu et al. [29] the effect of viscous dissipation on MHD stagnation point flow of casson fluid due to a linear stretching sheet has been investigated and the effects of the parameters involved has been presented. Abuzar et al. [30] have studied the effect of radiation and convective boundary condition on the enhancement of thermal conductivity of elastic viscous fluid filled with nanoparticles and the flow is considered impinging obliquely in the region of oblique stagnation point on the stretching surface. Vajravelu et al. [31] the mixed convective flow of casson fluid due to a vertical stretching sheet with variable conductivity has been studied and the governing parameters have been examined as well as the results have been reported. Eswara and Srenadh [32] the steady two dimensional MHD convective boundary layer flow of casson fluid due to a permeable stretching surface in the existence of thermal radiation and chemical reaction has been investigated along with the effects of velocity, thermal, solutal slips, thermal radiation chemical reaction and suction/blowing. The authors know that the problem of stagnation point flow of nano fluid due to an inclined stretching sheet is not yet investigated.

II. MATHEMATICAL FORMULATION

We consider the laminar boundary layer flow of nanofluid in the region of the stagnation point towards an inclined stretching sheet driven by a pressure gradient. The sheet stretched along the x-axis by the action of two equal and opposite forces as shown in Fig1 below, with velocity $U_w = U_0 e^{\frac{x}{L}}$. The sheet velocity varies as an exponential function of the distance from the origin where the slit is situated. L and U_0 are the references for length and velocity respectively. Let $T_w = T_\infty + Ae^{\frac{2x}{L}}$, $C_w = C_\infty + Be^{\frac{2x}{L}}$ be the temperature and the nanoparticle concentration at the sheet where T_∞ *and* C_∞ are the ambient temperature and concentration respectively. The boundary layer equations governing the conservation of mass, momentum, and energy and nanoparticle volume fraction are:

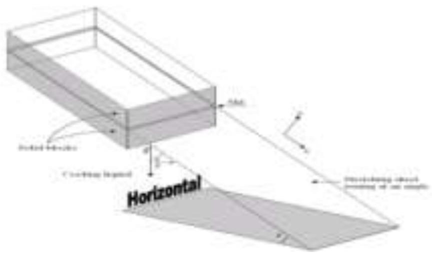

Fig.1: Schematic of the inclined stretching sheet problem.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon_f\frac{\partial^2 u}{\partial y^2}+g\beta_T(T-T_\infty)\sin\gamma+g\beta_C(C-C_\infty)\sin\gamma \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}(\frac{\partial T}{\partial y})^2] \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

With boundary conditions

$$\left.\begin{array}{l} u=U_w=U_0e^{\frac{x}{L}},\ v=0,\ T=T_w=T_\infty+Ae^{\frac{2x}{L}},\ C=C_w=C_\infty+Be^{\frac{2x}{L}} \quad at\ \ y=0 \\ u\to 0, \qquad T\to T_{\infty,} \qquad C\to C_\infty \qquad\qquad as\ \ y\to\infty \end{array}\right\} \tag{5}$$

Where u and v are the velocity components along x and y directions respectively, υ_f is the kinematic viscosity, g is the acceleration due to gravitation, β_T is the coefficient of thermal expansion, β_C is the solutal expansion coefficient γ is the angle of inclination, α is the thermal diffusivity, τ is the effective heat capacity of nanoparticle to the heat capacity of the fluid i.e. $\tau=\frac{(\rho c)_p}{(\rho c)_f}$, D_B is the Brownian motion coefficient, D_T is the thermophoresis diffusion coefficient.

Now let us introduce the following dimensionless variables

$$\left.(X,Y)=\frac{(x,y\mathrm{Re}^{\frac{1}{2}})}{L},\ (U,V)=\frac{(u,v\mathrm{Re}^{\frac{1}{2}}}{U_0}\quad \overline{T}=\frac{T-T_\infty}{T_w-T_\infty},\ \overline{C}=\frac{C-C_\infty}{C_w-C_\infty}\right\} \tag{6}$$

$\mathrm{Re}=\frac{U_0L}{\upsilon}$ is the Reynolds number.

With the help of the above dimensionless variables, the governing equations take the form

$$\left.\begin{array}{l} \frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \\ U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+e^{2X}\overline{T}Gr\sin\gamma+e^{2X}\overline{C}Gc\sin\gamma \\ U\frac{\partial\overline{T}}{\partial X}+V\frac{\partial\overline{T}}{\partial Y}=\frac{1}{\mathrm{Pr}}\frac{\partial^2\overline{T}}{\partial Y^2}+Nb\frac{\partial\overline{T}}{\partial Y}\frac{\partial\overline{C}}{\partial Y}+Nt(\frac{\partial\overline{T}}{\partial Y})^2 \\ U\frac{\partial\overline{C}}{\partial X}+V\frac{\partial\overline{C}}{\partial Y}=Le\frac{\partial^2\overline{C}}{\partial Y^2}+\frac{Nt}{Nb}\frac{\partial^2\overline{T}}{\partial Y^2} \end{array}\right\} \tag{7}$$

Where

$$Gr=\frac{g\beta_T AL}{U_0^2},\quad \mathrm{Pr}=\frac{\upsilon_f}{\alpha},\ Le=\frac{\upsilon_f}{D_B},\quad Nb=\frac{(\rho_c)_p D_B(\overline{C}_w-\overline{C}_\infty)}{(\rho_c)_f\upsilon_f}$$

$$Nt=\frac{(\rho_c)_p D_T(\overline{T}_w-\overline{T}_\infty)}{(\rho_c)_f\upsilon_f\overline{T}_\infty},\quad G_C=\frac{g\beta_C BL}{U_0^2}$$

And the boundary conditions take the following form

$$\left.\begin{array}{l} U = e^{X}, \quad V = 0, \quad \overline{T} = 1, \quad \overline{C} = 1 \quad at \quad Y = 0 \\ U \to 0, \quad \overline{T} \to 0, \quad \overline{C} \to 0, \quad as \quad Y \to \infty \end{array}\right\}$$

(8)

Using the stream function $\psi(X,Y)$ such that $U = \frac{\partial \psi}{\partial Y}, V = -\frac{\partial \psi}{\partial X}$, that satisfies the continuity equation in the dimensionless equations (7) and the boundary conditions (8), we have

$$\left.\begin{array}{l} \frac{\partial^3 \psi}{\partial Y^3} + \frac{\partial(\psi, \frac{\partial \psi}{\partial Y})}{\partial(X,Y)} + e^{2X}\overline{T}Gr\sin\gamma + e^{2X}\overline{C}Gc\sin\gamma = 0 \\ \frac{1}{\Pr}\frac{\partial^2 \overline{T}}{\partial Y^2} + \frac{\partial(\psi,\overline{T})}{\partial(X,Y)} + Nb\frac{\partial \overline{T}}{\partial Y}\frac{\partial \overline{C}}{\partial Y} + Nt(\frac{\partial \overline{T}}{\partial Y})^2 = 0 \\ Le\frac{\partial^2 \overline{C}}{\partial Y^2} + \frac{\partial(\psi,\overline{C})}{\partial(X,Y)} + \frac{Nt}{Nb}\frac{\partial^2 \overline{T}}{\partial Y^2} = 0 \end{array}\right\} \quad (9)$$

$$\left.\begin{array}{l} \frac{\partial \psi}{\partial Y} = e^{X}, \quad \frac{\partial \psi}{\partial X} = 0, \quad \overline{T} = 1, \quad \overline{C} = 1 \quad at\ Y = 0 \\ \frac{\partial \psi}{\partial Y} \to 0, \quad \overline{T} \to 0, \quad \overline{C} \to 0, \quad as\ Y \to \infty \end{array}\right\} \quad (10)$$

By applying the similarity transformation below, the coupled partial differential equations will be changed into a system of ordinary differential equations.

$$\psi(X,Y) = \sqrt{2}e^{\frac{X}{2}} f(\eta), \ \overline{T}(X,Y) = \theta(\eta), \ \overline{C}(X,Y) = \phi(\eta) \ and \ \eta = \frac{Ye^{\frac{X}{2}}}{\sqrt{2}} \quad (11)$$

Substituting equations (11) into equations (9) and (10), we get

$$\left.\begin{array}{l} f''' + ff'' - 2f'^2 + 2\theta Gr\sin\gamma + 2\phi Gc\sin\gamma = 0 \\ \frac{1}{\Pr}\theta'' + f\theta' + Nb\phi'\theta' + Nt\theta'^2 = 0 \\ Le\phi'' + f\phi' + \frac{Nt}{Nb}\theta'' = 0 \end{array}\right\}$$

(12)

With boundary conditions

$$\left.\begin{array}{l} f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad at\ \eta = 0 \\ f'(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0, \quad as\ \eta \to \infty \end{array}\right\} \quad (13)$$

Pr is the Prandtle number, Le is the Lewis Number, Gr is the local Grashof number, Gc is the local solutal Grashof number, Nb is the Brownian motion parameter and Nt is the thermophoresis parameter. Since we know that Nb and Nt are functions of x, the x-coordinate can't be deleted from the second and third equations of (12), therefore we search for the variability of the local similarity solution that allows us to investigate the behavior of these parameters at affixed location above the sheet.

The skin friction coefficient C_f, the local Nusselt number Nu and the local Sherwood number Sh are given by

$$C_f = \frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\rho u_w^2}, \ Nu = \frac{-x\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_w - T_\infty}, \ Sh = \frac{-x\left(\frac{\partial C}{\partial y}\right)_{y=0}}{C_w - C_\infty} \quad (14)$$

Substituting equations (11) into equations (14), the reduced form will be

$$\sqrt{2\,\mathrm{Re}}C_f = f''(0), \quad \frac{\sqrt{\frac{2L}{x}}Nu}{\mathrm{Re}_x^{\frac{1}{2}}} = -\theta'(0) = \mathrm{Nur}, \quad \frac{\sqrt{\frac{2L}{x}}Sh}{\mathrm{Re}_x^{\frac{1}{2}}} = -\phi'(0) = \mathrm{shr} \tag{15}$$

III. METHOD OF SOLUTION

Using the fourth order Runge-Kutta method with shooting technique, we solve the coupled ordinary differential equations (12), with their respective boundary condition equations (13) numerically.
The system of the ordinary differential equations (12) is highly non linear higher order differential equations. To solve these equations, we transform them into a system of first order differential equations by applying the following denotations.
Let

$$\left.\begin{array}{l} f = f(1), \quad f' = f(2), \quad f'' = f(3) \\ \theta = f(4), \quad \theta' = f(5) \\ \phi = f(6), \quad \phi' = f(7) \end{array}\right\} \tag{16}$$

Using equations 16 in equations (12), we have

$$\left.\begin{array}{l} f''' = -ff'' + 2f'^2 - 2Gr\theta\sin\gamma - 2Gc\phi\sin\gamma \\ \quad = -f(1)*f(3) + 2*f(2)*f(2) - 2*Gr*f(4)*\sin\gamma - 2*Gc*f(6)*\sin\gamma \\ \theta'' = -\Pr[f\,\theta' + Nb\phi'\theta' - Nt\theta'^2] \\ \quad = -\Pr[f(1)*f(5) + Nb*f(7)*f(5) - Nt*f(5)*f(5)] \\ \phi'' = -\dfrac{1}{Le}[f\phi' + \dfrac{Nt}{Nb}\theta''] \\ \quad = \dfrac{-1}{Le}[f(1)*f(7) + \dfrac{Nt}{Nb}*[-\Pr[f(1)*f(5) + Nb*f(7)*f(5) - Nt*f(5)*f(5)]] \end{array}\right\} \tag{17}$$

The boundary conditions are

$$\left.\begin{array}{l} f_a(1) = 0\,,\ f_a(2) = 1, \quad f_a(4) = 1, \ f_a(6) = 1 \\ f_b(2) \to 0, \quad f_b(4) \to 0 \quad f_b(6) \to 0 \\ where\ a = 0\ and\ b = \infty \end{array}\right\}$$

(18)

The boundary conditions $f''(0), \theta'(0)\ and\ \phi'(0)$ are not given in the system of ordinary differential equations. Therefore it is must for us to evaluate the approximate values of $f''(0), \theta'(0)\ and\ \phi'(0)$ with the help of shooting technique and then we apply the fourth order Runge-Kutta method to obtain $f\,,\ \ \theta\ and\ \ \phi$.
We choose the step size to be $\Delta h = 0.01$ with accuracy of 10^{-5}.

IV. RESULTS AND DISCUSSIONS

The influence of the different parameters included in the couple of ordinary differential equations such as local Grashof number Gr, local solutal Grashof number Gc, the angle of inclination γ, the prandtl number Pr, the Brownian motion parameter Nb, the thermophoresis parameter Nt, and the Lewis number Le on the stagnation point flow of nanofluid due to an inclined exponentially stretching sheet has been tabulated and their graphs sketched as well as discussed.

Table 1: Comparison table between the skin frictions of present and literature results for

Nt=GC=Gr=A=0, Pr=Le=1

λ	Skin friction or - $f''(0)$ of Meraj [10]	Present skin friction or - $f''(0)$
0	1.281810	1.281825
0.1	1.253580	1.253582
0.2	1.195120	1.195126
0.5	0.879835	0.879835
0.8	0.397771	0.397771

Table 2: Comparison table of Nusselt and Shrwood Numbers where Nb=Nt=0.1, λ=0.2

Pr	Nu or -θ'(0) of Meraj[10]	Present Nu or -θ'(0)	Sh or -ϕ'(0) of Meraj	Present Sh or -ϕ'(0)
0.4	0.74994	0.749944	0.97397	0.973989
0.7	1.03430	1.034259	0.77875	0.778747
1	1.26020	1.260245	0.61269	0.612686
1.2	1.39070	1.390719	0.51328	0.513278

In table 1 and 2 above it is clearly indicated that the present results have an excellent agreement with the results reported by Meraj [10]. In table 1, the skin friction decreases, the velocity of the fluid raises and the boundary layer thicken with the increment of λ as λ<1 that is the free stream velocity is smaller than the velocity of the stretching sheet. Table 2 above reveals that increasing the Prandtl number increases the reduced Nusselt number and decreases the reduced Sherwood number; it is due to the reduction of the thermal diffusivity which has an impact on the reduction of the thermal boundary layer and the Sherwood number but an increment on the Nusselt number.

Table 3

λ	Gr	Gc	γ	Pr	Nb	Nt	Le	Skin friction - $f''(0)$	Nusselt Number $-\theta'(0)$	Sherwood Number $-\phi'(0)$
0.1	0.3	0.2	45	4.5	0.1	0.1	10	1.0729	2.2626	4.0360
0.3								0.9370	2.2870	4.0914
0.5								0.7128	2.3228	4.1702
0.8								0.2414	2.3881	4.3109
0.2	0.1	0.2	45	4.5	0.1	0.1	10	1.0937	2.2644	4.0421
	0.3							1.0173	2.2730	4.0598
	0.4							0.9794	2.2772	4.0684
	0.5							0.9418	2.2813	4.0769
0.2	0.3	0.1	45	4.5	0.1	0.1	10	1.0480	2.2698	4.0533
		0.3						0.9866	2.2761	4.0663
		0.4						0.9562	2.2793	4.0727
		0.5						0.9258	2.2823	4.0790
0.2	0.3	0.2	30	4.5	0.1	0.1	10	1.4090	2.2283	3.9675
			35					1.2867	2.2427	3.9974
			40					1.0391	2.2706	4.0549
			45					1.0172	2.2730	4.0598
0.2	0.3	0.2	45	1	0.1	0.1	10	0.9590	1.2456	4.5080
				3				1.0045	1.9871	4.1897
				5				1.0202	2.3440	4.0277
				6.4				1.0264	2.5009	3.9582
0.2	0.3	0.2	45	4.5	0.1	0.1	10	1.0173	2.2730	4.0598
					0.3			0.9851	1.8855	3.2832
					0.4			0.9716	1.7415	3.0859
					0.5			0.9594	1.6215	2.9557
0.2	0.3	0.2	45	4.5	0.1	0.1	10	1.0727	2.2730	4.0598
						0.3		1.0146	1.5268	4.7255
						0.4		1.0102	1.2637	4.7866
						0.5		1.0059	1.0553	4.8149
							10	0.9212	0.4209	4.8246
0.2	0.3	0.2	45	4.5	0.1	0.1	14	0.9385	0.3537	5.7442

							18	0.9509	0.0311	6.5402
							20	0.9560	0.0294	6.9051

The first and fifth parts of table 3 above have the same results as table 1 and 2 respectively. In the second part of table 3 we observe that as Gr increases both the Nusselt and Sherwood numbers increase, it is because of, increasing Gr increases the buoyancy force and the thermal boundary layer thickness decays. The third part of table 3 illustrates that both the Nusselt number and Sherwood numbers enhanced with the increment of Gc. The Nusselt number and the Sherwood number increase as γ is enlarged, this is because of at γ=0, the sheet is horizontal and the buoyancy force is decreased and as $\gamma \to \frac{\pi}{2}$ the sheet is erecting to the vertical direction increased gravitational force act on the flow that enlarges the buoyancy force and reduces the thermal boundary layer thickness is described in the fourth part of table 3. As Nb increases in the sixth part of table both Nusselt and Sherwood numbers decrease by the increment of the Brownian motion a large extent of fluid enforced to become together so that the thermal boundary layer thickness increase. The seventh part of table 3 shows that with the increase of Nt the Nusselt number decreases and the Sherwood number increases, it is due to the fact that the thermophoresis diffusion penetrates deeper in to the fluid with the increase of Nt. The last part of table 3 illustrates that increasing Lewis number reduces Nusselt number and increases Sherwood number.

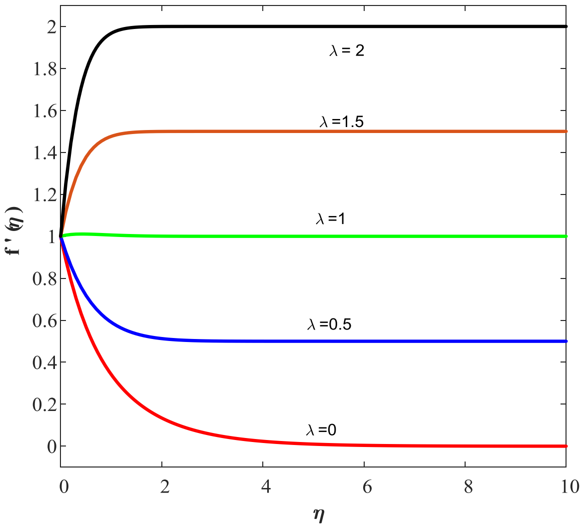

Figure 2: Variation of $f\,'(\eta)$ with the velocity ratio λ.

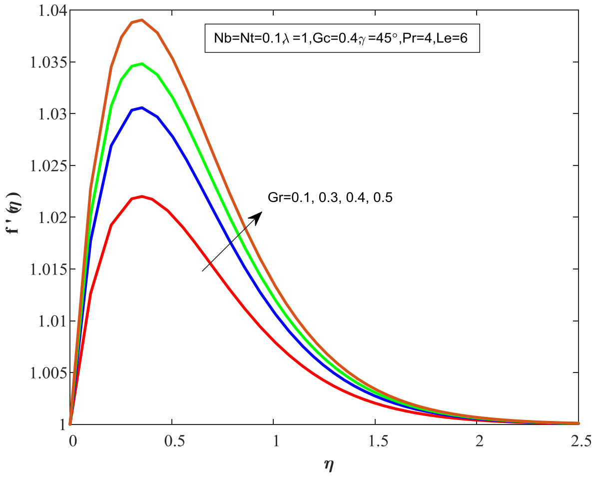

Figure 3: The effect of Gr on $f\,'(\eta)$.

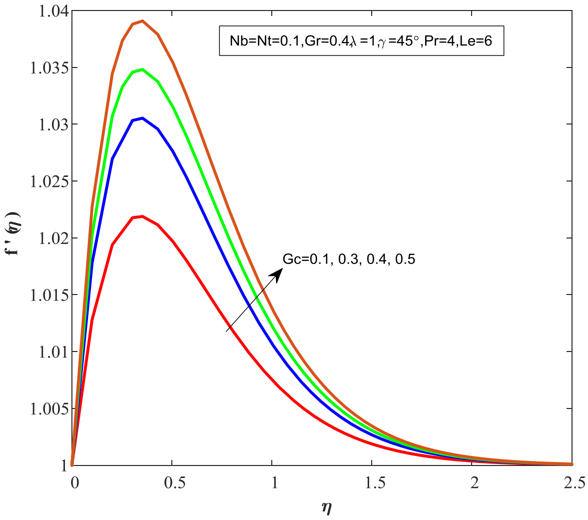

Figure 4: The influence of Gc on $f'(\eta)$.

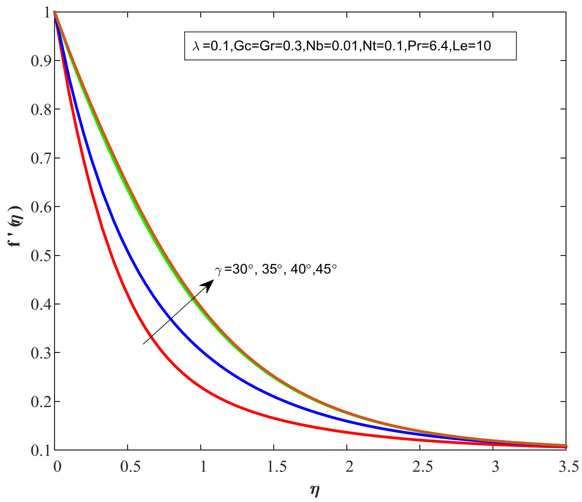

Figure 5: Variation of $f'(\eta)$ **with** γ.

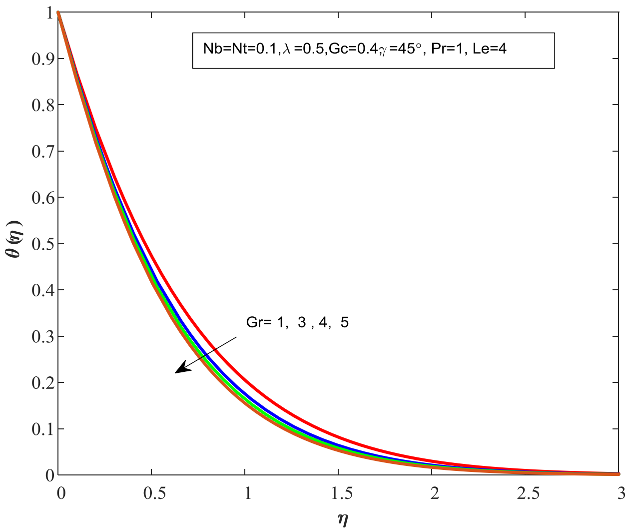

Figure 6: The effect of Gr on $\theta(\eta)$

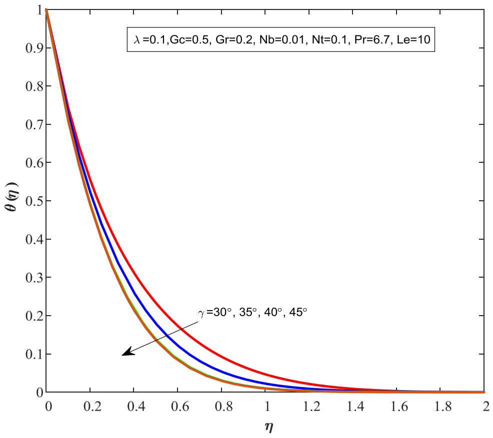

Figure 7: The influence of γ on $\theta(\eta)$.

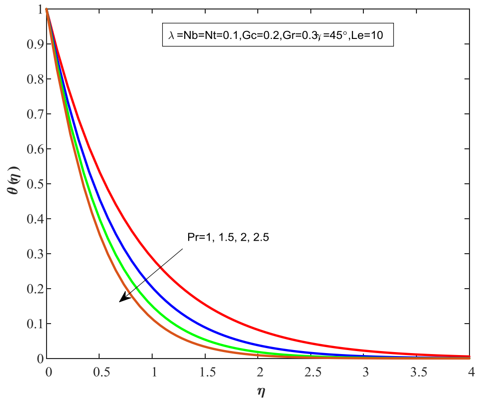

Figure 8: Variation of $\theta(\eta)$ with Pr.

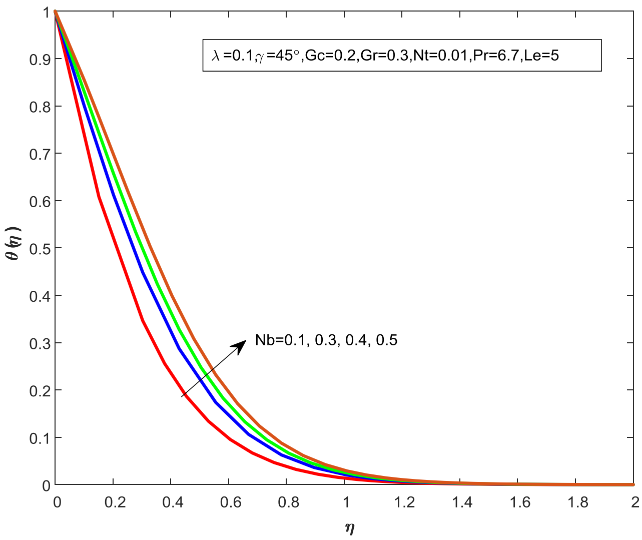

Figure 9: Influence of Nb on $\theta(\eta)$.

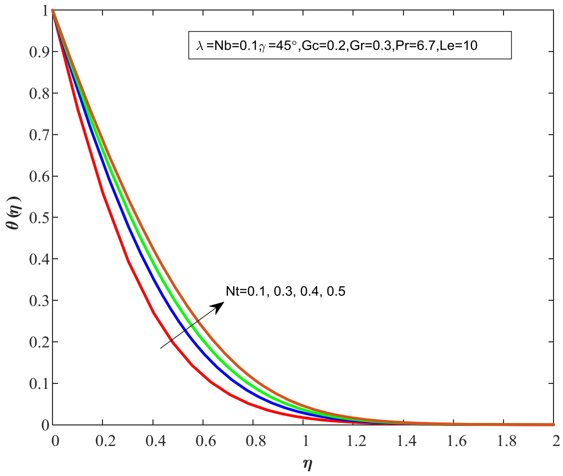

Figure 10: Effect of Nt on $\theta(\eta)$.

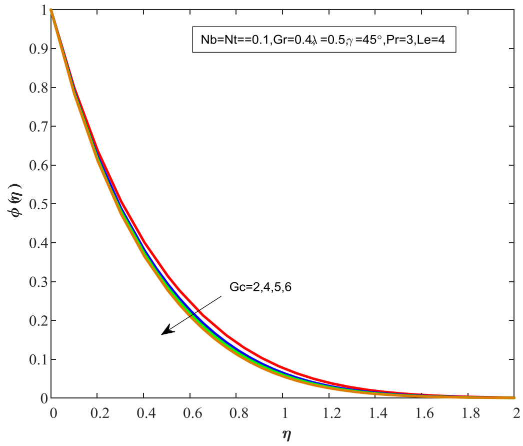

Figure 11: Variation of $\phi(\eta)$ with Gc.

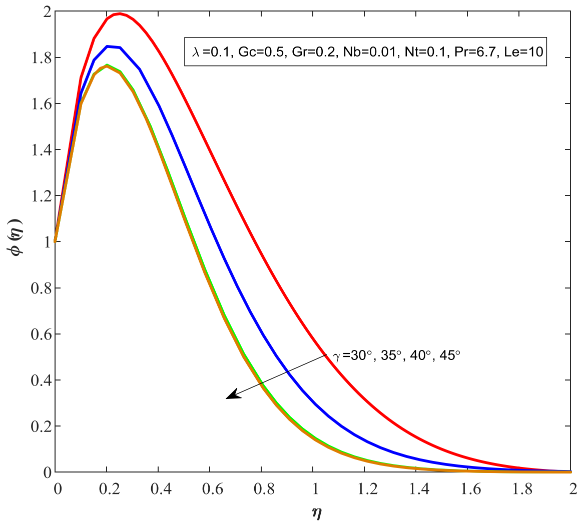

Figure 12: The effect of γ on $\phi(\eta)$.

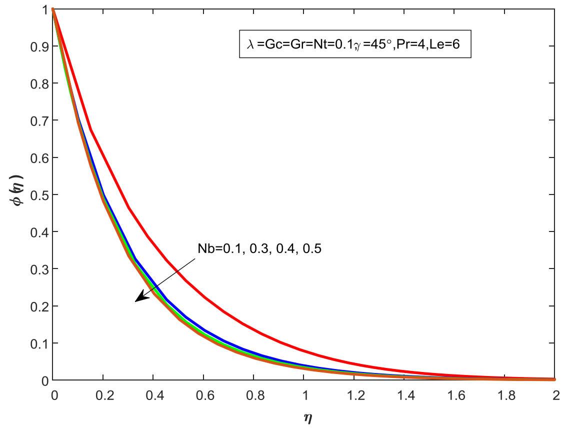

Figure 13: Influence of Nb on $\phi(\eta)$.

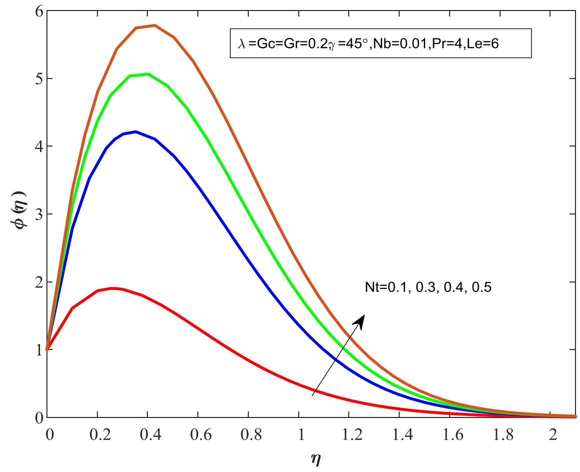

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Figure 14: Effect of Nt on $\phi(\eta)$.

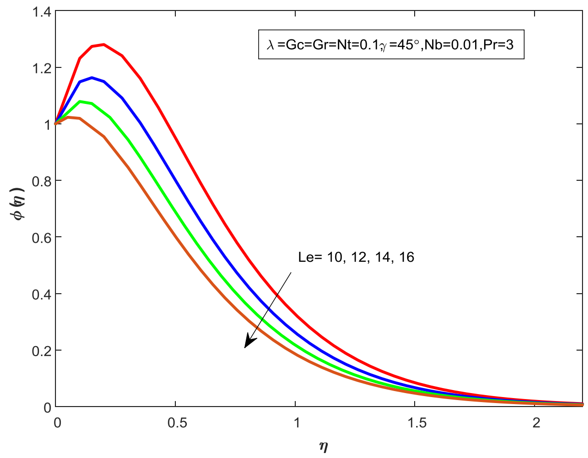

Figure 15: Variation of $\phi(\eta)$ with Le.

Figure 2 illustrates that as λ increases in the interval (0,1) the velocity of the fluid and the boundary layer thickness increase as the velocity of the stretching sheet is greater than the free stream velocity(i.e. $\lambda = \frac{b}{a} < 1$), for $\lambda > 1$, as λ increases the flow velocity increases and the boundary layer thickness decreases that is when the free stream velocity is larger than the velocity of the stretching sheet(i.e. $\lambda = \frac{b}{a}$>1). In the case where the velocity of the stretching sheet is equal to the free stream velocity, there is no boundary layer thickness of the nano fluid near the sheet. The velocity increases with the enlargement of the Grashof and the solutal Grashof numbers due to the effect of enhanced buoyancy force as indicated in Figure 3 and 4. An enlargement in the angle of inclination enhances the velocity field as shown in Figure 5 because at γ=0, the sheet is horizontal and the buoyancy force is decreased and as $\gamma \to \frac{\pi}{2}$ the sheet is erecting to the vertical direction increased gravitational force act on the flow that enlarges the buoyancy force and increases the velocity boundary layer thickness .The temperature profile decrease with the increase of Grashof number as given in Figure 6 and it is because of the enhancement of buoyancy force. In Figure 7 as the angle of inclination enlarged the thermal boundary layer diminished. Figure 8 reveals that the thermal boundary layer thickness reduces as the Prandtl number increases because the prandtl number increases as the thermal diffusivity decreases. The increase in Brownian motion parameter increases the thermal boundary layer thickness as shown in Figure 9, This is due to the increased Brownian motion increases the collision of the fluid molecules with the nano particles and this enforce a large extent of fluid molecules to wedge together make the thermal boundary layer thicken. Figure10 shows that as the thermophoresis parameter increases the thermal boundary layer thicken since the increased thermophoresis diffusion enforce the nano particles to shift from hot to cold area which increases the thermal boundary layer thickness . It is observed from Figure 11 that as solutal Grashof number increases the concentration boundary layer decreases because of the increased buoyancy force. The concentration boundary layer thickness reduces with the enlargement of the angle of inclination as it is shown in Figure 12, this is because of at γ=0, the sheet is horizontal and the buoyancy force is decreased and as $\gamma \to \frac{\pi}{2}$ the sheet is erecting to the vertical direction increased gravitational force act on the flow that enlarges the buoyancy force and reduces the concentration boundary layer thickness. The concentration boundary layer diminishes as the Brownian motion parameter enhanced obtained from Figure 13, the increased Brownian motion enable the nano particles to transfer the surface heat to the fluid and the nano particles gain higher kinetic energy that contributes to the thermal energy of the fluid that cause the movement of the nano particles from hot to the cold region. However the concentration boundary layer thickens with the increase of Nt as it is revealed in Figure 14, the nano particles move from a region of high temperature to a region of low temperature with the enlargement of thermophoresis diffusion. The molecular diffusivity decays as the Lewis number increases and the concentration boundary layer becomes thinner as it is illustrated in Figure 15.

V. CONCLUSIONS

Stagnation point flow of nanofluid due to an inclined stretching sheet is studied. The governing equations are numerically solved and the results obtained permit the computation of the flow, heat and mass transfer behaviors for various values of the velocity ratio λ, the Grashof number Gr, the solutal expansion parameter Gc, the Prandtl number Pr, the Brownian motion parameter Nb, the thermophoresis parameter Nt, and the Lewis Number Le . The results have been found that

1. The increase of the velocity ratio parameter λ, the Grashof number Gr, the solutal expansion parameter Gc and the angle of inclination γ increase the velocity boundary layer thickness.
2. The thermal boundary layer thickness increases with the increase of the Brownian motion parameter Nb and the thermophoresis parameter Nt but reduces with the increase of Prandtl number.
3. As the Prandtl number Pr and the thermophoresis parameter Nt increase the concentration boundary layer thickness increases.
4. Increasing the Brownian motion parameter and Lewis number reduces the concentration boundary layer thickness.

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