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# Optimal N-policy for Finite Queue with Server Breakdown and State-dependent Rate

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## ABSTRACT

In this paper, a finite Markovian queue with single server has been investigated under N-Policy. The server is subject to random breakdown and is restored to its previous state after repair. The customers arrive according to Poisson distribution to get service from the server. The server turns off when there is no customer in the system i.e. the system is empty and turns on when there are N or more than N customers present in the system. The server may break down when it is in working state. We derive queue size distribution by using generating function method. The optimum value of threshold parameter N is determined which minimizes the total average cost. To examine the effect of different parameters, sensitivity analysis is facilitated.

***Keywords*:** Queue size distribution, N-Policy, Markovian model, Finite capacity, State-dependent rate, Cost analysis

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## I. INTRODUCTION

Queueing model with server breakdown can be helpful in predicting the performance of various machining systems wherein machines are subject to random failure require attention of repair crew to restore it functionality. In N-policy server turns on only when there are N or more than N jobs present in the system so that N-policy is a cost effective measurement. Many researchers have worked on queueing system under N-Policy in different frameworks. A comprehensive survey on N-Policy queueing model can be found in Doshi (1986). Jain (1997) suggested an optimal N-Policy for a single server Markovian queue with break down, repair and state dependent arrival rate. Ke and Pearn (2004) analyzed optimal management policy for heterogeneous arrival queueing systems with server breakdowns and vacations.

In most engineering systems including computer, communication, production, manufacturing, etc., breakdown of component while processing the jobs, is common phenomenon. Avi-ltzhak and Naor (1963) studied some queueing problems with the service station subject to breakdowns. Neuts and Lucantoni (1979) described a Markovian queue with N servers subject to break down and repair. Jayarama, et al. (1994) discussed a general bulk service queue with arrival rate dependent on server breakdowns. Grey et al (2000) described a vacation queueing model with server breakdown. Recently Ke (2004) examined bi-level control for batch arrival queues with an early startup and un-reliable server. Dudina et al. (2013) analyzed retrial queuing system with phase service. Marin and Bulo (2014) studied a queue with hyper exponential service time and Poisson arrival. Yang and Wu (2015) obtained N-policy queue with unreliable server. Pradhan and Gupta (2017) discussed a of an infinite-buffer batch-arrival queue with batch-size-dependent service.

In the present paper we investigate single server finite Markovian queue with breakdown, repair and state dependent arrival rate under N policy by using recursive method. The steady state results for different states are obtained by using the probability generating technique. An attempt has been made to design optimal operating N-policy to minimize the total expected cost function.

## II. MODEL DESCRIPTION

Formulation of the mathematical model for a Markovian single server queue with server breakdown under N-policy has been considered. The customers arrive in Poisson process with rate dependent upon the states of the servers. The server may be one of the three states namely (i) state ‘0’ i.e. turn off state (ii) state ‘1’ i.e. turn on and in operating state and (iii) state ‘2’ i.e. turn on and under repair state. The server is subject to random breakdown and is restored to its previous state after repair. The server may breakdown only when it is in working state. The service time of customers as well as the, lifetime and repair time of servers are assumed to be exponential distributed. As soon as the repairing of the breakdown server completed, it starts service of the customers with the same strength as prior to breakdown. The duration for which the server is turned on and in operation known as busy period. The length of time for which the server is turned off and in breakdown condition is known as breakdown period. The sum of idle period, busy period and breakdown period is called a busy cycle.

The notations used for modeling purpose are as follows:

- $\lambda$ Arrival rate of customers when server is idle
- $\lambda_1$ ($\lambda_2$) Arrival rate of customers when server is turned on and in operation (under repair)
- $\alpha$ Failure rate of the server
- $\beta_0$ ($\mu_0$) Repair rate (Service rate) of the server for $1 \le n \le N$
- $\beta$ ($\mu_1$) Repair rate (Service rate) of the server for $N+1 \le n \le K$
- E[I], E[B] Expected length of the idle (busy) period.
- E[D], E[C] Expected length of the breakdown (cycle) period.
- $P_I$, $P_B$, $P_D$ The long-run fraction of time for which server is idle, busy and breakdown respectively.
- $P_i(n)$ The steady state probability that there are n customers present in the system when server is in state i.

**The Governing Equations And Analysis**

$$0 = -\lambda p_0(0) + \mu_0 p_1(1) \quad \ldots(1)$$

$$0 = -\lambda p_0(n) + \lambda p_0(n-1), \quad 1 \le n \le N-1 \quad \ldots(2)$$

$$0 = -(\lambda_1 + \alpha + \mu_0)P_1(1) + \mu_0 P_1(2) + \beta_0 P_2(1) \quad \ldots(3)$$

$$0 = -(\lambda_1 + \alpha + \mu_0)P_1(n) + \lambda_1 P_1(n-1) + \mu_0 P_1(n+1) + \beta_0 P_2(n), \quad 2 \le n \le N-1 \quad \ldots(4)$$

$$0 = -(\lambda_1 + \alpha + \mu_0)P_1(N) + \lambda_1 P_1(N-1) + \mu_1 P_1(N+1) + \beta_0 P_2(N) + \lambda P_0(N-1) \quad \ldots(5)$$

$$0 = -(\lambda_1 + \alpha + \mu_1)P_1(n) + \lambda_1 P_1(n-1) + \mu_1 P_1(n+1) + \beta P_2(n), \quad N+1 \le n \le K-1 \quad \ldots(6)$$

$$0 = -(\mu_1 + \alpha)P_1(k) + \lambda_1 P_1(k-1) + \beta P_2(k) \quad \ldots(7)$$

$$0 = -(\lambda_2 + \beta_0)P_2(1) + \alpha p_1(1) \quad \ldots(8)$$

$$0 = -(\lambda_2 + \beta_0)P_2(n) + \lambda_2 P_2(n-1) + \alpha p_1(n), \quad 2 \le n \le N \quad \ldots(9)$$

$$0 = -(\lambda_2 + \beta)P_2(n) + \lambda_2 P_2(n-1) + \alpha p_1(n), \quad N+1 \le n \le K-1 \quad \ldots(10)$$

$$0 = -\beta P_2(k) + \alpha P_1(k) + \lambda_2 P_2(k-1) \quad \ldots(11)$$

Solving equations (1)-(11) recursively, we find the analytical solution for $P_0(n)$, $P_1(n)$, and $P_2(n)$ as

$$P_0(n) = P_0(0) \qquad n=1, 2, \ldots, N-1 \qquad \ldots(12)$$

$$P_1(n)=\begin{cases}\dfrac{\lambda}{\mu_0}P_0(0), & n=1\\ AP_1(n-1)-B_1P_1(n-2)-\dfrac{\alpha\beta_0}{\mu_0\lambda_2}\sum_{i=1}^{n-3}B^{n-i}P_1(i), & 2\le n\le N-1\\ \left(\dfrac{B_1\mu_0-B_2\mu_1}{A\mu_0-A_1\mu_1}\right)P_1(N-1)+\dfrac{\alpha}{\lambda_2(A\mu_0-A_1\mu_1)}\left(\beta_0\sum_{i=1}^{N-2}B^{N-i+1}-\beta_0\sum_{i=1}^{N-2}B_3^{\,N-i+1}\right)P_1(i)+\dfrac{\lambda}{(A\mu_0-A_1\mu_1)}P_0(N-1)\\ A_1P_1(n-1)-B_2P_1(n-2)-\dfrac{\alpha\beta}{\mu_1\lambda_2}\sum_{i=1}^{n-3}B_3^{\,n-i}P_1(i), & N+1\le n\le K-1\\ \dfrac{1}{\mu_1}\left[(\lambda_1+\alpha B_3)P_1(K-1)+\alpha B_3^{\,2}P_1(K-2)+\alpha B_3^{\,3}P_1(K-3)+\alpha B_3^{\,3}\sum_{i=1}^{K-4}B^{K-i-3}P_1(i)\right]\end{cases}$$

…(13)

$$P_2(n)=\begin{cases}\dfrac{\alpha}{\lambda_2+\beta_0}P_1(n), & n=1\\ \dfrac{\alpha}{\lambda_2}\sum_{i=1}^{n}B^{n-i}P_1(i), & 2\le n\le N\\ \dfrac{\alpha}{\lambda_2}\sum_{i=1}^{n}B_3^{\,n-i}P_1(i), & N+1\le n\le K-1\\ \dfrac{\alpha}{\beta}\left[P_1(K)+B_3P_1(K-1)+B_3^{\,2}\sum_{i=1}^{K-2}B^{K-i-2}P_1(i)+P_1(n)\right]\end{cases}\quad\ldots$$

(14)

where $A=\left(\dfrac{(\mu_0+\lambda_1)(\lambda_2+\beta_0)+\alpha\lambda_2}{\mu_0(\lambda_2+\beta_0)}\right)$, $A_1=\left(\dfrac{(\mu_1+\lambda_1)(\lambda_2+\beta)+\alpha\lambda_2}{\mu_1(\lambda_2+\beta)}\right)$

$B=\left(\dfrac{\lambda_2}{\lambda_2+\beta_0}\right)$, $B_1=\left(\dfrac{\lambda_1(\lambda_2+\beta_0)^2+\alpha\beta_0\lambda_2}{\mu_0(\lambda_2+\beta_0)^2}\right)$,

$B_2=\left(\dfrac{\lambda_1(\lambda_2+\beta)^2+\alpha\beta\lambda_2}{\mu_1(\lambda_2+\beta)^2}\right)$, $B_3=\left(\dfrac{\lambda_2}{\lambda_2+\beta}\right)$

## III. GENERATING FUNCTION METHOD

Define the following generating function

$$G_0(z)=\sum_{n=0}^{N-1}P_0(n)z^n,\ G_i(z)=\sum_{n=1}^{K}P_i(n)z^n,\ i=1,2 \qquad ...(15)$$

Solving (1) and (11), we obtain

$$G_0(z)=\frac{1-z^N}{1-z}P_0(0) \qquad ...(16)$$

$$G_1(z)=\frac{\left[\,(\lambda_2 z-\lambda_2-\beta)\lambda z(1-z^N)P_0(0)-\lambda_2 z(z-1)\{\sum_{n=1}^{N}(\beta_0-\beta)z^n P_2(n)+\beta P_2(K)z^K\}+(\lambda_2 z-\lambda_2-\beta)(z-1)\{(\mu_0-\mu_1)\sum_{n=1}^{N}P_1(n)z^n+\lambda_1 P_1(K)z^{K+1}\}\,\right]}{\left[\,\{\lambda_1 z^2-(\lambda_1+\alpha+\mu_1)z+\mu_1\}(\lambda_2 z-\lambda_2-\beta)-\alpha\beta z\,\right]}$$

...(17)

$$G_2(z)=\frac{\left[\,\{\lambda_1 z^2-(\lambda_1+\mu_1)z+\mu_1\}\sum_{n=1}^{N}(\beta_0-\beta)z^n P_2(n)+\alpha\lambda z(z^N-1)P_0(0)+\{\lambda_1 z^2-(\lambda_1+\alpha+\mu_1)z+\mu_1\}\lambda_2 P_2(K)z^K(z-1)-\alpha(z-1)\{(\mu_0-\mu_1)\sum_{n=1}^{N}P_1(n)z^n+\lambda_1 P_1(K)z^{K+1}\}\,\right]}{\left[\,\{\lambda_1 z^2-(\lambda_1+\alpha+\mu_1)z+\mu_1\}(\lambda_2 z-\lambda_2-\beta)-\alpha\beta z\,\right]}$$

...(18)

The normalizing condition provides

$$P_0(0)=\frac{\gamma+(\alpha+\beta)\left(\sum_{n=1}^{N}(\mu_0-\mu_1)P_1(n)z^n+l\right)+(\lambda_2-\lambda_1+\mu_1)\sum_{n=1}^{N}(\beta_0-\beta)P_2(n)}{N\eta}$$ ...

(19)

where $\gamma=\beta\mu_1-\lambda_1\beta-\alpha\lambda_2$, $l=\lambda_1 P_1(K)+\lambda_2 P_2(K)$, $\eta=(\gamma+\lambda(\alpha+\beta))$

## IV. OPERATIONAL CHARACTERISTICS

Using generating function determined in previous section, we derive expressions for various operating characteristics as follows:

- The long-run fraction of time for which server is idle

$$P_I=\sum_{n=0}^{N-1}P_0(n)=G_0(1)=NP_0$$

$$=\left[\,\gamma+(\alpha+\beta)\{\sum_{n=1}^{N}(\mu_0-\mu_1)P_1(n)z^n+l\}+(\lambda_2-\lambda_1+\mu_1)\sum_{n=1}^{N}(\beta_0-\beta)P_2(n)\,\right]/\eta$$

… (20)

- The long-run fraction of time for which server is busy

$$P_B=\sum_{n=1}^{K}P_1(n)=G_1(1)=\left[\,\lambda\beta-\beta\{\sum_{n=1}^{N}(\mu_0-\mu_1)P_1(n)z^n+l\}+(\lambda-\lambda_2)\sum_{n=1}^{N}(\beta_0-\beta)P_2(n)\,\right]/\eta$$ ...

(21)

- The long-run fraction of time for which server breakdown

$$P_D=\sum_{n=1}^{K}P_2(n)=G_2(1)=\left[\,\alpha\lambda-\alpha\{\sum_{n=1}^{N}(\mu_0-\mu_1)P_1(n)z^n+l\}+(\lambda_1-\lambda-\mu_1)\sum_{n=1}^{N}(\beta_0-\beta)P_2(n)\,\right]/\eta$$

... (22)

- Expected number of customers when the server is turned off

$$E[N_0]=G_0'(1)=\frac{N(N-1)P_0(0)}{2}$$

$$=\frac{N-1\left[\,\gamma+(\alpha+\beta)\{\sum_{n=1}^{N}(\mu_0-\mu_1)P_1(n)z^n+l\}+(\lambda_2-\lambda_1+\mu_1)\sum_{n=1}^{N}(\beta_0-\beta)P_2(n)\,\right]}{2\eta}$$ ... (23)

- Expected number of customers when the server is turned on and in operation

$$E[N_1] = G_1'(1) = [\lambda\beta(N+1) - 2\lambda_1\lambda_2 + 2\lambda_2\{S^1 - S^2\} - \beta Q^1 - \lambda_2 Q^2 + K_1]/2\eta +$$
$$[-\lambda\beta X + \beta[\lambda(\alpha+\beta)(N+1) + X]S^1 + \{\lambda(\lambda_2 - \lambda_1 + \mu_1)[\beta(N+1) - 2\lambda_2]$$
$$-2\lambda\lambda_2(\alpha+\beta) + X(\lambda_2 - \lambda)\}S^2 - \lambda(\alpha+\beta)[\beta Q^1 + \lambda_2 Q^2] + \{\lambda(\alpha+\beta) \quad \ldots$$
$$[\beta(N+1) - 2\lambda_2] + X\beta\}l + \lambda(\alpha+\beta)K_1]/2\gamma\eta$$

(24)

where $S^1 = \sum_{n=1}^{N}(\mu_0 - \mu_1)P_1(n)z^n$ $\qquad S^2 = \sum_{n=1}^{N}(\beta_0 - \beta)P_2(n)$

$Q^1 = \sum_{n=1}^{N} n(\mu_0 - \mu_1)P_1(n)z^n$ $\qquad Q^2 = \sum_{n=1}^{N} n(\beta_0 - \beta)P_2(n)$

$X = 2(\lambda_1\lambda_2 - \lambda_1\beta - \alpha\lambda_2 - \mu_1\lambda_2)$

$K_1 = 2\lambda_1\lambda_2 P_1(K) - \lambda_1\beta(K+1)P_1(K) - 2\lambda_2\beta P_2(K) - \lambda_2\beta K P_2(K))$

- Expected number of customers when the server is turned on and broken

$$E[N_2] = G_2'(1) = [\lambda\alpha(N+1) + 2\lambda_1 S^2 - \alpha Q^1 + (\lambda_1 - \mu_1)Q^2 + K_2]/\eta +$$

$$[-\lambda\alpha X + [\alpha\lambda(\alpha+\beta)(N+1) + X\alpha](S^1 + l) + \{\alpha\lambda(\lambda_2 - \lambda_1 + \mu_1)(N+1) + 2\lambda\lambda_1$$
$$(\alpha+\beta) - X(\lambda_1 - \mu_1 - \lambda)\}S^2 - \lambda(\alpha+\beta)[\alpha Q^1 - (\lambda_1 - \mu_1)Q^2]l - K_2]/2\gamma\eta$$

where ... (25)

$K_2 = -\lambda_1\alpha(K+1)P_1(K) - \lambda_2 P_2(K)[2(\lambda_1 - \alpha - \mu_1) - \alpha K]$

- The Expected number of customers in the system is

$E[N] = E[N_0] + E[N_1] + E[N_2]$ ... (26)

## V. OPTIMAL OPERATING N-POLICY

According to memoryless property of the Poisson process, the length of the idle period is the sum of the N exponential random variables each having mean $1/\lambda$.

Thus $E[I] = N/\lambda$

Since $E[C] = E[I] + E[D] + E[B]$ and $P_I = \frac{E[I]}{E[C]}$, $P_D = \frac{E[D]}{E[C]}$, $P_B = \frac{E[B]}{E[C]}$

we get,

$$E[D] = N[\ \alpha\lambda - \alpha(S^1 + l) + (\lambda_1 - \lambda - \mu_1)\sum_{n=N+1}^{K}(\beta - \beta_0)P_2(n)\ ]/\xi \qquad \ldots (27)$$

$$E[B] = N[\ \beta\lambda - \beta(S^1 + l) + (\lambda - \lambda_2)\sum_{n=N+1}^{K}(\beta - \beta_0)P_2(n)\ ]/\xi \qquad \ldots (28)$$

$$E[C] = N(\gamma + \lambda(\alpha+\beta))/\xi \qquad \ldots (29)$$

where $\xi = \lambda[\gamma + (\alpha+\beta)\{S^1 + l\} + (\lambda_2 - \lambda_1 + \mu_1)S^2]$

## VI. COST ANALYSIS

Now we determine the optimal value of control parameter N with state dependent arrival rates so that the expected total cost per unit time could be minimized.

The expected total cost per unit time is given by

$E\{C(N)\} = (C_1+C_2)\frac{1}{E(C)} + C_3P_B + C_4P_I + C_5P_D + C_6E(N_s)$ ... (30) where

$C_1(C_2)$ = start-up cost when server is in turned-on (turned-off) state

$C_3$ ($C_4$) = cost per unit time for keeping server on (off)
$C_5$ = cost per unit time for a breakdown server
$C_6$ = holding cost per customer per unit time present in the system

The objective is to minimize E{C (N)} to determine an optimal value (say N*) of the decision variable N. In order to compute N* a heuristic approach can be employed.

## VII. SPECIAL CASES

I. When $\mu_0= \mu_1$, $\beta_0=\beta$, $\lambda=\lambda_1=\lambda_2$ i.e. when service rate and arrival rate are constant and $n \to \infty$ then our model reduces to a Markovian queueing system with a removable and non-reliable server.
II. When $\mu_0= \mu_1$, $\beta_0=\beta$, and n=1, we get results for a single unreliable server queue with arrival rate depending upon server's state.
III. When $\mu_0= \mu_1$, $\beta_0=\beta$ and $K \to \infty$ then our results coincide with Jain (1997).

## VIII. Numerical Illustration

In this section, we study the effect of various parameters on the system performance by taking the different combinations of the system parameters. The graphical presentation is provided in Figs. 1-6.
Fig. 1 (2) illustrates the effect of different values of $\lambda$ ($\mu$) on the average queue length E(N). We observe that the average queue length is higher for homogenous input rate ($\lambda$) in comparison of heterogeneous rate. Also we note that the average queue length increases (decreases) with the increase in the value of $\lambda$ ($\mu$); the increment (decrement) is significant for higher (lower) value of $\lambda$ ($\mu$). Fig. 3 (4) depicts the graphs for the average queue length E(N) vs. $\alpha$ ($\beta$). It is noted that the average queue length increases (decreases) with the increase in $\alpha$ ($\beta$). The effect of different values of N on the average queue length E(N) and arrival rate are shown in figure 5 and 6. It is seen from figures that the average queue length increases linearly with N, also we note that the average queue length is higher for heterogeneous rate in comparison of homogeneous rate ($\lambda_1=\lambda_2=\lambda=0.5$). In table 1 we summaries the minimum expected cost for different sets of $\lambda$ and ($\alpha$, $\beta$, $\mu$) by varying N. The optimal value of N can be determined by considering minimum costs which are displayed by bold letters in the table.

## IX. CONCLUSION

In this investigation, we have analyzed optimal N-policy for finite queue with server breakdown and state dependent rate. Using recursive method, we have determined steady state queue size distribution, which is employed to formulate other performance indices such as expected number of customers present in the different states and in the system, expected length of idle, busy and breakdown period, etc. We have determined the optimal N-policy so that expected total cost per unit time is minimized. The model investigated can be further extended to incorporate batch arrivals which is the subject of own future study.

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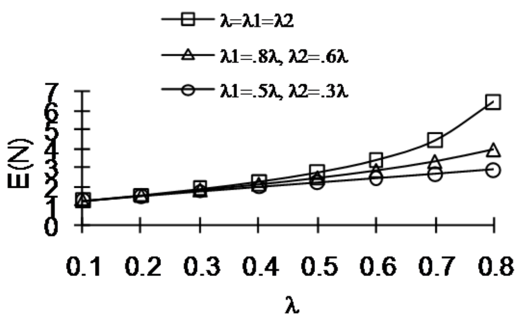


Fig. 1: Average queue length vs. λ
(N=5, α=.1, β=5, μ=1)

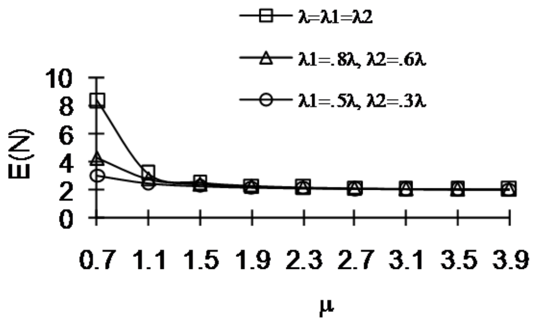


Fig. 2: Average queue length vs. μ
(N=5, α=.1, β=5, λ=.5)

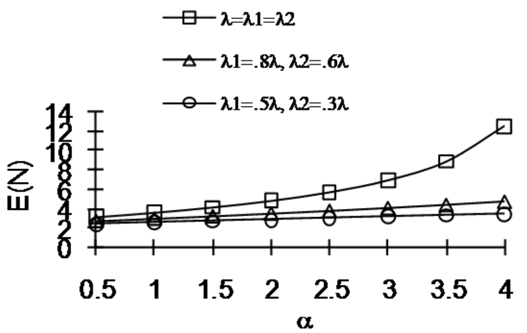


Fig. 3: Average queue length vs. α
(N=5, μ=1, β=5, λ=.5)

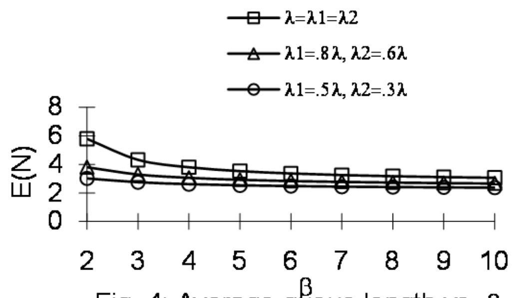


Fig. 4: Average queue length vs. β
(N=5, μ=1, α=1, λ=.5)

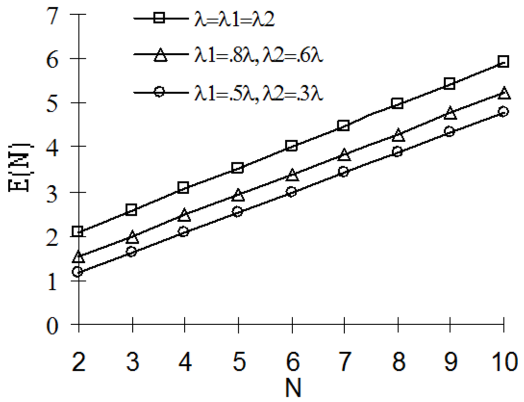


Fig. 5: Average queue length vs. N
($\beta$=5, $\mu$=1, $\alpha$=1, $\lambda$=.5)

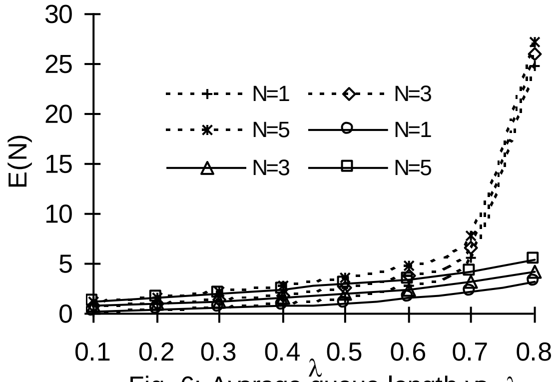


Fig. 6: Average queue length vs. $\lambda$
($\mu$=1, $\alpha$=1, $\beta$=5)

| $\lambda$ | N | ($\alpha$, $\beta$, $\mu$) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | (.5, 5, 1) | (1, 5, 1) | (0.5, 6, 1) | (1, 6, 1) | (0.5, 5, 2) | (1, 5, 2) | (0.5, 6, 2) | (1, 6, 2) |
| | 7 | 12.1369 | 11.9103 | 11.3033 | 11.1574 | 10.6647 | 10.3063 | 9.8527 | 9.5998 |
| | 8 | 11.9260 | 11.7773 | **11.1885** | **11.1015** | 10.4508 | 10.1556 | **9.7373** | **9.5311** |
| 0.5 | 9 | **11.8595** | **11.7765** | 11.1957 | 11.1590 | **10.3813** | **10.1373** | 9.7440 | 9.5758 |
| | 10 | 11.8940 | 11.8683 | 11.2884 | 11.2958 | 10.4128 | 10.2115 | 9.8361 | 9.6999 |
| | 11 | 12.0019 | 12.0275 | 11.4432 | 11.4904 | 10.5177 | 10.3530 | 9.9904 | 9.8817 |
| | 7 | 13.4276 | 13.2939 | 12.5791 | 12.5074 | 11.3173 | 10.9913 | 10.4996 | 10.2736 |
| | 8 | 13.2457 | **13.1941** | **12.4924** | **12.4834** | 11.1231 | **10.8632** | **10.4033** | **10.2265** |
| 0.6 | 9 | **13.2082** | 13.2267 | 12.5278 | 12.5728 | **11.0731** | 10.8674 | 10.4290 | 10.2928 |
| | 10 | 13.2716 | 13.3518 | 12.6486 | 12.7415 | 11.1241 | 10.9641 | 10.5401 | 10.4385 |
| | 11 | 13.4085 | 13.5443 | 12.8317 | 12.9680 | 11.2486 | 11.1283 | 10.7135 | 10.6418 |
| | 7 | 14.9187 | 14.9374 | 14.0473 | **14.0940** | 12.0035 | 11.7116 | 11.1797 | 10.9821 |
| | 8 | 14.7618 | **14.8663** | 13.9849 | 14.0975 | 11.8277 | **11.6048** | **11.1013** | **10.9553** |
| 0.7 | 9 | **14.7492** | 14.9274 | **14.0446** | 14.2143 | **11.7962** | 11.6302 | 11.1450 | 11.0420 |
| | 10 | 14.8376 | 15.0812 | 14.1898 | 14.4105 | 11.8657 | 11.7482 | 11.2742 | 11.2079 |
| | 11 | 14.9994 | 15.3022 | 14.3971 | 14.6644 | 12.0087 | 11.9336 | 11.4655 | 11.4317 |
| | 7 | 16.7374 | 17.0655 | 15.8241 | **16.1015** | 12.7279 | 12.4732 | 11.8975 | 11.7306 |
| | 8 | **16.6012** | **17.0179** | **15.7819** | 16.1277 | 12.5695 | **12.3862** | **11.8361** | **11.7230** |
| 0.8 | 9 | 16.6093 | 17.1026 | 15.8618 | 16.2672 | **12.5554** | 12.4316 | 11.8968 | 11.8287 |
| | 10 | 16.7184 | 17.2799 | 16.0272 | 16.4861 | 12.6424 | 12.5695 | 12.0429 | 12.0138 |
| | 11 | 16.9010 | 17.5246 | 16.2548 | 16.7626 | 12.8027 | 12.7747 | 12.2512 | 12.2566 |

**Table1**: Expected total cost E{C (N)} by varying different parameters