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$B_{g^{**}}$-Closed Sets In Topological Space

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ABSTRACT: In this paper we introduce and study new class of sets called $B_{g^{**}}$-closed sets in topological spaces.Also we discuss some of their properties and investigate the relations between other closed sets.
KEYWORDS: b-closed, bcl(A,)$B_{g^{**}}$-closed,$B_{g^{**}}$-open,g**-closed,g**-open,g*-open

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I. INTRODUCTION

In 1970, Levine introduced the concept of generalized closed set and discussed the properties of sets, closed and open maps, compactness, normal and separation axioms. Later in 1996 Andrjevic gave a new type of generalized closed set in topological space called b closed sets.. A.A.Omari and M.S.M. Noorani made an analytical study and gave the concepts of generalized b closed sets in topological spaces. In this paper, a new class of closed set called $B_{g^{**}}$-closed set is introduced to prove that the class forms a topology.Throughout this paper (X,τ) and (Y,σ) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned.Let A⊆X, the closure of A and interior of A will be denoted by cl (A) and int (A) respectively, union of all b-open sets X contained in A is called b-interior of A and it is denoted by bint (A),the intersection of all b-closed sets of X containing A is called b-closure of A and it is denoted by bcl (A).

II. PRELIMINARIES:

Before entering into our work we recall the following definitions which are due to Levine.

Definition 2.1:

(1) a pre-open set **[11]** if A ⊆ int(cl(A)) and a preclosed set if cl(int(A)) ⊆ A.
(2) a semi-open set [**9**] if A ⊆ cl(int(A)) and and semi-closed set if int (cl(A)) ⊆ A.
(3) a semi-preopen set **[2]** if A ⊆ cl(int(cl(A))) and a semi preclosed set **[1]** if int(cl(int(A)))⊆A
(4) an α-open set **[15]**if A ⊆ int(cl(int(A))) and an α-closed set **[17]** if cl(int(cl(A))) ⊆ A.
(5)a b-open set**[3]** if A⊆cl(intA)∪ int (cl(A)) and a b-closed set if (cl(intA))∩ (int(cl(A))⊆A
(6)a generalised closed set (briefly g-closed) **[8]** if cl(A)⊆U whenever A⊆U and U is open in(X, τ) .The complement of g-closed set is g-open in X.
(7)a g*-closed**[20]** if cl(A)⊆U whenever A⊆U and U is g-open in(X, τ). The complement of g*-closed set is g*-open in X.
(8)a g**-closed**[19]** if cl(A)⊆U whenever A⊆U and U is g*-open. The complement of g**-closed set is g**-open in X.
(9) an generalised semi pre-closed set (briefly gsp-closed) **[6]** if spcl(A)⊆U whenever A⊆U and U is open in (X,τ)
(10)a generalized b-closed set(briefly gb-closed)**[16]**if bcl(A)⊆U whenever A⊆U and U is open in X
(11)a generalized α closed set (briefly gα-closed)**[10]** if αcl(A)⊆U whenever A⊆U and U is α-open in X.
(12)a weakly closed set (briefly W-closed)**[18]** if cl(A)⊆U whenever A⊆U and U is semi-open in X
(13)a generalized pre-closed (briefly gp-closed)**[12]** if pcl(A)⊆U whenever A⊆U and U is open in X
(14) a semi generalized closed set(briefly sg-closed)**[5]**if scl(A)⊆U whenever A⊆U and U is semi open in X

(15)A subset A of a topological space (x,τ) is called b*-closed**[14]** set if int(cl(A)⊆U whenever A⊆U and U is b-open.

(16) A subset A of a topological space (X,τ) is called b**-open set**[17]** if A⊆int(cl(int(A)))∪cl(int(cl(A))) and b**-closed set if cl(int(cl(A))∩int(cl(int(A)) ⊆ A.

III. BASIC PROPERTIES OF B_{G**}-CLOSED SETS:

Definition 3.1: A subset A of a topological space (X,τ) is called B_{g**}-closed if bcl(A)⊆U whenever A⊆U and U is g**-open in X.The family of all B_{g**}-closed sets are denoted by B_{g**}-C(X).

Definition 3.2: The complement of B_{g**}-closed set is called B_{g**}-open set.The family of all B_{g**}-open sets of X are denoted by B_{g**}-O(X).

Example 3.3: Let X={1,2,3} with the topology τ={∅,X,{1},{1,2}} then {∅,X,{2},{3},{1,3},{2,3}}are B_{g**}-closed sets and {∅,X,{1},{2},{1,2},{1,3}} are B_{g**}-open sets in X.

Proposition 3.4: Every closed set is B_{g**}-closed set.

Proof: Let A be a closed set in X such that A ⊆ U.Let U be g**-open.Since A is closed,cl(A)=A.Since bcl(A)⊆cl(A)=A.Therefore bcl(A)⊆U.Hence A is a B_{g**}-closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example 3.5: Let X={1,2,3} with the topology τ={∅,X,{1},{2},{1,2}} then A={2} is a B_{g**}closed set but not a closed set of (X,τ).

Theorem 3.6: Every b-closed set is B_{g**}-closed set.

Proof: Let A be a b-closed set in X such that A⊆U and U is g**-open.Since A is b-closed,bcl(A)=A.Therefore bcl(A)⊆U.Hence A is B_{g**}-closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 3.7: Let X={1,2,3} with the topology τ={∅,X,{1},{1,2}} then A={1,3} is B_{g**}-closed but not a b-closed set.

Theorem 3.8: Every b*-closed set is B_{g**}-closed set.

Proof follows from the definition.

The converse of the above theorem need not be true as seen from the following example.

Example 3.9: Let X={1,2,3} with the topology τ={∅,X,{1,2}} then A={1,3} is B_{g**}-closed but not a b*-closed set.

Theorem 3.10: Every b**-closed set is B_{g**}-closed set.

Proof follows from the definition.

Example 3.11: Let X={1,2,3} with the topology τ={∅,X,{1},{1,3}} then A={1,2} is B_{g**}-closed but not a b**-closed set.

Theorem 3.12: Every sb*-closed set is B_{g**}-closed set

Proof follows from the definition.

The converse of the above theorem need not be true as seen from the following example.

Example 3.13: Let X={1,2,3} with the topology τ={∅,X,{2},{2,3}} then A={1,2} is B_{g**}-closed but not a sb*-closed set

Theorem 3.14: Every B_{g**}-closed set is gb-closed set.

Proof: Let A be a B_{g**}-closed set in X such that A⊆U and U is open.Since every open set is g**-open and A is B_{g**}-closed,bcl(A)⊆U.Hence A is gb-closed.

The reverse implication need not be true as seen from the following example.

Example 3.15: Let X={1,2,3} with the topology τ={∅,X,{1}}.Let A={1,3} be the subset of (X,τ).Here A is gb-closed but not B_{g**}-closed set of (X,τ).

Theorem 3.16: Every α-closed set is B_{g**}-closed .

Proof: Let A be a α-closed set in X such that A⊆U and U be g**-open.Since A is α-closed bcl(A)⊆αcl(A)⊆U.Therefore bcl(A)⊆U.Hence A is B_{g**}-closed.

The converse of the above theorem need not be true as seen from the following example.

Example 3.17: Let X={1,2,3} with the topology τ={∅,X,{2,3}}.Let A={3} be the subset of (X,τ).Here A is B_{g**}-closed but not a α-closed set of (X,τ).

Theorem 3.18: Every semi-closed set is B_{g**}-closed.

Proof:Let A be a semiclosed set in X such that A⊆U where U is g**-open.Since A is semiclosed bcl(A)⊆ scl(A)⊆U,.Therefore bcl(A)⊆U.Hence A is B_{g**}-closed set in X.

The reverse implication need not be true as seen from the following example.

Example 3.19: Let X={1,2,3} with the topology τ={∅,X,{1,3}}.Let A={2,3} be the subset of (X,τ).Here A is B_{g**}-closed but not semi-closed set of (X,τ).

Theorem 3.20: Every pre-closed set is B_{g**}-closed.

Proof:Let A be a preclosed set in X such that A⊆U where U is g**-open.Since A is preclosed,

bcl(A)⊆ pcl(A)⊆U,.Therefore bcl(A)⊆U.Hence A is B_{g**}-closed set in X.
The reverse implication need not be true as seen from the following example.
Example 3.22: Let X={1,2,3} with the topology τ={∅,X,{2},{2,3}} then A={1,2} is B_{g**}-closed but not a pre-closed set of (X,τ)

Theorem 3.21: : Every g*-closed set is B_{g**}-closed.
Proof:Let A be a g*-closed set in X such that A⊆U where U is g**-open.Since A is g*-closed, bcl(A)⊆ cl(A)⊆U,.Therefore bcl(A)⊆U.Hence A is B_{g**}-closed set in X.
The converse of the above theorem need not be true as seen from the following example.
Example 3.22: Let X={1,2,3} with the topology τ={∅,X,{1,2},{3}} then A={1,3} is B_{g**}-closed but not a g*-closed set of (X,τ)

Theorem 3.23: : Every gα-closed set is B_{g**}-closed.
Proof:Let A be a gα-closed set in X such that A⊆U where U is g**-open.Since A is gα-closed, bcl(A)⊆αcl(A)⊆U,.Therefore bcl(A)⊆U.Hence A is B_{g**}-closed set in X.
The converse of the above theorem need not be true as seen from the following example.
Example 3.24: In example 3.11,A={1,2} is B_{g**}-closed but not α-closed.
Theorem 3.25: : Every B_{g**}-closed set is gsp-closed set.
Proof:Let A be a B_{g**}-closed set in X such that A⊆U where U is open in X.Since every open set is g**-open and A is B_{g**}-closed,spcl(A)⊆bcl(A)⊆U.Hence A is gsp-closed.
The converse of the above theorem need not be true as seen from the following example.
Example 3.26: Let X={1,2,3} with the topology τ={∅,X,{1}} then A={1,3} is gsp-closed but not B_{g**}-closed set of (X,τ).
Theorem 3.27:A set A is B_{g**}-closed iff bcl(A)-A contains no nonempty g**-closed set
Proof:**Necessity:**Let F be a g**-closed set of (x,τ) such that F⊆bcl(A)-A.Then A⊆ X-F.Since A is B_{g**}-closed and X-F is g**-open,then bcl(A)⊆X-F.This implies F⊆X-bcl(A) so F⊆(X-bcl(A)) ∩(bcl(A)-A)⊆(X-bcl(A)∩ (bcl(A))=∅.Therefore F=∅.

Sufficiency: Assume that bcl(A)-A contains no nonempty g**-closed set.Let A⊆U and U is g**-open.Suppose that bcl(A) is not contained in U,bcl(A)∩U^c is a nonempty g**-closed set of bcl(A)-A which is a contradiction.Therefore bcl(A)⊆U and hence A is B_{g**}-closed.

Theorem 3.28:A B_{g**}-closed set A is b-closed iff bcl(A)-A is b-closed.
Proof: If A is b-closed then bcl(A)-A =∅.Conversely suppose bcl(A)-A is b-closed in X.Since A is B_{g**}-closed by theorem3.27,bcl(A)-A contains no nonempty g**-closed set in X.Then bcl(A)-A=∅.
Hence A is b-closed
Theorem 3.29: If A and B are B_{g**}-closed then A∩B is also B_{g**}-closed.
Proof:Given that A and B are two B_{g**}-closed sets in X.Let A∩B⊆U,U is g**-open set in X.Since A is B_{g**}-closed bcl(A)⊆U,whenever A⊆U,U is g**-open in X.Since B is B_{g**}-closed,bcl(A)⊆U,whenever B⊆U,U is g**-open in X.
Corallary 3.30: The intersection of a B_{g**}-closed set and a closed set is a B_{g**}-closed set.
The above corallary can be proved by the following example.
Example 3.31: Let X={1,2,3,4} with the topology τ={∅,X,{1,2,4}}.{3} is the only closed set of (X,τ).
B_{g**}-closed sets of (X,τ) are {1},{2},{3},{4}{1,2},{1,3},{1,4}{2,3},{2,4},{3,4}{1,2,3}{1,3,4},{2,3,4}
The intersection of a B_{g**}-closed set and {3} is again a B_{g**}-closed set.
Remark 3.32: If A and B are B_{g**}-closed then their union need not be B_{g**}-closed.
The above Remark can be proved by the following Example.
Example3.33: Let X={1,2,3} with the topology τ={∅,X,{1,2}} Here A={1} and B={2} are B_{g**}-closed. But A∪B={1,2} is not B_{g**}-closed set.
Theorem 3.34: If A is both g**-open and B_{g**}-closed then A is b-closed.
Proof: Since A is g**-open and B_{g**}-closed in X,bcl(A)⊆A.But always A⊆bcl(A).Then A=bcl(A).Hence A is b-closed.
Theorem 3.35: For x ϵX,the set X-{x} is B_{g**}-closed or g**-open.
Proof: Suppose X-{x} is not g**-open,then X is the only g**-open set containing X-{x}.This implies bcl(X-{x})⊆ X.Then X-{x}is B_{g**}-closed in X.
Theorem 3.36: If A is B_{g**}closed and A⊆B⊆bcl(A) then B is B_{g**}-closed.
Proof: Let U be a g**-open set of X such that B⊆U.Then A⊆U.Since A is bg**-closed then bcl(A)⊆U.Now bcl(B)⊆bcl(bcl(A))=bcl(A)⊆U.Therefore B is B_{g**}closed in X.
Theorem 3.37: Let A ⊆Y ⊆X and suppose that A is B_{g**}closed in X,then A is B_{g**}-closed relative toY.

Proof: Given that A ⊆Y ⊆X and A is $B_{g^{**}}$-closed in X. To show that A is $B_{g^{**}}$-closed relative to Y. Let A ⊆Y ∩U, where U is g**-open in X. Since A is $B_{g^{**}}$-closed, A ⊆U, implies bcl(A) ⊆U. It follows that
Y ∩bcl(A) ⊆Y ∩ U.Thus A is $B_{g^{**}}$-closed relative to Y.

Theorem 3.31 Suppose that B ⊆ A ⊆ X, B is $B_{g^{**}}$-closed set relative to A and that A is both g**-open and $B_{g^{**}}$-closed subset of X, then B is $B_{g^{**}}$-closed set relative to X.

Proof: Let B ⊆G and G be an open set in X. But given that B⊆ A ⊆ X,therefore B ⊆A and B ⊆ G. This implies B ⊆ A ∩ G. Since B is $B_{g^{**}}$-closed relative to A, A ∩bcl(B) ⊆ A∩G. Implies (A ∩ bcl(B)) ⊆A∩G. Thus (A∩bcl(B))∪(bcl(B))^c ⊆G∪(bcl(B))^c. Implies A∪(bcl(B)^c ⊆ G∪(bcl(B))^{c}Since A is $B_{g^{**}}$-closed in X, we have bcl(A) ⊆ G ∪(bcl(B))^c. Also B ⊆ A
implies bcl(B) ⊆ bcl(A). Thus bcl(B) ⊆ bcl(A) ⊆G ∪ (bcl(B))^c. Therefore bcl(B) ⊆ G, since bcl(B) is not contained in bcl(B)^c. Thus B is $B_{g^{**}}$-closed set relative to X.

IV. $B_{G^{**}}$-CLOSED SET IS INDEPENDENT OF OTHER CLOSED SETS

Remark 4.1: The following example shows that the concept of W-closed and $B_{g^{**}}$-closed sets are independent.

Example 4.2: Let X={1,2,3} with the topology τ={∅,X,{2}}.In this topological space the subset A={2} is W-closed but not $B_{g^{**}}$-closed set.Also the subset B={1} is $B_{g^{**}}$- closed but not W-closed.

Remark 4.3: The following example shows that the concept of sg-closed and $B_{g^{**}}$-closed sets are independent.

Example 4.4: Let X={1,2,3} with the topology τ_1={∅,X,{1},{2},{1,2}}.In this topological space the subset A={1,2} is sg-closed but not $B_{g^{**}}$-closed set.For the topology τ_2={∅,X,{1},{1,2}}. In this topological space the subset B={1,3} is $B_{g^{**}}$-closed but not sg-closed set.

Remark 4.5: The following example shows that the concept of gp-closed and $B_{g^{**}}$-closed sets are independent.

Example 4.6: Let X={1,2,3} with the topology τ_1={∅,X,{1}}.In this topological space the subset A={1,3} is gp-closed but not $B_{g^{**}}$-closed set.For the topology τ_2={∅,X,{1,3}}. In this topological space the subset B={1} is $B_{g^{**}}$-closed but not gp-closed set.

Remark 4.7: The following example shows that the concept of g-closed and $B_{g^{**}}$-closed sets are independent.

Example 4.8: Let X={1,2,3} with the topology τ_1={∅,X,{1,3}}.In this topological space the subset A={1} is $B_{g^{**}}$-closed but not g-closed set.For the topology τ_2={∅,X,{1}}. In this topological space the subset B={1,2} is g-closed but not $B_{g^{**}}$-closed set

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