ISSN (e): 2250 – 3005 || Volume, 07 || Issue, 04|| April – 2017 ||
International Journal of Computational Engineering Research (IJCER)

Modeling the Frictional Effect on the Rip Current on a Linear Depth Profile

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ABSTRACT
We develop an analytical theory for the interaction between waves and currents induced by breaking waves on a depth profiles. Here we examine the effect of friction on the rip currents and so are not particularly concerned with frictionally determined rip currents. The near shore is characterized by the presence of breaking waves, and so we develop equations to be used outside the surf zone, based on small-amplitude wave theory, and another set of equations to be used inside the surf zone, based on an empirical representation of breaking waves. Suitable matching conditions are applied at the boundary between the offshore shoaling zone and the near shore surf zone. Both sets of equation are obtained by averaging the basic equations over the wave phase. Thus the qualitative solution constructed is a free vortex defined in both shoaling and surf zone where in the surf zone the free vortex is perturbed by a long shore component.
***Keywords*:** Wave-current interactions, set down, set up, surf zone, shoaling zone, breaker, matching conditions and rip current

I. INTRODUCTION

Wave transformation in the surf zone is the dominant factor in the hydrodynamics of the near shore circulation, and consequent sediment transport. A global description in terms of the spatial variation of such quantities such as wave action, wave action flux and wave radiation stress are the driving entities we have used to describe the generation by waves of mean currents in the near shore zone. Studies on the interaction between waves and currents span nearly half a century. Mainly driven by a combination of engineering, shipping and coastal interests, there has been much research on shoaling nonlinear waves, on how currents affect waves and how waves can drive currents. This last aspect is the main concern in this paper.

The basis for this subject was laid down by Longuet-Higgins & Stewart(1960, 1961), who analyzed the nonlinear interaction between short waves and long waves (or currents), and showed that variations in the energy of the short waves correspond to work done by the long waves against the *radiation stress* of the short waves. In the shoaling zone this radiation stress leads to what are now known as wave setup and wave set down, surf beats, the generation of smaller waves by longer waves, and the steepening of waves on adverse currents, see Longuet-Higgins & Stewart (1962, 1964). The divergence of the radiation stress was shown to generate an alongshore current by obliquely incident waves on a beach (Longuet-Higgins 1970).

As wave groups propagate towards the shore, they enter shallower water and eventually break on beaches. The important process here is the wave breaking and dissipation of energy. The focusing of energy and the wave height variation across the group forces low frequency long waves that propagates with the group velocity (Longuet-Higgins & Stewart 1960). These long waves may be amplified by continued forcing during the shoaling of the short wave group into shallower water (Longuet-Higgins & Stewart 1962; Battjes 1988; List 1992; Herbers *et al*. 1994). In sufficiently shallow water, the short waves within the group may break at different depths leading to further long wave forcing by the varying breaker-line position (Symonds *et al*. 1982; Schaffer 1993). This means that the shoreward propagating waves may reflect at the shoreline and subsequently propagate offshore (Munk 1949).

Wave breaking leads to a transfer of the incoming wave energy to a range of different scales of motions, and particularly to lower frequencies (see, for instance, Wright, Guza& Short 1982). Thus waves called surf beat (Munk 1949; Tucker 1950), may propagate in the cross-shore direction (called leaky waves). Waves may be trapped refractively as edge waves (Huntley, Guza & Thorton 1981). Wave breaking may occur for two reasons; firstly, due to natural variation in the wave direction and amplitude. These changes occur in space and time. Secondly wave may break due to topographic influences. When this is the case, as in the near shore zone, the location and form of the wave breaking is influenced by the bottom depth profile, so the wave forcing and consequent motion may be different for different depth profiles, viz; linear, quadratic or other bottom depth dependence on the onshore coordinate; this theme will be developed in this thesis. Wave breaking can generate both irrotational low frequency waves and rotational low frequency circulation (Kennedy, A.B. 2005).

Topographically controlled breaking produces circulation about a stationary location (Yu &Slinn 2003, Kennedy, A.B. *et al*, 2003, 2006). Rip currents due to topographic influence are more dangerous with longer period and swell waves (Lascody 1998). Rip currents are shore-parallel feeder currents, joined at a rip neck and then becoming narrower and offshore directed (Shepard, Emery & La Fond 1941). They can be generated by alongshore variability in the breaking waves, or by alongshore variations in the bottom topography. In this paper we shall describe some analytical solutions for rip currents forced by alongshore periodicity in the shoaling waves, for two bottom depth profiles viz; linear and quadratic. We shall use the concept of radiation stress forcing with periodic alongshore variability to describe a rip current model. Rip currents interaction with bottom sediment transport implies that their hydrodynamics help to shape the topography that drives them in ways not completely understood. People are caught and drowned in rip currents every year so predicting rip currents and identifying their locations is important (Kennedy 2003).

Essentially we derive a model for the interaction between waves and currents. The aim is to provide analytical solution for waves in the near shore zone on time scales longer than an individual wave. This is possible on long-time scales using the wave-averaging procedure often employed in the literature (see the textbook by Mei, 1983).

The structure of the mathematical model is based on the Euler equations for an inviscid incompressible fluid. We then employ an averaging over the phase of the waves, exploiting the difference in time scales between the waves and the mean flow, which is our main interest. The near shore zone is divided into regions, a shoaling zone where the wave field can be described by linear sinusoidal waves, and the surf zone, where the breaking waves are modelled empirically. The breakerline is fixed at $x = x_b$ but in general could vary.

In the shoaling zone wave field, we use an equation set consisting of a wave action equation, combined with the local dispersion relation and the wave kinematic equation for conservation of waves. The mean flow field is then obtained from a conservation of mass equation for the mean flow, and a momentum equation for the mean flow driven by the wave radiation stress tensor. In the surf zone, we use a standard empirical formula for the breaking wave field, together with the same mean flow equations.

II. FORMULATION

The mathematical formulation and principles used here are those found in [29], [30], [31] and [35], where we ignored frictional effects due to turbulence in the surf zone for details see [29] and [30]. We constructed the longshore current driven by the radiation stress in the surf zone when this wave forcing has no alongshore modulation, that is, there is no *y*-dependence in the radiation stress. In that case the frictional effects are necessary for the longshore current to exist, and the weaker the friction the stronger is the current. But in the inviscid rip-current model constructed we demonstrated that there is a longshore component of the rip current, namely *Z(x)* which does not need friction in order to exist. Hence in this paper we are not concerned with any frictionally determined currents, but only with how the frictional terms modify the already constructed inviscid solution in [29] and [30]. We note that the friction terms are not invoked in the shoaling zone, and so the solution there remains unchanged.

3 Effect of friction on rip current

We have so far ignored frictional effects due to turbulence in the surf zone in [29] where our results are purely for an inviscid regime. Also in [30] section (3.2) we constructed the longshore current driven by the radiation stress in the surf zone when this forcing has no alongshore modulation, that is there is no *y*-dependence in the radiation stress. In that case the frictional effects are necessary for the longshore current to exist, and the weaker the friction the stronger is the current. But in the inviscid rip-current model constructed in [29], we have demonstrated that there is a longshore component of the rip current system, namely **Z**(**x**) which does not need friction in order to exist. Hence here in the viscous regime we are not concerned with any frictionally determined currents, but only with how the frictional terms might modify the already constructed inviscid solution. Before proceeding note that the friction terms are not invoked in the shoaling zone, and so the solution there remains unchanged.The full momentum equations with the frictional terms included are in [28]

$$H[U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y}] = -g\,H\frac{\partial \zeta}{\partial x} - [\tau_x] + \tau_x' \qquad \text{(25a, 25b)}$$

$$H[U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y}] = -g\,H\frac{\partial \zeta}{\partial y} - [\tau_y] + \tau_y'$$

Here the radiation stress terms are as before, namely

$$\tau_x = \frac{\partial S_{11}}{\partial x} + \frac{\partial S_{12}}{\partial y} \quad \text{and} \quad \tau_y = \frac{\partial S_{21}}{\partial x} + \frac{\partial S_{22}}{\partial y}. \qquad (26)$$

The terms τ_y' and τ_x' are lateral mixing terms which describe the aforementioned frictional effects, and are expressed as follows in [14] and [28] ,

$$\tau_y' = \nu_e v_{xx}, \tau_x' = \nu_e u_{xx} \text{ where } \nu e = \nu_0 g^{1/2} h^{3/2}. \qquad (27)$$

Next we recall that the steady-state mean mass equation can be solved using a transport stream function $\psi(x,y)$ so that

$$U = -\frac{1}{H}\frac{\partial\psi}{\partial y} \quad \text{and} \quad V = \frac{1}{H}\frac{\partial\psi}{\partial x}. \tag{28}$$

Next, again eliminating the pressure, we obtain the mean vorticity equation in the frictional regime

$$\psi_x(\frac{\Omega}{H})_y - \psi_y(\frac{\Omega}{H})_x = [\frac{\tau_x}{H}]_y - [\frac{\tau_y}{H}]_x + \mu_0[[h^{\frac{3}{2}}(\frac{\psi_x}{h})_{xx}]_x + [h^{\frac{3}{2}}(\frac{\psi_y}{h})_{xx}]_y], \tag{29}$$

where $\tau_x = \dfrac{3}{2}E_x\,, \quad \tau_y = \dfrac{1}{2}E_y.$

For convenience we have set $\mu_0 = g^{1/2}\nu_0$. The radiation stress terms are evaluated as before, and so finally equation (29) becomes

$$\psi_x(\frac{\Omega}{H})_y - \psi_y(\frac{\Omega}{H})_x = \frac{(h^{1/2}E_y)_x}{h^{3/2}} + \mu_0[[h^{\frac{3}{2}}(\frac{\psi_x}{h})_{xx}]_x + [h^{\frac{3}{2}}(\frac{\psi_y}{h})_{xx}]_y]\,. \tag{30}$$

As before Ω is defined by

$$\Omega = V_x - U_y = (\frac{\psi_x}{H})_x + (\frac{\psi_y}{H})_y. \tag{31}$$

As before the wave forcing is given by the expression, that is,

$$E = E_0\cos Ky + F_0\sin Ky + G_0(x), \tag{32}$$

so that the wave forcing term in (30) again simplifies to

$$(K\cos Ky)\frac{(h^{1/2}F_0)_x}{h^{3/2}} - (K\sin Ky)\frac{(h^{1/2}E_0)_x}{h^{3/2}}. \tag{33}$$

Thus again we observe that the unmodulated term $G_0(x)$ plays no role here at all. In order to match at $x = x_b$ with the expression $Y = \sin Ky$, for the stream function in the shoaling zone, we should try for a solution of (29) of the form

$$\psi = E(x)\cos Ky + F(x)\sin Ky + G(x). \tag{34}$$

Note that, comparing this with the analogous expression (**??**) in the friction-free case we see that here the term $E(x)$ is a purely due to friction. Next, equation (34) yields

$$\Omega = \tilde{E}\cos Ky + \tilde{F}\sin Ky + \tilde{G}, \tag{35}$$

where $\tilde{F}$, $\tilde{E}$ and $\tilde{G}$ are the differential operators

$$\tilde{E} = (\frac{E_x}{h})_x - \frac{K^2E}{h} \tag{36}$$

$$\tilde{F} = (\frac{F_x}{h})_x - \frac{K^2F}{h} \tag{37}$$

$$\tilde{G} = Z_x, \quad Z = \frac{G_x}{h} \tag{38}$$

The left-hand side of equation (30) contains terms in cos $2Ky$,sin $2Ky$,cos Ky, sin Ky,1 , while the right-hand side contains only terms in cosKy, sin Ky,1. Equating the appropriate coefficients on each side we get that

$$F_x\frac{\tilde{F}}{h} - E_x\frac{\tilde{E}}{h} - F(\frac{\tilde{F}}{h})_x + E(\frac{\tilde{E}}{h})_x = 0, \tag{39a}$$

$$E_x\frac{\tilde{F}}{h} - F(\frac{\tilde{E}}{h})_x - E(\frac{\tilde{F}}{h})_x + F_x\frac{\tilde{E}}{h} = 0, \tag{39b}$$

$$G_x\frac{\tilde{F}}{h} - (\frac{\tilde{G}}{h})_xF = \frac{(h^{1/2}F_0)_x}{h^{3/2}} + \frac{\mu_0}{K}[[h^{\frac{3}{2}}(\frac{E_x}{h})_{xx}]_x - K^2[h^{\frac{3}{2}}(\frac{E}{h})_{xx}]] \tag{39c}$$

$$E(\frac{\tilde{G}}{h})_x - G_x\frac{\tilde{E}}{h} = -\frac{(h^{1/2}E_0)_x}{h^{3/2}} + \frac{\mu_0}{K}[[h^{\frac{3}{2}}(\frac{F_x}{h})_{xx}]_x - K^2[h^{\frac{3}{2}}(\frac{F}{h})_{xx}]], \quad (39d)$$

$$E(\frac{\tilde{F}}{h})_x - F_x\frac{\tilde{E}}{h} + E_x\frac{F}{h} - F(\frac{\tilde{E}}{h})_x = \frac{2\mu_0}{K}[h^{\frac{3}{2}}(\frac{G_x}{h})_{xx}]_x \quad (39e)$$

The boundary conditions are analogous to those imposed in section 3.2 see [29]. That is, at $x = 0$ where $h = 0$ both mass transport fields U,V should vanish, that is, $U = -\frac{1}{H}\frac{\partial\varphi}{\partial y}$ and $V = \frac{1}{H}\frac{\partial\varphi}{\partial x}$, ψ = constant and $\psi_x/h = 0$, which implies that

$$\frac{(E,F)}{h} = \frac{(E_x,F_x)}{h} = 0\,,\ G = \text{constant},\ \frac{G_x}{h} = 0\,,\ \text{at } x = 0. \quad (40)$$

As before there are also the matching conditions E, F, G and G separately at the breakerline, that is, we now have that

$$\frac{F_x(x_b)}{F(x_b)} = \frac{X_x(x_b)}{X(x_b)}\,, \quad E_x(x_b) = G_x(x_b) = 0 \quad (41)$$

Equations (39a, 39b, 39c, 39d, 39e) form five equations for only three unknowns. Hence in general there is unlikely to be an exact solution, and instead we seek an approximate solution. First, we note that when $\mu_0 = 0$ (the frictionless case) there is an exact solution, as then we can satisfy (39c) with $E = 0$, and then provided we also set $E_0 = 0$, equations (39d, 39e) are also satisfied, leaving only (39a) for F and (39c) for G. This of course is just the procedure we have followed above. Hence we shall regard the frictional terms as a perturbation on this and so treat ν_0 as a small parameter, noting that ν_0 is dimensionless. Thus we infer that $E = 0(\nu_0)$, and for consistency we must then also choose $E_0 = O(\nu_0)$; indeed we will set $E_0 = 0$ for simplicity.

It now follows that to leading order in ν_0, F is given again by the frictionless solution, so that it satisfies (**55**) see [29] again. Indeed, since $E = O(\nu_0)$, the error incurred for F is $O(\nu_0^2)$ from (39a). Next we see that the frictional term in (39c) is $O(\nu_0^2)$, so that also G is again given by the frictionless solution (64), see [29] with an error of $O(\nu_0^2)$. It remains to determine the leading order term for E. For this purpose we can use (39e), since the alternative equation (39d) generates only a term for E which is $O(\nu_0^2)$. Then, using the above estimates for F, G we see that (39e) becomes

$$ChE - \tilde{E} = \frac{2\mu_0}{KF}h^{5/2}Z_{xx}, \quad (42)$$

that is

$$ChE - (\frac{E_x}{h})_x + \frac{K^2E}{h} = \frac{2\mu_0}{KF}h^{5/2}Z_{xx}, \quad (43)$$

on using (36), where a constant of integration has been set to zero. Here the right-hand side can be regarded as known, and is given by the expression (**53**) see [29] for $Z = G_x/h$.

Thus we see that the essential outcome of the frictional terms is to introduce into the surf zone an extra component $E(x)\cos Ky$ which, in the longshore direction, is out-of-phase with the component proportional to sinKy in the shoaling zone.

4 Application to linear depth profile

Now we consider the linear depth $h = \alpha x$, in which case F is given by (**61**) see [29] and Z is given by (**64**) see [29]. Although the complete solution can be found using the method of variation of parameters, it will be simpler to seek a more manageable solution in which the right-side is approximate in the limit $x \to 0$. In that case $F, Z \propto x^2$ and so the right-hand side is proportional to $x^{1/2}$, to the lowest order, that is

$$\frac{h^{5/2}Z_{xx}}{F} \approx \frac{5g2^{1/2}\alpha^{9/2}x^{1/2}}{6A_0^2x_b\sin(2^{1/2})} \quad (44)$$

Then the particular solution of (43) is proportional $x^{7/2}$,

$$E_p \approx -\frac{20\nu_o g^{3/2}2^{1/2}\alpha^{11/2}x^{7/2}}{63KA_0^2x_b\sin(2^{1/2})}. \quad (45)$$

Equation (43) contains an analogous homogeneous equation to (**57**) see [29]

$$E_{xx} - \frac{1}{x}E_x - K^2E = Ch^2E, \quad (46)$$

thus the homogeneous solution is found by putting $u = x^2$ and seeking solutions of

$$4E_{uu} - \lambda E - \frac{K^2E}{u} = 0,$$

where $\lambda = C\alpha^2$ as in the inviscid solution. This can be solved in terms of Whittaker functions. However, again for simplicity we seek just an approximation as $x \to 0$, in which case the solution which satisfies the boundary condition (40) is

$E_h \approx u = x^2$.

Finally we need to combine E_h, E_p to satisfy the boundary condition at (41) at $x = x_b$ so that

$$E \approx \frac{20\nu_0 g^{3/2} 2^{1/2} \alpha^{11/2} x_b^{5/2}}{63 K A_0^2 \sin(2^{1/2})} \{-(\frac{x}{x_b})^{7/2} + \frac{7}{4}(\frac{x}{x_b})^2\}. \quad (47)$$

Thus we get from (34, 47) and (61, 65) see [29] in $x < x_b$ that the normalized stream function ψ_n is given by

$$\psi_n \;=\; \frac{F(x)}{F(x_b)} \sin Ky + R\frac{G(x)}{G(0)} + S\frac{E(x)}{E(x_b)} \cos Ky \text{ , for } \quad 0 < x < x_b \quad (48)$$

where $R = G(0)/F(x_b)$ as before (**68**) see [29] and $S = E(x_b)/F(x_b)$ is a new parameter. Using the expressions (**61, 68**) as in [29] and (47) we can write

$$S = \frac{5\nu_0 g^{3/2} \alpha^{11/2} x_b^{1/2} \sqrt{2}}{14 K A_0^3 \sin\sqrt{2}}. \quad (49)$$

But we recall from (**68**) that

$$R = -\frac{0.16 g \alpha^3 x_b}{A_0^2},$$

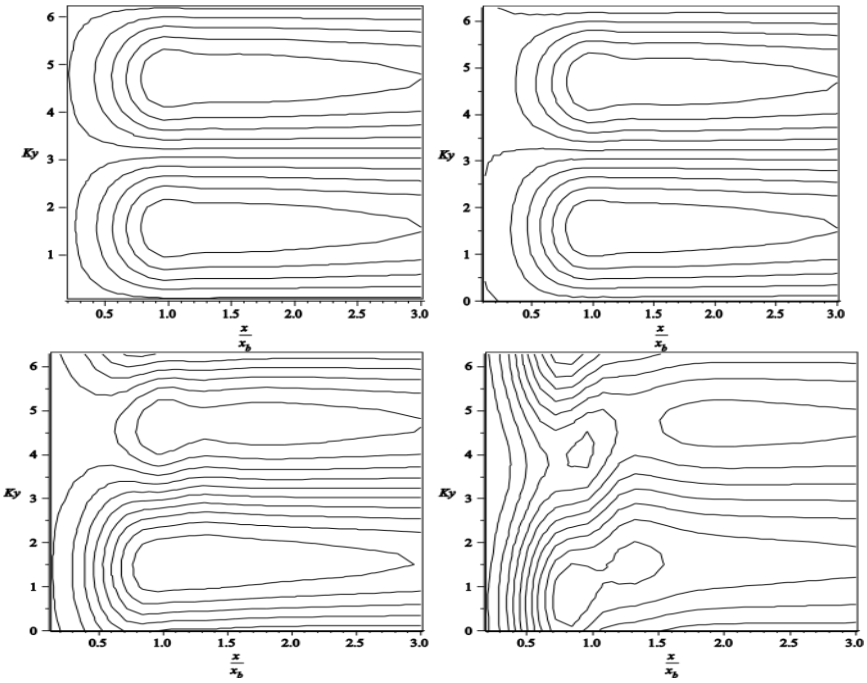
and measures the free parameter A_0. Hence it is convenient to write the expression (49) in the form

$$S = \tilde{S}|R|^{3/2,\ 122} \text{ where } \tilde{S} = \frac{8.0(\mathrm{sign}A_0)\nu_0\alpha}{K x_b}, \quad (50)$$

and is independent of the magnitude of A_0. Thus $\tilde{S}$ is the new parameter which measures the effects of friction. Note that it increases with the slope α, but decreases with the parameter Kx_b measuring the long shore modulation wavenumber. Numerical values for ν_0 are not readily available, but we note that from Mei (1983) section 10.6.2 we can estimate that $\nu_0\alpha \approx 0.01$. Using $Kx_b = 0.2$ as in the inviscid solution, we infer that a suitable value is $\tilde{S} = 0.4$ (assuming without loss of generality that $A_0 > 0$).

The normalized stream functions (48) are plotted for the same values of $R = -0.02$, $R = -0.1$ $R = -0.5$ and $R = -2$ used in the inviscid solution respectively shown in figures below. In contrast to the figures in the invsicid case, we see that the rip-currents in the frictional regime [1, 2, **3**] are modified. There is a noticeable effect of the friction terms, in that there is a phase shift of the surf zone component relative to the shoaling zone component. In particular, note the case as $R = -2.0$ as shown in [**4**], contrasted with the inviscid case shown in figure [**4**]. In general, as shown by the expression (50), the frictional effects increase with $|R|$

As wave forcing is large there is an improved output of the long shore component of the rip-currents. Overall this means that with large R the near-shore current system and its circulation becomes noticeable in the region.



REFERENCES

[1]. Baldock, T.E, (2006), Long wave generation by the shoaling and breaking of transient wave groups on a beach, Proceedings of the Royal society, vol.462, pp 1853-1876.

[2]. Baldock, T.E., O'Hare, T.J. & Huntley, D.A, (2004), Long wave forcing on a barred beach, Journal of Fluid Mechanics, vol 503, pp. 321-343.

[3]. Battjies, J. A., (1988), Surf zone dynamics, Annual Rev. Fluid Mechanics, vol. 20, pp.257-293.

[4]. Billigham, J & King, A.C., (2000), Wave Motion- Cambridge Texts in Applied Mathematics, Cambridge University Press.

[5]. Brocchini, M., Kennedy, A., Soldini, L & Mancinelli, A., (2004) Topographically controlled, breaking-wave-induced macrovortices. Part 1. Widely separated breakwaters, J. Fluid Mechanics, vol. 507, pp. 289307.

[6]. Buhler, O., (2000), On the vorticity transport due to dissipating or breaking waves in shallow-water flow, Journal of Fluid Mechanics, vol 407, pp 235-263.

[7]. Buhler, O. & Jacobson, T. E., (2001), Wave-driven currents and vortex dynamics on barred beaches, Journal of Fluid Mechanics, vol 449, pp 313-339.

[8]. Damgaard, J., Dodd, N., Hall, L. &Chesher, T., (2002), Morphodynamic modelling of rip channel growth, Coastal Engineering, vol.45, pp 199-221.

[9]. Davies, A. M. & Lawrence, J. (1995), Modelling the effect of wavecurrent interaction on the three-dimensional wind driven circulation of the Eastern Irish Sea, Journal of Physical Oceanography, vol. 25, pp. 29-45.

[10]. Falques, A., Calvete, D.&Monototo, A. (1998a), Bed-flow instabilities of coastal currents. In Physics of Estuaries and Coastal Seas, pp. 417424. A. A. Balkema.

[11]. Falques, A., Montoto, A. & Vila, D. (1999), A note on hydrodynamic instabilities and horizontal circulations in the surf zone, Journal of Geophysical Research, vol. 104(C9), pp 20605-20615

[12]. Falques, A., Coco, G & Huntley, D.A (2000), A mechanism for the generation of wave-driven rhythmic patterns in the surf zone, Journal of Geophysical Research, vol. 105, No. C10, pp 24701-20615.

[13]. Guza, R. T. & Davis, R. E. (1974), Excitation of edge waves by waves incident on a beach, Journal of Geophysical Research, vol. 79, C9, pp. 1285-1291.

[14]. Horikawa, K (1978), An Introduction to Ocean Engineering, University of Tokyo Press.

[15]. Kennedy, A.B. (2003), A circulation description of a rip current neck, Journal of Fluid Mechanics, vol. 497, pp. 225-234

[16]. Kennedy, A.B. (2005), Fluctuating circulation forced by unsteady multidirectional breaking waves, Journal of Fluid Mechanics, vol. 538, pp. 189-198

[17]. Kennedy, A.B., Brocchini, M., Soldini, L., & Gutierrez. E., (2006) , Topographically controlled, breaking-wave-induced macrovortices. Part 2. Changing geometries, J. Fluid Mechanics, vol. 559, pp. 57-80.

[18]. Lane, E.M &Restrepo, J.M (2007), Shoreface-connected ridges under the action of waves and currents, Journal of Fluid Mechanics, vol. 582 , pp. 23-52

[19]. Lascody, L. L (1998), East central Florida rip current program, National Weather Digest, vol. 22, pp. 25-30.

[20]. List, J.H. (1992), A model for the generation of two dimensional surf beat, Journal of Geophysical Research, vol. 97, pp. 5623-5635.

[21]. Longuet-Higgins, M.S. (1970), Longshore currents generated by obliquely incident sea waves, 1, J. Geophysical Research, vol. 75, pp 6778-6789.

[22]. Longuet-Higgins, M.S. (1970), Longshore currents generated by obliquely incident sea waves, 2, J. Geophysical Research, vol. 75, pp 6790-6801.

[23]. Longuet-Higgins, M.S & Stewart, R.W. (1960), The changes in the form of short gravity waves on long waves and tidal currents, Journal of Fluid Mechanics, vol. 8, pp 565-583

[24]. Longuet-Higgins, M.S & Stewart, R.W. (1961), The changes in amplitude of short gravity waves on on steady non-uniform currents, Journal of Fluid Mechanics, vol. 10, pp 529-5549

[25]. Longuet-Higgins, M.S & Stewart, R.W. (1962), Radiation stress and mass transport in gravity waves, with application ro surf beats, Journal of Fluid Mechanics, vol. 13, pp 481-504

[26]. Longuet-Higgins, M.S & Stewart, R.W. (1964), Radiation stress in water waves: a physical discussion with applications, Deep-sea Research, vol. 11, pp. 529-562.

[27]. Longuet-Higgins, M.S. (2005), On wave set-up in shoaling water with a rough sea bed, Journal of Fluid Mechanics, vol. 527, pp. 217-234

[28]. Mei, C.C. (1983), Applied Dynamics of Ocean Surface Waves, John Wiley and Sons, Inc.

[29]. Osaisai, E. F (2013), An analytical model of the rip current flow International Journal of Computational Engineering Research, vol. 3, issue 9, pages 1-12

[30]. Osaisai, E. F (2013), Modeling the rip current flow on a quadratic depth profile International Journal of Computational Engineering Research, vol. 3, issue 9, pages 47-62

[31]. Osaisai, E. F (2008), The interaction between waves and currents in the nearshore zone, PhD thesis, Loughborough University.

[32]. Peregrine, D.H. (1998), Surf zone currents, Theoretical and Computational Fluid Dynamics, vol. 10, pp 295-309.

[33]. Rahman, M. (2002), Mathematical methods with applications, WIT Press, Southampton.

[34]. Restrepo, J. M. (2001), Wave-current interactions in shallow waters and shore-connected ridges, Continental Shelf Research, vol. 21, pp. 331-1360.

[35]. Roger Grimshaw and Evans Osaisai (2012), Modeling the effect of bottom sediment transport on beach profiles and wave set-up, ocean Modelling, 59-60 (2012) 24-30

[36]. Shepard, F. P., Emery, K. O. & La Fond, E. C (1941), Rip currents: a process of geological importance, Translation of American Geophysical Union, vol. 31, pp. 555-565.

[37]. Tucker, M.J., (1950), Surf beats: sea waves of 1 to 5 min. period, Proc. R. Soc. London, vol. 202, pp. 565-573.

[38]. Yu, J. & Mei, C.C. (2000), Formation of sand bars under surface waves, Journal of Fluid Mechanics, vol. 416, pp. 315 - 348.

[39]. Yu, J. & Mei, C.C. (2000), Do longshore bars shelter the shore? Journal of Fluid Mechanics, vol. 404, pp. 251 - 268.

[40]. Yu, J. & Slinn, D.N. (2003), Effects of wave-current interaction on rip currents, Journal of Geophysical Research, vol 108 (C3), 3088.

[41]. Yu, J. (2006), On the instability leading to rip currents due to wave-current interaction, Journal of Fluid Mechanics, vol. 549, pp. 403-428.