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The Comparison of Neighborhood Set and Degrees of a Circular-Arc Graph G Using an Algorithm

Dr. A. Sudhakaraiah[1], K. Narayana[2], Dr. T.Venkateswarlu[3]
1 Asst. Professor, Department of Mathematics, S V University, Tirupati, Andhra Pradesh, India- 517502
Corresponding author: Dr. A. Sudhakaraiah

ABSTRACT
A circular-arc graph is the intersection graph of family arcs on a circle. Many projects involving graphs even pure graph theory itself, involve algorithms. Most of the real life problems when transformed into graph problems exhibit some special inequality properties. This has given rise to special classes of graphs such as circle graphs, circular-arc graphs. In this chapter, we present algorithms for finding the comparison of neighborhood set and degrees of a circular-arc graph G.
KEYWORDS: Circular-Arc Family, Circular-Arc Graph, Maximum Degree, Minimum Degree, Neighborhood Set.

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I. INTRODUCTION

Circular-arc graphs are introduced as generalization of interval graphs. The class of interval graphs is properly contained in the class of circular-arc graphs. In fact every interval graph is a Circular-arc graph and the converse need not be true. However both these classes of graphs have received considerable attention in the literature in recent years and have been studied extensively. Circular-arc graphs are rich in combinatorial structures and have found applications in several disciplines such as biology, ecology, psychology, traffic control, genetics, computer science and particularly useful in cyclic scheduling and computer storage allocation problems etc. (many standard graphs theoretic problems known to be NP-complete for general graphs can be solved in polynomial time for these classes of graphs).

Let $A = \{A_1, A_2, \ldots, A_n\}$ be a circular-arc family on a circle, where each A_i is an arc. Without loss of generality assume that the end points of all arcs are distinct and no arc covers the entire circle. Denote an arc i that begins at end point p and ends at endpoint q in the clock wise direction by (p, q). Define p to be head, denoted by h(i) of the arc i, q to be the tail, denoted by t(i) Thus i = (h(i), t(i)).

The continuous part of the circle that begins with an end point c and ends with d in the clockwise direction is referred to as segment (c, d) of the circle. We use “arc” to refer to a member of A and “segment” to refer to a part of the circle between two end points. A point on the circle is said to be an arc (p, q) if it is contained in segment (p, q). An arc (p, q) of A is also referred as the segment (p, q). An arc i = (a, b) is said to be contained in another arc j = (c, d) if segment (a, b) is contained in the segment (c, d). A circular- arc family A is said to be proper if no arc in A is contained in another arc.

Let G (V, E) be a graph. Let $A = \{A_1, A_2, \ldots, A_n\}$ be a family of arcs on a circle. Then G is called a Circular-arc graph if there is one-to-one correspondence between V and A such that two vertices in V are adjacent if and only if their corresponding arcs in A intersects. We denote this Circular-arc graph by G[A]. Circular-arc family is proper then the corresponding graph is called a proper Circular-arc graph.

Usually we deal with intervals or arcs instead of vertices. Further if there are in intervals or arcs in the existing interval or arc family, then we denote its corresponding vertex set by {1, 2,….., n }. So alternatively, depending on the convenience, we use intervals or arcs as vertices and vice- versa.

II. MAIN THEOREMS

Theorem: If for any finite circular-arc Graph G to A, then

$$\left[\frac{n}{1+\Delta}\right] \leq |MNS| \leq n - \Delta.$$

Proof: Suppose a finite Circular-arc graph G, corresponding to a Circular-arc family A. Now we have to show that $\left[\frac{n}{1+\Delta}\right] \leq |MNS| \leq n - \Delta$.

In this theorem it will arise three cases.

a) The vertex degree of a Circular-arc graph G corresponding to A.

b) The neighborhood set of S corresponding to A.

c) The MNS of a Circular-arc graph G corresponding to A.

Case (a): Let $A = \{A_1, A_2, \ldots, A_n\}$ be a Circular-arc family and let G be a Circular-arc graph corresponding to A. In this we can find the maximum degree of a vertex from a Circular-arc graph G. The degree of a vertex V of a Circular-arc graph G is the number of edges of G incident with v and it is denoted by deg(v). The **maximum** or **minimum** degree among the vertices of G is defined by max{deg(v)/v$\in G$} or min{deg(v)/v$\in G$} and is denoted by $\Delta(G)$ **or** $\delta(G)$.

In this connection we should find the maximum degree of vertex v from G. Generally we call that a vertex is called odd or even depending on whether degree is odd or even. From a Circular-arc graph G, we have deg(v) = 0 then v is called an isolated vertex of G. If deg (v) = 1, then v is called an end vertex or pendent vertex of G and an edge incident with an end vertex is called an end edge or pendent edge.

The degree of an edge xy is defined to be deg(x)+ deg(y) – 2. The maximum or minimum degree among the edges of G denoted by $\Delta(G)$ **or** $\delta(G)$. An edge is called an isolated edge if deg(xy) = 0 and then graph G be disconnected but we should consider only connected graphs. In this way we can find the maximum vertex degree or minimum vertex degree of a Circular-arc graph G corresponding to a Circular-arc family A.

Case (b): In this case we will find the neighborhood set S of a circular-arc graph G corresponding to A.

From G we have a neighborhood of a vertex V in G is defined as the set of vertices adjacent with V (including v) and is denoted by nbd[v].

A Set S of vertices in G is called a neighbourhood set of G if, G = $\bigcup_{v\in s} G[nbd[v]]$ where $G[nbd[v]]$is the vertex induced sub graph of G. We know that G be a circular-arc graph and v_1 is a subset of V. Then the subgraph of G whose vertex set is $v_1$and edge set is the set of those edges in E whose both ends are in $v_1$is called the vertex induced subgraph and it is denoted by $<v_1>$. And also a neighbourhood of a vertex v $\in$ V is a set consisting all vertices adjacent to v including v and is denoted by nbd[v] that is nbd[v] = { the set of all vertices adjacent to v } $\cup$ {v} . The neighbourhood number of G is defined as the minimum cardinality of neighbourhood set S(G). Now we have to find the minimum neighbourhood set towards an algorithm of a circular-arc graph G corresponding to A.

Let S=$\{A_1, A_2, \ldots, A_n\}$ where A_1=min[1], A_n=min(max(next(A_{n-1}))) where n≥ 2 be the set constructed by the algorithm. Let the arcs be A_i and $A_j$are any two arcs in A such that $A_i < A_j <$ min (A_i) then $A_j \cap$ min(A_i), all the arcs from 1 to min[1] belonging to nbd[min[1]] and also the subset $\{A_j, \ldots, A_k \text{ is contained in } nbd[A_j] \cup nbd[A_k]\}$ that is any arc between $A_j$and $A_k$is covered by either $A_j$or $A_k$that is all the arcs $A_j$such that $A_s < A_j < A_{s+1}$are contained in nbd[A_s]$\cup$nbd[A_{s+1}] for all $1\leq s \leq A_{n-1}$Where A_{s+1} = min(max(next(A_s))) by the algorithm next (A_n)= null. Therefore the arcs from A_k *to* n belongs to nbd[A_k]. All the arcs in A belonging to

nbd[A_1] $\cup$ nbd[A_2] $\cup$nbd[A_3] $\cup$………. $\cup$nbd[A_k]andA=nbd[A_1] $\cup$nbd[A_2] $\cup$nbd[A_3] $\cup$………. $\cup$nbd[A_k].

Further an induced subgraph of G corresponding to circular-arc family A on $\{A_s, \ldots, A_{s+1}\}$ is contained in G[nbd(A_s)] $\cup$ G[nbd(A_{s+1})].

Hence the neighborhood set $\bigcup_{\{A_s,\ldots,A_{s+1}\}\subseteq A} G\{A_s,\ldots,A_{s+1}\} \subseteq \bigcup_{A_s\in s} G[nbd[A_s]]$

But clearly G $\subseteq \bigcup_{\{A_s,\ldots,A_{s+1}\}\subseteq A} G\{A_s,\ldots,A_{s+1}\}$.

Case (c): Next we will find the minimum neighborhood set (MNS) of G.

We consider $A = \left\{ A_i, A_j, \ldots, A_k \right\}$ be a circular-arc family and G is a circular-arc graph corresponding to A. Where A_1= min [1], A_k= min(max(next(A_{k-1}))) where $k \geq 2.$ The left end points of circular-arcs in S are ordered such that $b_{A1} < b_{A2} < \cdots < b_{AK}$. Let A_j and A_k two arcs in A such that A_k= min(max(next(A_J))). We consider If 'l' is any arc such that $b_l > b_{A_{s+1}}$, than it does not intersect max(next(A_s)) this implies that the edge[max(next(A_s)), next(A_s)]does not belonging to nbd[m]. Since MNS is a neighborhood set of A, there exists an arc y $\in MNS$ such that $b_s \leq b_{A_{s+1}}$ and the edge[max(next(A_s)), next(A_s)]$\in$ G[nbd[y]]. That there exists an arc $y \in MNS$ such that $b_y \leq b_{A_{s+1}}$ and the edge[max(next(A_s)), next(A_s)]$\in$ G[nbd[y]]that is there exit an arc y $\in MNS$ such that $A_s < y \leq A_{s+1}$ and the edge[max(next(A_s)), next(A_s)]$\in$ G[nbd[y]]. Thus in between any two consecutive arcs in s we get an arc y$\in$MNS such that $A_s < y \leq A_{s+1}$ where 1$\leq s \leq m-1$. Therefore besidesA_1, there are at least m-1 subintervals in S. Hence $|NS| \geq M = |S|$. Where S is the minimum neighbourhood set of G.

ALGORITHM:

Input: Circular-arc family $A = \{A_1, A_2, \ldots, A_n\}$.

Step1: Set S={min(nbd[1])}.
Step 2 : LI= The largest interval in S.
Step 3 : N= next(LI).
Step 4: If N$\in$ S, then go to step 11.
Step 5: If N $\notin$ S then
    5.1: If N< LI then go to step11.
    5.2: If N> LI then go to step6.
Step 6: J= max (min[N]).
Step7 : If J$\notin$ nbd[S] then S= S$\cup${J},go to step2.
Step8: If J$\in$ nbd[s] then i= min(max(nbd[N])).
Step9: If i$\notin$ nbd[s], then S= S$\cup${i},then go to step2.
Step10: If i$\in$ nbd[s] then S= S$\cup${N}, go to step2.
Step11: End and MNS= S.

Out put: S= minimum neighbourhood set (MNS) of the circular-arc graph G.

Note: In the above algorithm, we use the closed neighbourhood set of v is simply denoted by [v] in graph theory.

III. ILLUSTRATIONS

Experimental problem1:

If for any finite circular-arc Graph G to A, then $\left\lceil \frac{n}{1+\Delta} \right\rceil \leq |MNS| \leq n - \Delta$.

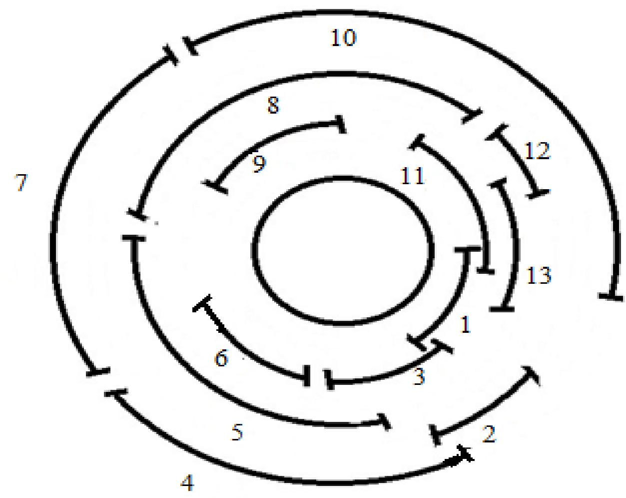

Figure 1.Circular-Arc Family A

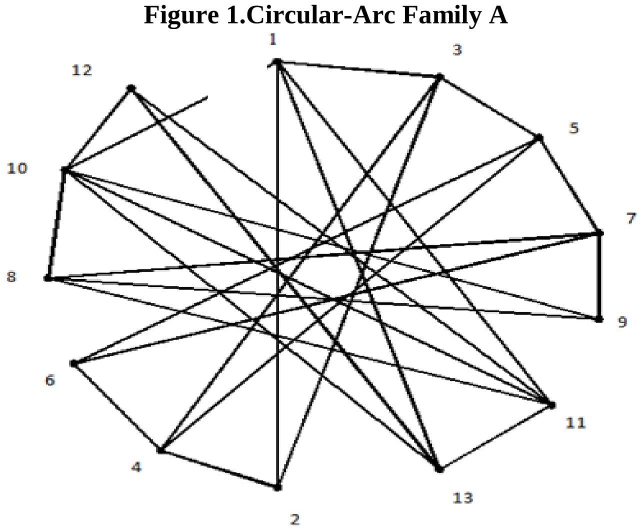

Figure 2.Circular-Arc Graph G

Degree of vertices:

deg(1)= 5	deg(2)= 3	deg(3)=4
deg(4)= 4	deg(5)= 4	deg(6)= 3
deg(7)= 4	deg(8)= 4	deg(9)= 3
deg(10)= 6	deg(11)= 5	deg(12)=3
deg(13)=4		

Neighbourhood of vertices:

nbd(1)= {1,2,3,10,11,13}, min(1)= 1 , max(1)=13, next(1)= 4
nbd(2)= {1,2,3,4},min(2)= 1 , max(2)= 4, next(2)= 5
nbd(3)= {1,2,3,4,5}, min(3)= 1 , max(3)= 5, next(3)= 6
nbd(4)= {2,3,4,5,6}, min(4)= 2 , max(4)= 6, next(4)= 7
nbd(5)= {3,4,5,6,7}, min(5)= 3 , max(5)= 7, next(5)= 8
nbd(6)= {4,5,6,7}, min(6)= 4 , max(6)= 7, next(6)= 8
nbd(7)= {5,6,7,8,9}, min(7)= 5 , max(7)= 9, next(7)= 10
nbd(8)= {7,8,9,10,11}, min(8)= 7 , max(8)= 11, next(8)= 12
nbd(9)= {7,8,9,10}, min(9)= 7 , max(9)= 10, next(9)= 11
nbd(10)= {1,8,9,10,11,12,13}, min(10)= 1, max(10)= 13, next(10)= 2
nbd(11)= {1,8,10,11,12,13},min(11)= 1 , max(11)=13, next(11)= 2
nbd(12)= {10,11,12,13}, min(12)= 10, max(12)=13, next(12)= 1
nbd(13)= {1,10,11,12,13},min(13)= 1 , max(13)=13, next(13)= 2

Algorithm:

Input: Circular-arc family A= {1,2,….,13}

Step 1: S= {min(nbd[1])}={min(1,2,3,10,11,13)}= {1}

Step 2: LI= the largest interval in S= 1
Step 3: N= Next(1)= 4
Step5: If N= 4$\notin$ S then
Step5.2: N= 4> LI=1, go to step6
Step 6: J= max(min(4))= max(2)= 4
Step7: If J= 4$\notin$ nbd[s] then S= S$\cup${4}= {1,4}, go to step2
Step 2: LI= the largest interval in S= 4
Step 3: N= Next(4)= 7
Step5: If N= 7$\notin$ S then
Step5.2: N=7> LI=4,go to step6
Step 6: J= max(min(7))= max(5)= 7
Step7: If J= 7$\notin$ nbd[s] then s= S$\cup${7}= {1,4,7}, go to step2
Step 2: LI= the largest interval in S= 7
Step 3: N= Next(7)= 10
Step5: If N= 10$\notin$ S then
Step5.2: N= 10> LI=7,go to step6
Step 6: J= max(min(10))= max(1)= 13
Step8: If J= 13$\in$ nbd[s] then i= min(max(10))min(13)=1
Step10: If i$\in$ nbd[S] then S= S$\cup${N}= {1,4,7}$\cup${10}={1,4,7,10}
Step 2: LI= the largest interval in S=10
Step 3: N= Next(10)= 2
Step5: If N= 2$\notin$ S then
Step5.1: N= 2< LI=10, go to step11
Step11: End and MNS= {1,4,7,10}.
Out put: S = {1,4,7,10} is the minimum neighbourhood set of G.

Now, here n= 13, Δ= 6, δ=3, MNS= {1,4,7,10}

Therefore $\left[\frac{n}{1+\Delta}\right] \leq |MNS| \leq n - \Delta$

i.e., $[\frac{13}{7}] \leq 4 \leq 7$

there fore $1 \leq 4 \leq 7$ (true)

Experimental problem1 is verified.

Theorem2: For any circular-arc graph G of order n corresponding to a circular-arc family A, then $|MNS| \leq \frac{1}{2}\left[(n+1) - (\delta - 1)\frac{\Delta}{\delta}\right]$.

Proof: Let $A = \{A_1, A_2, \ldots, A_n\}$ be a circular-arc family and G is circular-arc graph corresponding to A. we have already proved minimum neighbourhood set as well as maximum and minimum degrees in theorem1.As follows the experimental problem2 with an algorithm.

Experimental problem2:

For any circular-arc graph G of order n corresponding to a circular-arc family A, then $|MNS| \leq \frac{1}{2}\left[(n+1) - (\delta - 1)\frac{\Delta}{\delta}\right]$.

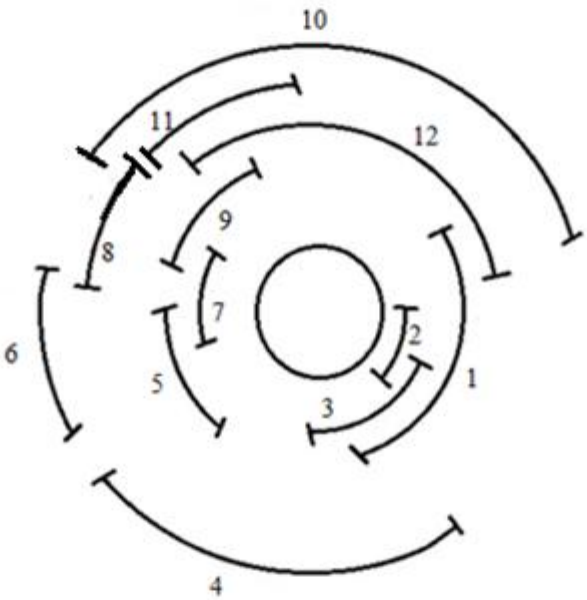

Figure 3.Circular- Arc Family A

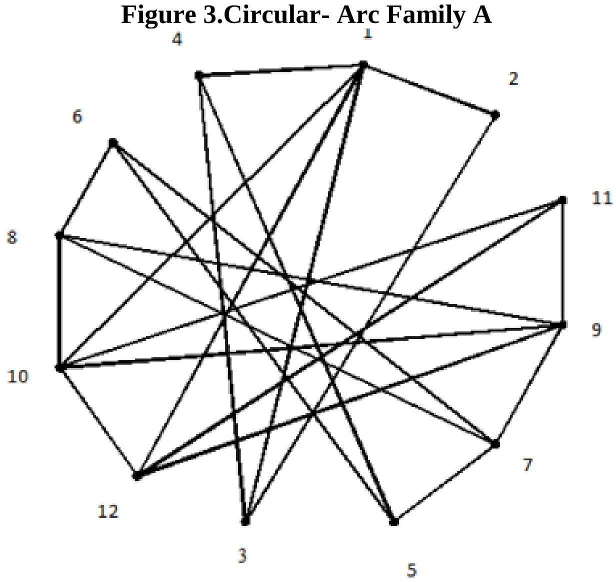

Figure 4.Circular-Arc Graph G

Degree of vertices:

deg(1)= 5	deg(2)= 2	deg(3)=3
deg(4)= 3	deg(5)= 3	deg(6)= 3
deg(7)= 4	deg(8)= 4	deg(9)= 5
deg(10)= 5	deg(11)= 3	deg(12)=4

Neighbourhood of vertices:

nbd(1)= {1,2,3,4,10,12}, min(1)= 1 , max(1)=12, next(1)= 5
nbd(2)= {1,2,3}, min(2)= 1 , max(2)= 3, next(2)= 4
nbd(3)= {1,2,3,4}, min(3)= 1 , max(3)= 4, next(3)= 5
nbd(4)= {1,3,4,5}, min(4)= 1 , max(4)= 5, next(4)= 6
nbd(5)= {4,5,6,7}, min(5)= 4 , max(5)= 7, next(5)= 8
nbd(6)= {5,6,7,8}, min(6)= 5 , max(6)= 8, next(6)= 9
nbd(7)= {5,6,7,8,9}, min(7)= 5 , max(7)= 9, next(7)= 10
nbd(8)= {6,7,8,9,10}, min(8)= 6 , max(8)= 10, next(8)= 11
nbd(9)= {7,8,9,10,11,12}, min(9)= 7 , max(9)= 12, next(9)= 1
nbd(10)= {1,8,9,10,11,12}, min(10)= 1 , max(10)= 12, next(10)= 2
nbd(11)= {9,10,11,12}, min(11)= 9 , max(11)=12, next(11)= 1
nbd(12)= {1,9,10,11,12}, min(12)= 1, max(12)=12, next(12)= 1

Algorithm:
Input: Circular-arc family A= {1,2,….,12}
Step 1: S= {min(nbd[1])}={min(1,2,3,4,10,12}= {1}
Step 2: LI= the largest interval in S= 1
Step 3: N= Next(1)= 5
Step5: If N= 5$\notin$ S then
Step5.2: N= 5> LI=1,go to step6
Step 6: J= max(min(5))= max(4)= 5
Step7: If J= 5$\notin$ nbd[s] then S= S$\cup${5}= {1,5}, go to step2
Step 2: LI= the largest interval in S= 5
Step 3: N= Next(5)= 8
Step5: If N= 8$\notin$ S then
Step5.2: N=8> LI=5,go to step6
Step 6: J= max(min(8))= max(6)= 8
Step7: If J= 8$\notin$ nbd[s] then s= S$\cup${8}= {1,5,8}, go to step2
Step 2: LI= the largest interval in S= 8
Step 3: N= Next(8)= 11
Step5: If N= 11$\notin$ S then
Step5.2: N= 11> LI=8,go to step6
Step 6: J= max(min(11))= max(9)= 12
Step8: If J= 12$\in$ nbd[s] then i= min(max(11))= min(12)=1
Step10: If i$\in$ nbd[S] then S= S$\cup${N}= {1,5,8}$\cup${11}={1,5,8,11}
Step 2: LI= the largest interval in S=11
Step 3: N= Next(11)= 1
Step5: If N= 1$\in$ S, thengo to step11
Step11: End and MNS= {1,5,8,11}.
Out put: S= {1,5,8,11} is the minimum neighbourhood set of G.
Now, here n= 12, Δ= 5, δ=2, MNS= {1,5,8,11}

Therefore $|\boldsymbol{MNS}| \leq \frac{1}{2}\left[(\boldsymbol{n}+\mathbf{1}) - (\boldsymbol{\delta}-\mathbf{1})\frac{\Delta}{\delta}\right]$

i.e., $\mathbf{4} \leq \frac{1}{2}\left[\mathbf{13} - \frac{5}{2}\right]$

$4 \leq \frac{1}{2}[10.5]$

Therefore $4 \leq 5$(true)

Experimental problem2 is verified.

Theorem3: Let $A = \{A_1, A_2, \dots, A_n\}$ be a finite circular-arc family of order n and G be a circular-arc graph corresponding to A, then

$$|\boldsymbol{MNS}| \leq \frac{1}{2}[(\boldsymbol{n}+\mathbf{2}) - \boldsymbol{\delta}].$$

Proof: we consider a circular-arc family $A = \{A_1, A_2, \dots, A_n\}$. We have to show that the inequalities $|\boldsymbol{MNS}| \leq \frac{1}{2}[(\boldsymbol{n}+\mathbf{2}) - \boldsymbol{\delta}]$. Already proved in theorem1.

Experimental problem 3:
Let G be a circular-arc graph corresponding to A,

Then $|MNS| \leq \frac{1}{2}[(n+2) - \delta]$.

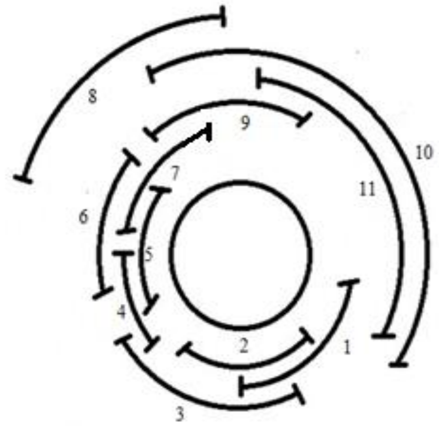

Figure 5.Circular-Arc Family A

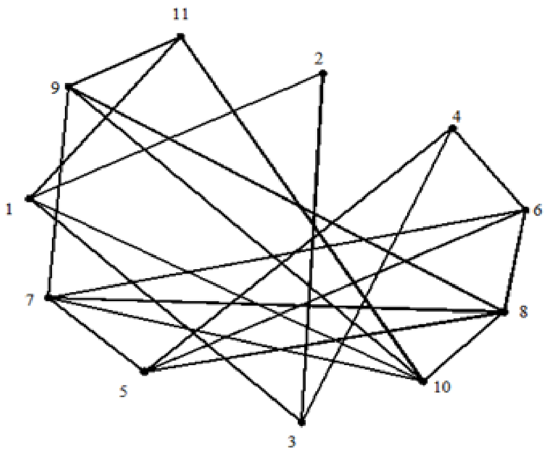

Figure 6.Circular-Arc Graph G

Degree of vertices:

deg(1)= 4	deg(2)= 2	deg(3)=3
deg(4)= 3	deg(5)= 4	deg(6)= 4
deg(7)= 5	deg(8)= 5	deg(9)= 4
deg(10)=5	deg(11)= 3	

Neighbourhood of vertices:

nbd(1)= {1,2,3,10,11}, min(1)= 1 , max(1)=11, next(1)= 4
nbd(2)= {1,2,3}, min(2)= 1 , max(2)= 3, next(2)= 4
nbd(3)= {1,2,3,4}, min(3)= 1, max(3)= 4, next(3)= 5
nbd(4)= {3,4,5,6}, min(4)= 3 , max(4)= 6, next(4)= 7
nbd(5)= {4,5,6,7,8}, min(5)= 4 , max(5)= 8, next(5)= 9
nbd(6)= {4,5,6,7,8}, min(6)= 4 , max(6)= 8, next(6)= 9
nbd(7)= {5,6,7,8,9,10}, min(7)= 5 , max(7)= 10, next(7)= 11
nbd(8)= {5,6,7,8,9,10}, min(8)= 5 , max(8)= 10, next(8)= 11
nbd(9)= {7,8,9,10,11}, min(9)= 7, max(9)= 11, next(9)= 1
nbd(10)= {1,7,8,9,10,11}, min(10)= 1, max(10)= 11, next(10)= 2
nbd(11)= {1,9,10,11}, min(11)= 1, max(11)=11, next(11)= 2

Algorithm:

Input: Circular-arc family A= {1,2,….,11}
Step 1: S= {min(nbd[1])}={min(1,2,3,10,11)}= {1}
Step 2: LI= the largest interval in S= 1
Step 3: N= Next(1)= 4
Step5: If N= 4∉ S then
Step5.2: N= 4> LI=1,go to step6

Step 6: J= max(min(4))= max(3)= 4

Step7: If J= 4$\notin$ nbd[s] then S= S$\cup${4}= {1,4}, go to step2

Step 2: LI= the largest interval in S= 4
Step 3: N= Next(4)= 7
Step5: If N= 7$\notin$ S then
Step5.2: N=7> LI=4,go to step6
Step 6: J= max(min(7))= max(5)= 8

Step7: If J= 8$\notin$ nbd[s] then s= S$\cup${8}= {1,4,8}, go to step2

Step 2: LI= the largest interval in S= 8
Step 3: N= Next (8)= 11
Step5: If N= 11$\notin$ S then
Step5.2: N= 11> LI=8,go to step6
Step 6: J= max (min(11))= max(1)= 11

Step8: If J= 11$\in$ nbd[s] then i= min(max(11)), min(11)=1

Step10: If i$=$ 1 $\in$ nbd[S] then S= S$\cup${N}= {1,4,8}$\cup${11}
={1, 4, 8, 11}, go to step2

Step 2: LI= the largest interval in S=11
Step 3: N= Next(11)= 2
Step5: If N= 2$\notin$ S then
Step5.1: N= 2< LI=11, go to step11
Step11: End and MNS= {1,4,8,11}.
Out put: S= {1,4,8,11} is the minimum neighborhood set of G.

Now, here n= 11, Δ= 5, δ=2, MNS= {1,4,8,11}

Therefore $|\boldsymbol{MNS}| \leq \frac{1}{2}[(\boldsymbol{n}+2)-\boldsymbol{\delta}]$

i.e., **4**$\leq$ 5.5 (true)

Experimental problem3 is verified.

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