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Motion Around The Equilibrium Points In The Planar Magnetic-Binary Problem

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ABSTRACT
This article deals with the motion near the equilibrium points in the planar magnetic-binary problem including the effect of the gravitational forces of the primaries on the small body.
Key words: Space dynamics, magnetic-binaries problem

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I. INTRODUCTION

Much attention has been paid in the past of study of motion in the vicinity of the equilibrium points of the restricted three body problem such as the families of the periodic orbits around the collinear equilibrium points have been studied by Moulton [1], in the planer case of the problem , these families have been computed by (e.g. Henon) [2] , and V. Szebehely [3]. Motion around the triangular equilibrium points of the restricted three body problem under angular velocity variation has been study by Papadakis [4]. The several cases of the magnetic binaries problem have been studied by A. Mavragnais [5]---[7]. and Bhatnagar and Mohd. Arif [8].
In this article we have studied the motion around the equilibrium points in the planar magnetic-binary problem.

II. EQUATION OF MOTION

Two bodies (the primaries), with magnetic fields move under the influence of gravitational force and a charged particle P of charge q and mass m moves in the vicinity of these bodies. The question of the magnetic-binary problem is to describe the motion of this particle fig (i). The equation of motion and the integral of relative energy in the rotating coordinate system written as:

$$\ddot{x} - \dot{y}\, f = U_x \qquad (1)$$

$$\ddot{y} + \dot{x}\, f = U_y \qquad (2)$$

$$\dot{x}^2 + \dot{y}^2 = 2\mathrm{U} - \mathrm{C} \qquad (3)$$

Where

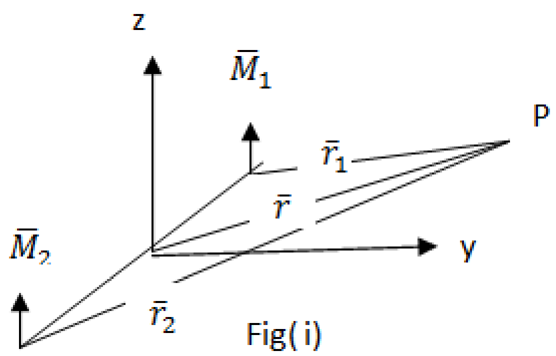
$$f = 2 - \left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right),\quad U_x = \frac{\partial U}{\partial x} \text{ and } U_y = \frac{\partial U}{\partial y}$$

$$U = (x^2 + y^2)\left\{\frac{1}{2} + \frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right\} + x\left\{\frac{\mu}{r_1^3} - \frac{\lambda(1-\mu)}{r_2^3}\right\} + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2} \qquad (4)$$

Here we assumed

1. Primaries participate in the circular motion around their centre of mass
2. Position vector of P at any time t be $\overline{r} = (xi + yj + zk)$ referred to a rotating frame of reference $\mathrm{O}(x, y, z)$ which is rotating with the same angular velocity $\overline{\omega} = (0, 0, 1)$ as those the primaries.
3. Initially the primaries lie on the x-axis.
4. The distance between the primaries as the unit of distance and the coordinate of one primary is $(\mu, 0, 0)$ then the other is $(\mu-1, 0, 0)$.
5. The sum of their masses as the unit of mass. If mass of the one primaries μ then the mass of the other is $(1-\mu)$.
6. The unit of time in such a way that the gravitational constant G has the value unity and q= mc where c is the velocity of light.

$r_1^2 = (x-\mu)^2 + y^2$, $r_2^2 = (x+1-\mu)^2 + y^2$, $\lambda = \frac{M_2}{M_1}$ (M_1, M_2 are the magnetic moments of the primaries which lies perpendicular to the plane of the motion).



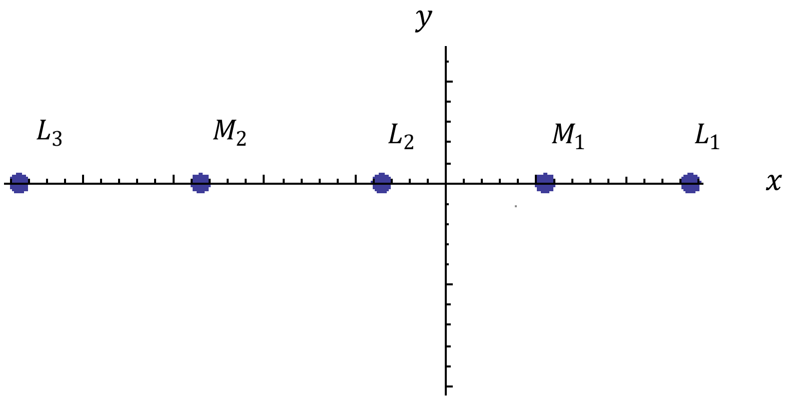
Fig(i)

To find the locations of the equilibrium points, we must solve the following equations

$$U_x = 0 \text{ ------------------------ } (5)$$

$$U_y = 0 \text{ ------------------------ } (6)$$

The solution of equations (5) and (6) results the equilibrium points, three on the x-axis, called the collinear equilibrium points (fig(ii)) and other are on xy-plane called the non-collinear equilibrium points(ncep).



Fig(ii)

To study analytically the solutions in the neighborhood of the equilibrium points $L_{(a,b)}$ we write

$$x = x_0 + \xi, \quad y = y_0 + \eta$$

Where ξ and η are coordinates relative to the equilibrium points. Now the equations (1) and (2) become

$$\ddot{\xi} - \dot{\eta}\, f = \xi\, U_{xx}^0 + \eta U_{xy}^0 \quad \text{--} \quad (7)$$

$$\ddot{\eta} + \dot{\xi}\, f = \xi\, U_{xy}^0 + \eta U_{yy}^0 \quad \text{--} \quad (8)$$

The characteristic equation of (7) and (8) is

$$\lambda^4 + \lambda^2\{f^2 - U_{xx}^0 - U_{xy}^0\} + U_{xx}^0\, U_{yy}^0 - \left(U_{xy}^0\right)^2 = 0 \quad \text{-------------------------} \quad (9)$$

III. MOTION AROUND COLLINEAR EQUILIBRIUM POINTS

Since the general solution of the equations (10) is of the form

$$\xi = \sum_{i=1}^{4} A_i e^{\lambda_i t}$$

$$\eta = \sum_{i=1}^{4} B_i e^{\lambda_i t}$$

------------------------- (10)

Contains one term increasingly monotonically for $t \geq t_0$ which gives unbounded values for ξ and η as $t \to \infty$. The solution is unstable.

The coefficients A_i, B_i are not independent and related to one another as:

$$B_i = \frac{\{\lambda_i{}^2 - U_{xx}^0\}}{\lambda_i f + U_{xy}^0} A_i = \alpha_i A_i \quad \text{-------------------------} \quad (11)$$

and these coefficients are completely determined by the initial conditions as below

$$\xi_0 = \xi(t_0) = \sum A_i\, e^{\lambda_i t_0}$$
$$\dot{\xi}_0 = \dot{\xi}_0(t_0) = \sum A_i \lambda_i\, e^{\lambda_i t_0}$$
$$\eta_0 = \eta(t_0) = \sum \alpha_i A_i\, e^{\lambda_i t_0}$$
$$\dot{\eta}_0 = \dot{\eta}_0(t_0) = \sum A_i \lambda_i\, \alpha_i e^{\lambda_i t_0}$$

------------------------- (12)

The inversion of this equation gives the coefficients

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = A^{-1} \begin{bmatrix} \xi_0 \\ \eta_0 \\ \dot{\xi}_0 \\ \dot{\eta}_0 \end{bmatrix}$$

Where

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ \alpha_1\lambda_1 & \alpha_2\lambda_2 & \alpha_3\lambda_3 & \alpha_4\lambda_4 \end{bmatrix}$$

With det $A \neq 0$

For collinear equilibrium points $U_{xy}^0 = 0$. The coefficients A_1 and A_2 are associated with the real exponents (λ_1 and λ_2). So for these values of A_1 and A_2 the first two terms on the right side of equation (10) in the solution represent exponential increase and decay with time. Choose the condition such that $A_1 = A_2 = 0$, and evaluate A_3 and A_4 as function of λ_3, α_3 and initial conditions t_0, ξ_0, η_0 and substitute the result in equation (10), we have

$\xi = \xi_0 \cos s(t - t_0) + \frac{\eta_0}{\beta_3} \sin s(t - t_0)$ ------------------------- (13)

$\eta = \eta_0 \cos s(t - t_0) - \xi_0\, \beta_3 \sin s(t - t_0)$ ------------------------- (14)

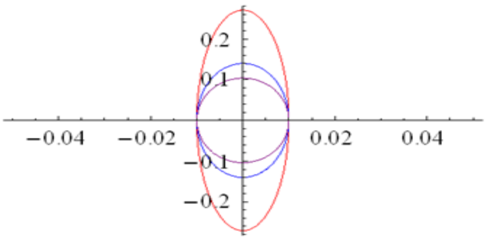
Where

$$\lambda_{3,4} = \pm\sqrt{\left(\frac{-\Delta - \sqrt{\Delta^2 - 4c}}{2}\right)},\quad \Delta = f^2 - U_{xx}^0 - U_{xy}^0\ , c = U_{xx}^0\, U_{yy}^0, \qquad \lambda_3 = is,\ \alpha_3 = i\beta_3$$

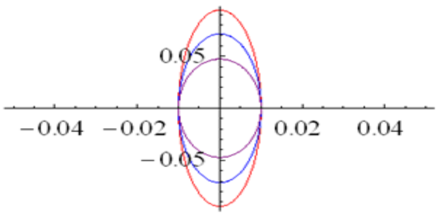
From equation (13) and (14), we can obtains

$$\xi^2 + \frac{\eta^2}{{\beta_3}^2} = {\xi_0}^2 + \frac{{\eta_0}^2}{{\beta_3}^2} \text{ ------------------------- } (15)$$

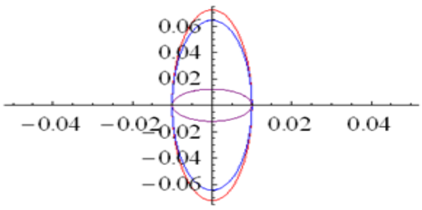
This shows that the orbit is an ellipse whose semimajor axis is ${\xi_0}^2{\beta_3}^2 + {\eta_0}^2$ which is parallel to the y-axis, the center of this ellipse at the equilibrium point and eccentricity e is $\sqrt{(1 - {\beta_3}^{-2})}$. The motion is periodic with respect to the rotating frame of reference with the synodic period $T = \frac{2\pi}{s}$.



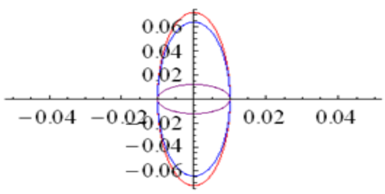
Fig(iii) μ=.05



Fig(iv) μ=.15

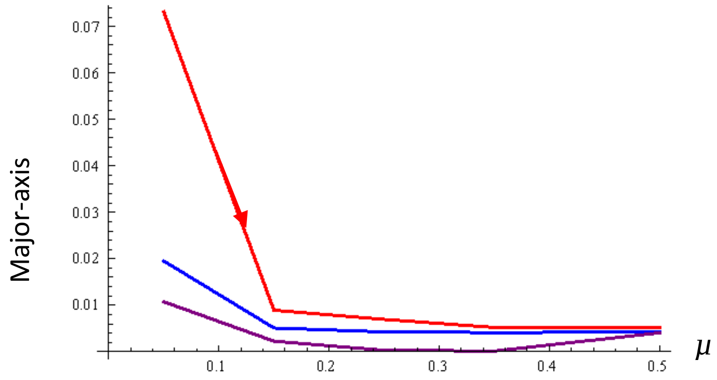


Fig(v) μ=.25

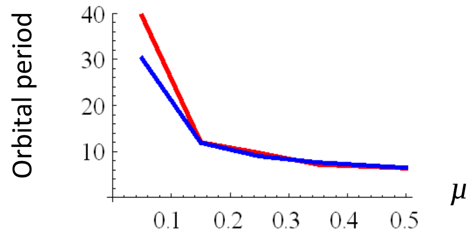


Fig(vi) μ=.35

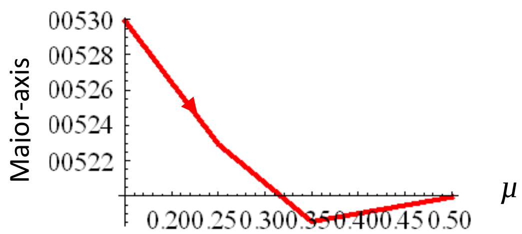
In above figs((iii),….(vi)), we have drawn some orbits around L_1 for different values of μ and $\lambda = 1$ in three different cases { red color for magnetic-binaries problem including the effect of the gravitational forces of the primaries on the charged particle, blue color for magnetic-binaries problem and purple color only for gravitational forces}. Here we have observed that the length of the major-axis fig(viia) and orbital period fig(viib) of all the orbits decreases as μ increases, around L_1 . Almost the same result exist around L_3 figs((viii)......(x)).



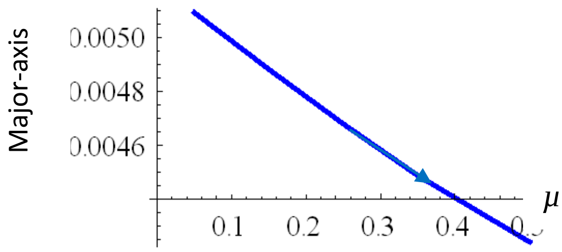
fig(vii a)



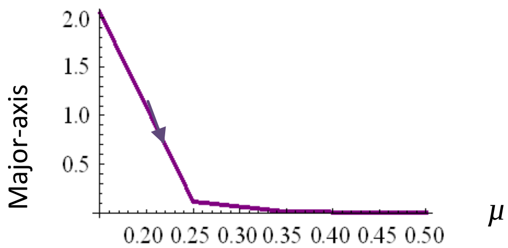
fig(vii b)



fig(viii)



fig(ix)

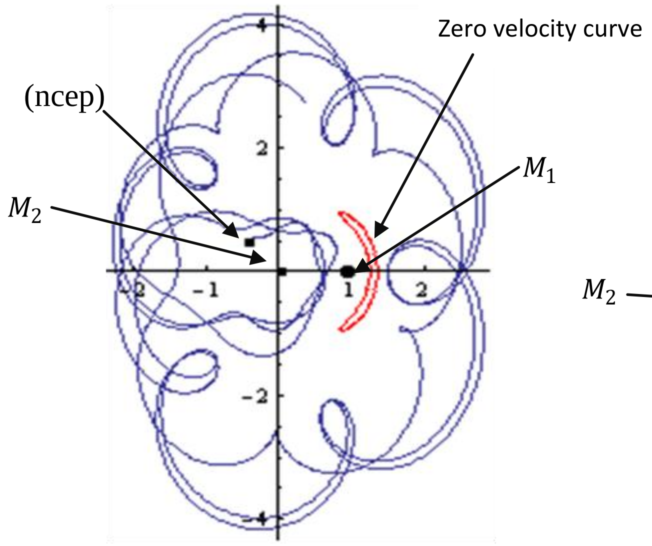


fig(x)

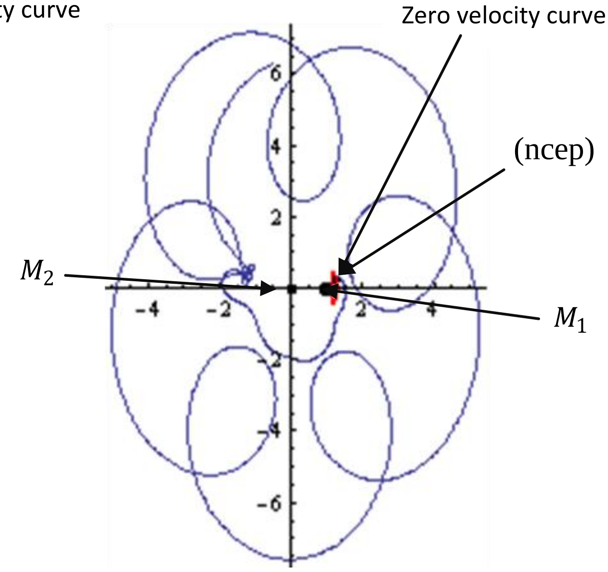
IV. MOTION AROUND NON-COLLINEAR EQUILIBRIUM POINTS(NCEP)

The peculiar, looping nature of the particle's path shown in following fig (xi) and (xii) results from the two different types of motion that contribute to the perturbed orbit about the equilibrium point for $\mu = .05$ and $\mu = .001$.

These types of motion described by the solution of the equations for perturbed motion around the equilibrium point L_4 are only valid in the vicinity of this equilibrium point and only for small displacement .075 in fig(xi) and .0065 in fig(xii) from the equilibrium point L_4 and these trajectories obtained by integrating the full equations of motion (1) and (2).



fig(xi) $\mu = .05, \lambda = 1, L_4 + .075$



fig(xii) $\mu = .001, \lambda = 1, L_4$ +.0065

Therefore, instead of dealing with the full equations of planar magnetic-binaries problem including the effect of the gravitational forces of the primaries on the small body, it makes more sense to work with a system of equations that describe the motion of the small body in the vicinity of the secondary mass, this type of system was derived by Hill in 1878. By making some assumptions and transferring the origin of the coordinate system to the second mass the equations of motion (1) and (2) , become

$\ddot{\zeta} - \dot{\nu} f = U_\zeta$ --- (16)

$\ddot{\nu} + \dot{\zeta} f = U_\nu$ --- (17)

Where

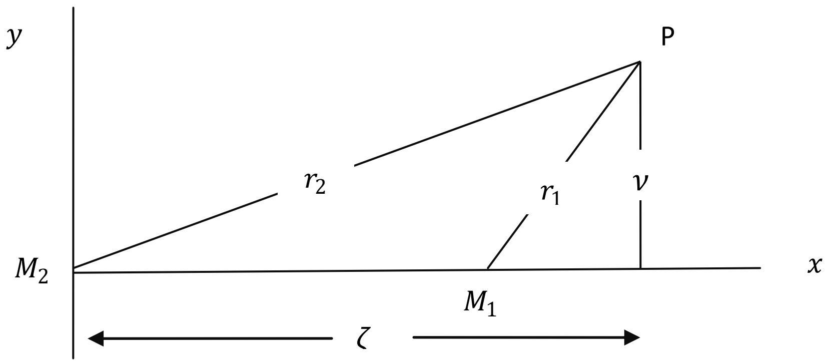
$f = 2 - \left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right)$, $U_\zeta = \frac{\partial U}{\partial \zeta}$ and $U_\nu = \frac{\partial U}{\partial \nu}$

$U = \{(\zeta + \mu - 1)^2 + \nu^2\}\left\{\frac{1}{2} + \frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right\} - (\zeta + \mu - 1)\left\{\frac{\mu}{r_1^3} - \frac{\lambda(1-\mu)}{r_2^3}\right\} + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2}$ ……..(18)

$r_1^2 = (\zeta - 1)^2 + \nu^2$, $r_2^2 = \zeta^2 + \nu^2$

And new Jacobi constant, C_n, is given by

$\dot{\zeta}^2 + \dot{\nu}^2 = 2U - C_n$ -- (19)



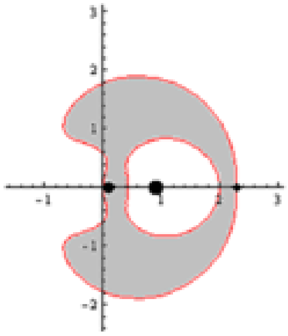
fig(xiii)

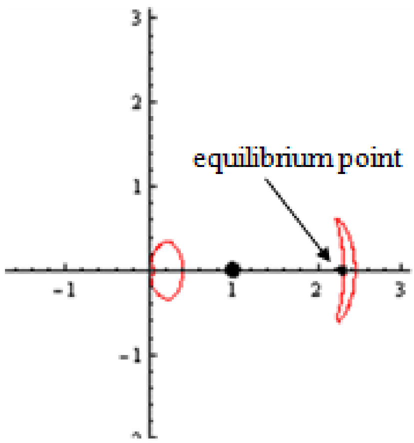
By setting $\dot{\zeta} = \ddot{\zeta} = \dot{\nu} = \ddot{\nu} = \nu = 0$ and $\zeta \neq 0$ in equation (16) and (17) we can find the location of the collinear equilibrium points. We have observed that there exisist only two collinear equilibrium points for $\mu = 0$ while for $\mu \neq 0$ we have three collinear equilibrium points Table(i).

μ	L_1	L_2	L_3
0	2.3053	0.55943	
.05	2.2995	0.9486	0.5668
.005	2.3047	0.9949	0.56008
.1	2.2940	0.89392	0.57663
.15	2.2889	0.83401	0.5908
.2	2.2840	0.76259	0.61596

Table(i) $\lambda = 1$

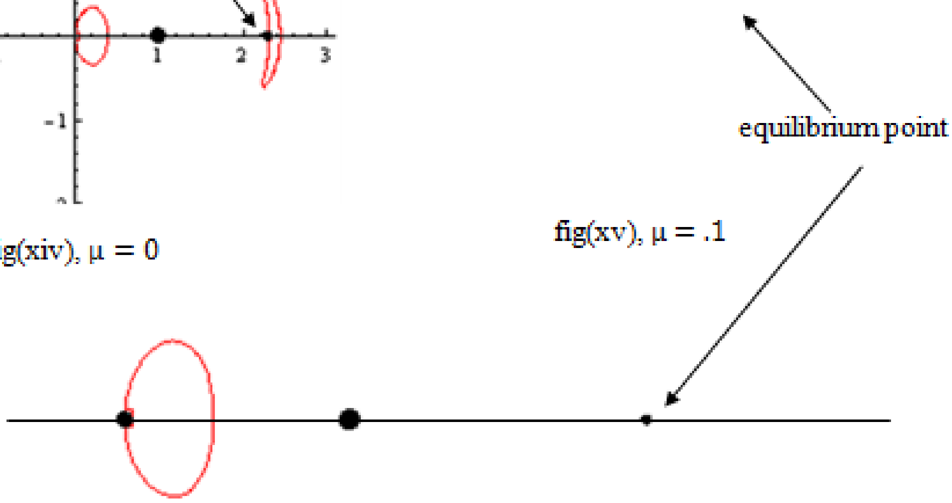
The shapes of the resulting zero velocity curves in the vicinity of collinear equilibrium points are showen in the following figures for different values of μ.





fig(xiv), $\mu = 0$

fig(xv), $\mu = .1$



fig(xvi), $\mu = .005$

low relative energy levels, and they either move one of the small binari star in a small oval or neighbourhood of this star or will stay outside the large oval which enclose both stars. Those small body for which $\boldsymbol{C_3} < C < \boldsymbol{C_2}$, possess enough relative energy to communicate between the stars. Let us now decrease the C so that $\boldsymbol{C_1} < C < \boldsymbol{C_3}$. The physical meaning of this step is that the small body is not confined to its motion around the small star, but it's allowed to move near the second star.

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