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Two-Warehouse Inventory Model for Non-Instantaneous Deterio rating Items with Exponential Demand Rate

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ABSTRACT

Atwo-warehouseinventoryproblemfornon-instantaneousdeterioratingitemswithexponentialdemandrateunderdifferent dispatching policies is planned. While formulating theinventory model for deteriorating items, usually it is assumedthat the items start deteriorating as soon as they enter into thewarehouse.However,therearenumerousproductslikedryfruits,foodgrainsetc.thathaveashelf-lifeandstartdeterioratingafteratimelagthatistermedasnon-instantaneous deterioration. Moreover, the price discount, lowcost storage, huge demand etc. and under such a situation onemay decide to procure large quantity of the items which wouldarise the problem of storing. As the capacity of own warehouseis limited, therefore one has to hire another rented warehouse tostore the excess quantity.A numerical example is given toillustrate the model. Sensitivity analysis of the optimal solutioniscarriedout.

Keywords:Inventory,non-instantaneousdeterioration,two-warehouse,exponential demandrate.

INTRODUCTION

In recent years, inventory problems for deteriorating items havebeen widely studied. Deterioration is defined as decay, changeor spoilage such that the items are not in a condition of beingusedforitsoriginalpurpose.Electronicgoods,radioactivesubstances,grains,blood,alcohol,gasoline,turpentinea reexamples of deterioration items. For any business organization,it is major concern to control and maintain the inventories ofdeterioratingitems.

The problems on classical inventory models which are found inthe existing literature generally deal with single storage facility.ButwhentheoptimallotsizedictatedbytheEOQmodelbecomes more than the total amount that can be stored in theexisting storage facility (Warehouse owned by the managementOW) the question of acquiring some extra storage facility tostoretheseexcessquantityarises.Thisadditionalstoragefacility may be a rented warehouse (RW) with sophisticatedpreservationfacilityandabundantspace.

A model with exponentially decaying inventory was initiallyproposedby GhareandSchrader(1963). CovertandPhilip(1973)formulatedmodelwithvariabledeterioratingrateoftwo-parameter Weibull distribution. Philip (1974) generalizedthismodelbytakingthree-parameterWeibulldistribution.After that many researchers such as Goyal(1987), Raafat et al.(1991),Wee(1993),andothersdevelopedmodelson deterioratingitems.AdetailedreviewofdeterioratinginventoryliteraturesisgivenbyGoyalandGiri(2001).Thereisa vast inventory literature on deteriorating items under differentconditions,theoutlinewhichcanbefoundinarticles(2001),(2003),(2006),(2008),(2011)and theirreferences.

However,inmanyinventorysystems,thedeteriorationofgoods is a realistic phenomenon. It is well known that certainproducts such as refrigerated food, fruit and vegetable, freshseafood and many others have a high deterioration rate. Mandaland Phaujdar (1989) developed a production inventory modelfordeterioratingitemswithuniform rateofproductionandlinearlystock-dependentdemand.Giri,Pal,Goswami,andChaudhuri (1996) studied the model of Datta and Pal (1990) fordeterioratingitems.Meanwhile,GiriandChaudhuri(1998)extended Goh's model (1994) to consider the inventory modelwithaconstantdeteriorationrate.PadmanabhanandVrat(1995) considered an EOQ model for perishable items with aconstantsellingpriceand linearlystockdependentdemand.

Inventory model with double storage facility OW and RW wasfirstdevelopedbyHartley(1976).SahuandBishi(2017)extended the inventory deteriorating Items under permissibledelay in payments. After his pioneering contribution, severalother researchers have attempted to extend his work to variousother realistic situations. In this connection, mention may bemadeofthestudiesundertakenbySarma(1983,1990),MurdeshwarandSathe(1985), Pakkala andAchary (1992),Dave (1988), Bhunia and Maity (1997) Yang (2004), Singh andSahu(2012),Lee (2006), Yang (2006), Dey et al.(2008) tonameonlyafew.

In this paper we consider a two-warehouse inventory problemfornon-instantaneousdeterioratingitemswithexponentialdemand rate under different dispatching policies is planned.While formulating the inventory model for deteriorating items,usuallyitisassumedthattheitemsstartdeterioratingassoonas they enter into the warehouse. However, there are numerousproducts like dry fruits, food grains etc. that have a shelf-lifeand start deteriorating after a time lag that is termed as non-instantaneous deterioration. Moreover, the price discount, lowcost storage, huge demand etc. and under such a situation onemay decide to procure large quantity of the items which wouldarise the problem of storing. As the capacity of own warehouseis limited, therefore one has to hire another rented warehouse tostorethe excessquantity.

1. ASSUMPTIONANDNOTATIONS

Thefollowingassumptionsandnotationshavebeenusedintheentirepaper.

Assumption

(i) Replenishmentrateisinstantaneous.
(ii) Lead-timeisnegligible.
(iii) Theplanninghorizonoftheinventorysystemisinfinite.
(iv) $t_d$is the length of time during which the product has nodeterioration.
(v) TheOW hasafixedcapacityofW units;theRWhas
unlimitedcapacity.
(vi) TheunitinventoryholdingcostperunittimeinRWishigher thanthatinOW andthe deterioration
rateinRW islessthanthatinOW.
(vii) Unsatisfied demand/shortages are allowed. Unsatisfieddemandispartiallybackloggedand thefractionofshortages

I_r (t)TCF$_i$TCL$_i$B(t)
L(t)
where0 t T
inventorylevelintheRWatanytimet
where0 t T
totalrelevantcostperunittimeforcase
i1,2
totalrelevantcostperunittimeforcase
i1,2
backloggedlevel atanytimetwhere
t_w t T
numberoflostsalesatanytimetwhere
t_w t T
backloggedis adifferentiableand decreasingfunctionoftimet
,denotedbyg(t),wheretisthewaitingtimeuptothenextreplenishment. We have defined the partial backlogging rateg(t) e^{t},where isapositiveconstant.

Notations

Inaddition,thefollowing notationsareusedthroughoutthispaper.

A orderingcostper order

c purchasingcostperunit

W capacityoftheownedwarehouse

$\lambda e^{\lambda t}$ demandrateperunittime

Q_t orderquantityper cycle

S_f maximuminventorylevelpercycle

H holdingcostperunitperunittimeinOW

F holdingcostperunitperunittimeinRW,

where $F>H$

s thebackloggingcostperunitperunittime, ifshortageisbacklogged

c_1 unitopportunitycostdueto lostsale,iftheshortageislost

α deteriorationrateinO W, where $0\leq\alpha<1$

β deteriorationrateinRW,where $0\leq\beta<1;\beta<\alpha$.

t_d timeperiodduringwhichnodeteriorationoccurs

t_r timeat whichtheinventorylevelreaches zeroinRW

t_w timeatwhichtheinventorylevelreacheszero inOW

where $0\leq t\leq T$

I_r inventorylevelintheRWatanytime t where $0\leq t\leq T$

$(t)T$ totalrelevantcostperunittimeforcase $i=1,2$

CF_iT totalrelevantcostperunittimeforcase $i=1,2$

CL_i backloggedlevel atanytime t where $t_w\leq t\leq T$

$B(t)$ numberoflostsalesatanytime t where $t_w\leq t\leq T$

$L(t)$

MATHEMATICALMODELFORMULATION

In the present study a two warehouse inventory model has beendeveloped, where the OW has a fixedcapacity of W units andthe RWhas unlimited capacity. The units in RW are storedonly when the capacity of OW has been utilized completely.Demand is assumed to be constant. Shortages are allowed butarepartiallybacklogged.ThegoodsarestoredinOwnedWarehouse (OW) initially after satisfying the OW; remaininggoods are stored in Rented Warehouse (RW) but uses the goodsof RW prior to the goods of OW to satisfy the demand in orderto reduce the inventory carrying charge (holding cost). Whereasthose goods are sold that are stored first in order to maintain thefreshnessofproductwhichresultsingreatercustomersatisfaction. Which ultimately boost the sales and increase thevalueoftheorganizationinthelongterm.Thefollowingsectionsdiscussthemodelformulation forboththepolicies.

Case1:Whent_d □ t_w

whereF□ H

s thebackloggingcostperunitperunittime,

ifshortageisbacklogged

c_1 unitopportunitycostdueto lostsale,iftheshortageislost

During the time interval[0,t_d] , there is no deterioration. So,theinventoryinOW $I_0(t)$ isdepletedonlyduetodemandwhereasinRW,inventorylevelremainsthesame.Further,

duringthetimeinterval[t_d,t_w]theinventorylevelinOW

□ deteriorationrateinOW,where0□ □ □ 1

□ deteriorationrateinRW,where

$I_0(t)$isdroppingtozeroduetothecombinedeffectof

demandanddeteriorationandtheinventoryinRW$I_r(t)$gets

0   1; .

t_d timeperiodduringwhichnodeteriorationoccurs

t_r timeat whichtheinventorylevelreaches zeroinRW

depletedduetodeteriorationonly.Now,duringthetimeinterval[t_w,t_r] depletion of inventoryI_r (t) occurs in RW duetothecombinedeffectofdemandanddeteriorationanditreachestozeroattime t_r .Moreover,duringtheinterval[t_r,T]thedemandisbacklogged.So,B(t)representsthe

t_w timeatwhichtheinventorylevelreacheszero inOW

T thelengthofthereplenishmentcyclein year

levelofnegativeinventoryattimetduringtheinterval[t_r,T]

.Thebehaviorofthemodeloverthetimeinterval[0,T]hasbeenrepresentedgraphicallyinfigure1.

I_0(t)

inventorylevelintheOWatanytimet

I_r(t) (S f

 W)e (t_d t),t

 t t_w

(9)

d

I(t) e(   )t   t e t ,t

r

   w

[ ]

 t t_r

(10)

B(t) D e (T t) e (T t_r) ,t

 t T

δ

r

(11)

Thenumbers oflost sales attimetisL(t)given by

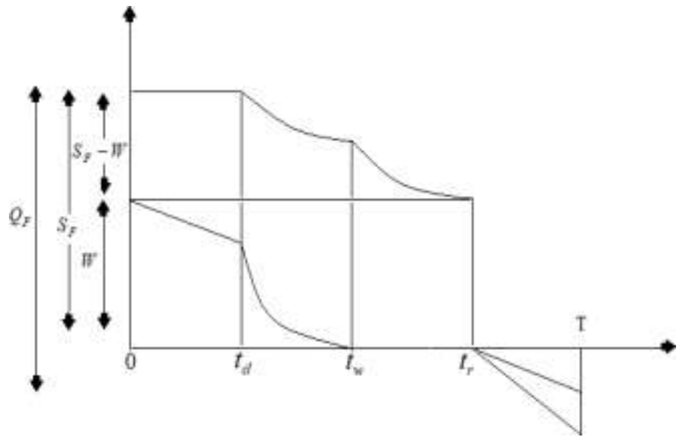


Figure1: Two-warehouseinventorysystem,

Therefore,thedifferentialequationsthatdescribetheinventoryleve l in the RW and OW at time t over the Period $(0,T)$ aregivenby:

$$\frac{dI_0(t)}{dt}=-\lambda e^{\lambda t}, \quad 0\le t\le t_d \tag{1}$$

$$\frac{dI_0(t)}{dt}+\alpha I_0(t)=-\lambda e^{\lambda t} \quad ,t_d \le t\le t_w \tag{2}$$

$$\frac{dI_r(t)}{dt}=0,0\le t\le t_d \tag{3}$$

$$\frac{dI_r(t)}{dt}+\beta I_r(t)=0 \quad , t_d \le t\le t_w \tag{4}$$

$$\frac{dI_r(t)}{dt}+\beta I_r(t) =-\lambda e^{\lambda t} \quad ,t_w \le t\le t_r \tag{5}$$

$$\frac{dB(t)}{dt} =De^{-\delta(T-t)} \quad ,t_r\le t\le T \tag{6}$$

ThesolutionoftheaboveWiththeboundarycondition
$I_0(0)=W,I_0(t_w)=0,I_r(t_d)=S_f-W,I_r(t_r)=0,$
$B(t_r)=0$

$$I_0(t)=(w+1)-e^{\lambda t} \quad , 0\le t\le t_d \tag{7}$$

$$I_0(t)=\frac{\lambda}{\lambda+\alpha}\left[e^{(\lambda+\alpha)t_w-\alpha t}-e^{\lambda t}\right], t_d \le t\le t_w \tag{8}$$

$$I_r(t)=(S_f -W)e^{\beta(t_d-t)}, t_d \le t\le t_w \tag{9}$$

$$I_r(t)=\frac{\lambda}{\lambda+\beta}\left[e^{(\lambda+\beta)t_r-\beta t}-e^{\lambda t}\right], t_w \le t\le t_r \tag{10}$$

$$B(t)=\frac{D}{\delta}\left\{e^{-\delta(T-t)}-e^{-\delta(T-t_r)}\right\}, t_r \le t\le T \tag{11}$$

Thenumbers oflost sales attime$t$is$L(t)$given by

$$L(t) =\int_{t_r}^{T} D \left\{1- e^{-\delta(T-t)}\right\}dt \qquad t_r<t<T$$

$$=D\left[(t-t_r)-\frac{1}{\delta}\left\{e^{-\delta(T-t)}-e^{-\delta(T-t_r)}\right\}\right], t_r \le t\le T$$

(12)

$I_r(t)=S_f$ with $I_r(0)=S_f$, $0\le t\le t_d$ (13)

Consideringthecontinuityof $I_0(t)$ at $t=t_d$,itfollowsfrom equation(7)and(8),weget

$$W+1-e^{\lambda t_d}=\frac{\lambda}{\lambda+\alpha}e\left[^{(\lambda+\alpha)t_w-\alpha t_d}-e^{\lambda t_d}\right]$$

$$t_w=\frac{\alpha}{\lambda+\alpha}t_d+\frac{1}{\lambda+\alpha}\ln\left|e^{\lambda t_d}+\frac{\lambda+\alpha}{\lambda}(1+W-e^{\lambda t_d})\right|$$

(14)

Consideringthecontinuityof $I_r(t)$ at $t=t_w$,itfollowsfromequation(9) and(10),we get

$$(S_f-W)e^{\beta(t_d-t_w)}=\frac{\lambda}{\lambda+\beta}\left[e^{(\lambda+\beta)t_r-\beta t_w}-e^{\lambda t_w}\right]$$

$$S_f=W+\frac{\lambda}{\lambda+\beta}\left[e^{(\lambda+\beta)t_r-\beta t_d}-e^{\beta(t_w-t_d)-\lambda t_w}\right]$$

(15)

Putting $t=T$ inequation(11),Themaximumamountof demandbackloggedpercycle is

$$B(T)=\frac{D}{\delta}\left(1-e^{-\delta(T-t_r)}\right) \quad (16)$$

Thereforetheorderquantityoverthereplenishmentcyclecanbedete rminedaseqn. (14) &(16),we get

$$Q_f = S_f + B(T)$$

$$Q_f = W + \frac{\lambda}{\lambda+\beta}\left[e^{(\lambda+\beta)t_r-\beta t_d} - e^{\beta(t_w-t_d)-\lambda t_w}\right]$$

$$+\frac{D}{\delta}\left(1-e^{-\delta(T-t_r)}\right) \qquad (17)$$

Hence, $(0,T)$thecostsduringthecycleareevaluatedas follows

(a) Orderingcostpercycle$=A$

(b) TheInventoryholdingcost percycleinRW

$$HC_{RW} = F\left[\int_0^{t_d} I_r(t)dt + \int_{t_d}^{t_w} I_r(t)dt + \int_{t_d}^{t_r} I_r(t)dt\right]$$

Byusingthevalue of$S_f - W$inequation(9) weget

$$I_r(t) = \frac{\lambda}{\lambda+\beta}\left[e^{(\lambda+\beta)t_r-\beta t_d} - e^{\beta(t_w-t_d)-\lambda t_w}\right]e^{\beta(t_d-t)} \qquad (18)$$

Then

$$HC_{RW} = FS_f t_d \left[\frac{\lambda}{\lambda+\beta}\left\{e^{(\lambda+\beta)t_r} - e^{(\beta-\lambda)t_w}\right\}\left(\frac{-1}{\beta}\right)e^{\beta(t_w-t_d)} + \frac{\lambda}{\lambda+\beta}\left\{\frac{-1}{\beta}e^{(\lambda+\beta)t_r-\beta(t_r-t_w)} - \frac{1}{\lambda}e^{\lambda(t_r-t_w)}\right\}\right]$$

(c)TheInventoryholdingcost percycleinOW

$$HC_{OW} = H\left[\int_0^{t_d} I_0(t)dt + \int_{t_d}^{t_w} I_0(t)dt\right]$$

$$HC_{OW} = H\left[(W+1)t_d - \frac{1}{\lambda}(e^{\lambda t_d} - 1)\right]$$

$$-\frac{\lambda}{\lambda+\alpha}\left\{\frac{1}{\alpha}e^{(\lambda+\alpha)t_w-\alpha(t_w-t_d)}+\frac{1}{\lambda}e^{\lambda(t_w-t_d)}\right\}\Bigg]$$

(d) Thebackloggedcostpercycleis $=s\int_{t_r}^{T}B(t)dt$

$$SC=\frac{sD}{\delta}\left[\frac{1}{\delta}-\left\{\frac{1}{\delta}+T-t_r\right\}e^{-\delta(T-t_r)}\right]$$

(e) Theopportunitycostduetolostsaleis

$$=c_1D\int_{t_r}^{T}\left\{1-e^{-\delta(T-t)}\right\}dt$$

$$=c_1D\left[T-t_r-\frac{1}{\delta}\left\{1-e^{-\delta(T-t_r)}\right\}\right]$$

(f)Thedeteriorationcostpercycle

$$=c_d\left[\beta\int_{t_d}^{t_r}I_r(t)dt+\alpha\int_{t_d}^{t_w}I_0(t)dt\right]$$

$$= c\beta \left[\frac{\lambda}{\lambda+\beta} \left\{ \left(e^{(\beta-\lambda)t_w} - e^{(\lambda+\beta)t_r} \right) \frac{e^{-\beta(t_w - t_d)}}{\beta} \left(\frac{1}{\beta} e^{(\lambda+\beta)t_r - \beta(t_r - t_{rw})} + \frac{1}{\lambda} e^{\lambda(t_r - t_{rw})} \right) \right\} \right]$$

$$+ \frac{\alpha\lambda}{\lambda+\alpha} \left\{ \frac{1}{\alpha} e^{(\lambda+\alpha)t_w - \alpha(t_w - t_d)} + \frac{1}{\lambda} e^{\lambda(t_w - t_d)} \right\} \Bigg]$$

Now,theTotalrelevantcostpertimeunittimeduringthe Cycle$(0,T)$ usingequationisgivenby

$$TCF1(t_r, T) = \frac{1}{T}[OC + HC + BC + OC + DC]$$

$$= \frac{1}{T}\left[A + FS \quad f t_d \left[\frac{\lambda}{\lambda+\beta} \left\{ e^{(\lambda+\beta)t_r} - e^{(\beta-\lambda)t_w} \right\} \left(\frac{-1}{\beta} \right) e^{\beta(t_w - t_d)} + \frac{\lambda}{\lambda+\beta} \left\{ \frac{-1}{\beta} e^{(\lambda+\beta)t_r - \beta(t_r - t_w)} - \frac{1}{\lambda} e^{\lambda(t_r - t_w)} \right\} \right] + H \left[(W \quad +1)t_d - \frac{1}{\lambda}(e^{\lambda t_d} - 1) \right. \right.$$

$$
-\frac{\lambda}{\lambda+\alpha}\left\{\frac{1}{\alpha}e^{(\lambda+\alpha)t_w-\alpha(t_w-t_d)}+\frac{1}{\lambda}e^{\lambda(t_w-t_d)}\right\}\Bigg]\Bigg]
+\frac{SD}{\delta}\left[\frac{1}{\delta}-\left\{\frac{1}{\delta}+T-t_r\right\}e^{-\delta(T-t_r)}\right]
+c_1D\left[T-t_r-\frac{1}{\delta}\left\{1-e^{-\delta(T-t_r)}\right\}\right]
+c\beta\left[\frac{\lambda}{\lambda+\beta}\left\{\left(e^{(\beta-\lambda)t_w}-e^{(\lambda+\beta)t_r}\right)\frac{e^{-\beta(t_w-t_d)}}{\beta}
-\left(\frac{1}{\beta}e^{(\lambda+\beta)t_r-\beta(t_r-t_w)}+\frac{1}{\lambda}e^{\lambda(t_r-t_w)}\right)\right\}
-\frac{\alpha\lambda}{\lambda+\alpha}\left\{\frac{1}{\alpha}e^{(\lambda+\alpha)t_w-\alpha(t_w-t_d)}+\frac{1}{\lambda}e^{\lambda(t_w-t_d)}\right\}\Bigg]\Bigg] \quad (19)
$$

Case2:When$t_d > t_w$

In thiscase,timeduringwhichnodeterioration occursisgreater thanthe time duringwhichinventoryinOWbecomes zeroandthebehaviorofthemodeloverthetimeinterval $[0,T]$hasbeengraphicallyrepresentedbelowinFigure2.

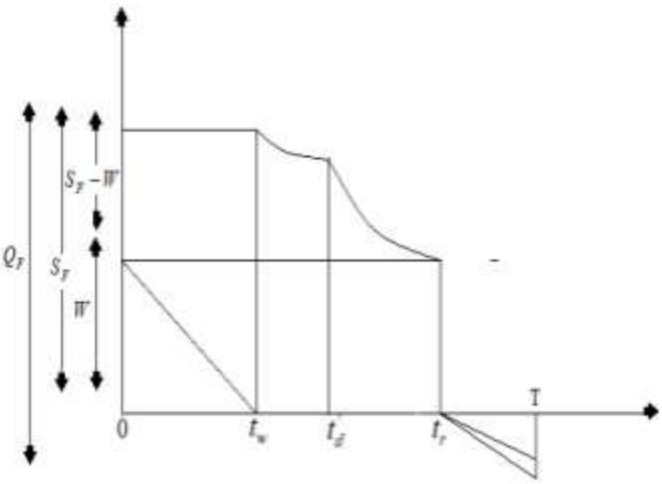


Figure2: Two-warehouseinventorysystem,whent_d □ t_w.

Therefore,thedifferentialequationsthatdescribetheinventory levelintheRWandOWattime$t$overtheperiod$(0,T)$aregivenby:

$$\frac{dI_0(t)}{dt}=-\lambda e^{\lambda t} \quad ,0\le t\le t_w \tag{20}$$

$$\frac{dI_r(t)}{dt}=0,0\le t\le t_d \tag{21}$$

$$\frac{dI_r(t)}{dt}=-\lambda e^{\lambda t} \quad ,t_w\le t\le t_d \tag{22}$$

$$\frac{dI_r(t)}{dt}+\beta I_r(t)=-\lambda e^{\lambda t} \quad ,t_d\le t\le t_r \tag{23}$$

$$\frac{dB(t)}{dt}=De^{-\delta(T-t)} \quad ,t_r\le t\le T \tag{24}$$

ThesolutionoftheabovedifferentialequationWiththe boundary condition $I_0(0)=W$, $I_r(t_w)=S_f-W$, $I_r(t_r)=0,B(t_r)=0$

$$I_0(t)=(W+1)-e^{\lambda t} \quad ,0\le t\le t_w \tag{25}$$

$$I_r(t)=S_f-W-e^{\lambda t}+e^{\lambda t_w},t_w\le t\le t_d \tag{26}$$

$$I(t)=\left(\frac{-\lambda}{\lambda+\beta}\right)e^{\lambda t}+\left(\frac{-\lambda}{\lambda+\beta}\right)e^{(\lambda+\beta)t_r-\beta t},$$

$$t_d\le t\le t_r \tag{27}$$

$$B(t)=\frac{D}{\delta}\left\{e^{-\delta(T-t)}-e^{-\delta(T-t_r)}\right\},t_r\le t\le T \tag{28}$$

$$I_r(t)=W,0\le t\le t_d \tag{29}$$

The Numbers oflostsales at time$t$is$L(t)$given by

$$L(t)=\int_{t_r}^{T} D\left\{1-e^{-\delta(T-t)}\right\}dt \quad ,t_r<t<T$$

$$=D\left[(t-t_r)-\frac{1}{\delta}\left\{e^{-\delta(T-t)}-e^{-\delta(T-t_r)}\right\}\right], \quad t_r\le t\le T \tag{30}$$

Now at $t=t_w$ when $I_0(t)=0$ we get $t_w=\frac{W}{D}$

Considering the continuity of $I_r(t)$ at $t=t_d$, it follows from equation (26) and (27), we get

$$S=W+\frac{\lambda}{\lambda+\beta}e^{\lambda t_d}+e^{\lambda t_w}+\frac{\lambda}{\lambda+\beta}e^{(\lambda+\beta)t_r-\beta t_d} \tag{31}$$

Putting $t=T$ in equation (28), The maximum amount of demand backlogged per cycle is

$$B(T)=\frac{D}{\delta}\left(1-e^{-\delta(T-t_r)}\right) \tag{32}$$

Therefore the order quantity over the replenishment cycle can be determined as

$$Q_f=S_f+B(T)$$

Using equation (31) & (32) we get

$$Q_f=W+\frac{\beta}{\beta+\lambda}e^{\lambda t_d}+e^{\lambda t_w}+\frac{\lambda}{\lambda+\beta}e^{(\lambda+\beta)t_r-\beta t_d}+\frac{D}{\delta}\left(1-e^{-\delta(T-t_r)}\right) \tag{33}$$

The total costs per cycle cost of the following

(a) Ordering cost per cycle $=A$

(b) Holding cost per cycle in RW

$$HC_{RW}=F\left[\int_0^{t_w}I_r(t)dt+\int_{t_w}^{t_d}I_r(t)dt+\int_{t_d}^{t_r}I_r(t)dt\right]$$

$$=Wt_d+(S_f-W+e^{\lambda t_w})(t_d-t_w)-\frac{1}{\lambda}e^{\lambda(t_d-t_w)}-\frac{e^{\lambda(t_r-t_d)}}{\lambda+\beta}-\frac{\lambda}{\beta(\lambda+\beta)}e^{\lambda t_r+\beta t_d}$$

(c) Holding cost per cycle in OW

$$=HC_{OW}=H\int_0^{t_d}I(t)\,dt$$

$$HC_{OW}=H\left[(W+1)t_w-\frac{1}{\lambda}(e^{\lambda t_w}-1)\right]$$

(d) The backlogged cost per cycle is $= s\int_{t_r}^{T} B(t)dt$

$$= \frac{sD}{\delta}\left[\frac{1}{\delta} - \left\{\frac{1}{\delta} + T - t_r\right\} e^{-\delta(T-t_r)}\right]$$

(e) The opportunity cost due to Sales is

$$c_1 D \int_{t_r}^{T} \left\{1 - e^{-\delta(T-t)}\right\} D dt$$

$$= c_1 D\left[T - t_r - \frac{1}{\delta}\left\{1 - e^{-\delta(T-t_r)}\right\}\right]$$

(f) The deterioration cost per cycle $= c\beta \int_{t_d}^{t_r} I_r(t)dt$

$$= \frac{-c\beta}{\lambda+\beta}\left[e^{\lambda(t_r-t_d)} + \frac{\lambda}{\beta} e^{\lambda t_r - \beta t_d}\right]$$

Now, the Total relevant cost per time unit time is given by

$$TCF2(t_r, T) = \frac{1}{T}\left[OC + HC + BC + OC + DC\right]$$

$$= \frac{1}{T}\Bigg[A + \Bigg[W t_d + (S_f - W + e^{\lambda t_w})(t_d - t_w) - \frac{1}{\lambda} e^{\lambda(t_d - t_w)} - \frac{e^{\lambda(t_r - t_d)}}{\lambda+\beta} - \frac{\lambda}{\beta(\lambda+\beta)} e^{\lambda t_r + \beta t_d}\Bigg] + H\left[(w+1)t_w - \frac{1}{\lambda}(e^{\lambda t_w} - 1)\right]$$
$$+ \frac{SD}{\delta}\left[\frac{1}{\delta} - \left\{\frac{1}{\delta} + T - t_r\right\} e^{-\delta(T-t_r)}\right] + c_1 D\left[T - t_r - \frac{1}{\delta}\left\{1 - e^{-\delta(T-t_r)}\right\}\right]$$
$$+ \left(\frac{-c\beta}{\lambda+\beta}\right)\left[e^{\lambda(t_r-t_d)} + \frac{\lambda}{\beta} e^{\lambda t_r - \beta t_d}\right]\Bigg] \quad (34)$$

Case3: When $t_d < t_r$

During the time interval $[0, t_d]$ there is no deterioration so the inventory in RW is depleted only due to demand whereas in OW inventory level remains the same. Further, during the time interval $[t, t]$ the inventory level in RW is dropping to zero due to the combined effect of demand and deterioration and the inventory in OW gets depleted due to deterioration alone. Now, during the time interval $[t_r, t_w]$ depletion of inventory occurs in OW due to the combined effect of demand and deterioration and it reaches to zero at time t_w. Moreover, during the interval $[t_w, T]$ the demand is backlogged. The behavior of the model over the time interval $[0, T]$ has been graphically represented below in Figure 3.

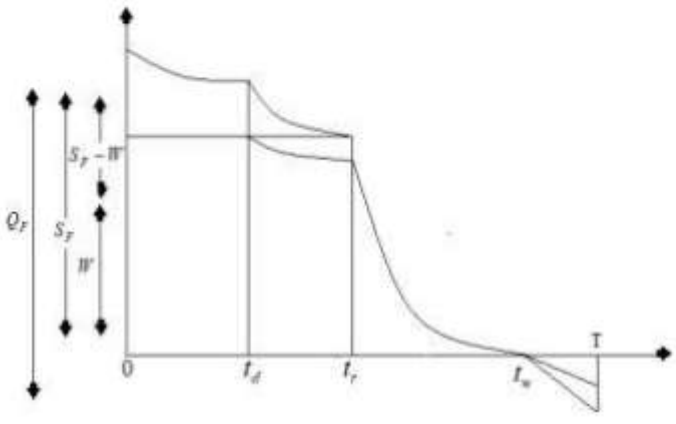


Figure3: Two-warehouse inventory system, when t_d □ t_r.

Therefore, the differential equations that describe the inventory level in the RW and OW at time t over the period $(0, T)$ are given by:

$$\frac{dI_r(t)}{dt} = -\lambda e^{\lambda t} \quad , 0 \le t \le t_d \tag{35}$$

$$\frac{dI_r(t)}{dt} + \beta I_r(t) = -\lambda e^{\lambda t} \quad , t_d \le t \le t_r \tag{36}$$

$$\frac{dI_0(t)}{dt} + \alpha I_0(t) = 0 \quad , t_d \le t \le t_r \tag{37}$$

$$\frac{dI_0(t)}{dt} + \alpha I_0(t) = -\lambda e^{\lambda t}, \quad t_r \le t \le t_w \tag{38}$$

$$\frac{dB(t)}{dt} = De^{-\delta(T-t)}, t_w \le t \le T \tag{39}$$

The solution of the above differential equation With the boundary condition $I_r(0) = S_F - W$, $I_r(t_r) = 0$, $I_0(t_d) = W, I_0(t_w) = 0, B(t_w) = 0$

$$I_r(t) = S_F - W + 1 - e^{\lambda t} \quad , 0 \le t \le t_d \tag{40}$$

$$I_r(t) = \left(\frac{-\lambda}{\lambda+\beta}\right) e^{\lambda t} + \left(\frac{\lambda}{\lambda+\beta}\right) e^{(\lambda+\beta)t_r - \beta t}, \quad t_d \le t \le t_r \tag{41}$$

$$I_d(t)=We^{-\alpha(t-t_d)},t_d\le t\le t_r \quad (42)$$

$$I_e(t)=\left(\frac{-\lambda}{\lambda+\alpha}\right)e^{\lambda t}+\left(\frac{\lambda}{\lambda+\alpha}\right)e^{(\lambda+\alpha)t_w-\alpha t},$$

$$t_r\le t\le t_w \quad (43)$$

$$B(t)=\frac{D}{\delta}\left\{e^{-\delta(T-t)}-e^{-\delta(T-t_w)}\right\},t_w\le t\le T \quad (44)$$

Thenumbersofcostsalesattime$t$is$t=t_d$ itfollowsfrom equation(40) &(41)

$$S_f-W+1-e^{\lambda t}=I_r(t)=\left(\frac{-\lambda}{\lambda+\beta}\right)e^{-\lambda t_d}$$

$$+\left(\frac{\lambda}{\lambda+\beta}\right)e^{(\lambda+\beta)t_r-\beta t_d} \quad (45)$$

Thenumberoflost salesattime$t$is

$$L(t)=\int_{t_w}^{T}D\left(1-e^{-\delta(T-t)}\right)dt,\quad t_w\le t\le T$$

$$=D\left[(t-t_w)-\frac{1}{\delta}\left\{e^{-\delta(T-t)}-e^{-\delta(T-t_w)}\right\}\right] \quad (46)$$

$$S_f=W-1+\frac{\beta}{\lambda+\beta}e^{\lambda t_d}+e^{\lambda t_w}+\frac{\lambda}{\lambda+\beta}e^{(\lambda+\beta)t_r-\beta t_d} \quad (47)$$

Consideringthecontinuity $I_0(t)$at$t=t_r$,itfollowsfrom ofequation(42)and(43),weget

$$We^{-\alpha(t_r-t_d)}=\frac{-\lambda}{\lambda+\alpha}e^{\lambda t_r}+\frac{\lambda}{\lambda+\alpha}e^{(\lambda+\alpha)t_w-\alpha t_r}$$

$$t_w=\frac{1}{\lambda+\alpha}\ln\left[\frac{(\lambda+\alpha)W}{\lambda}e^{\alpha t_d}+e^{(\lambda+\alpha)t_r}\right] \quad (48)$$

Putting$t=T$inequation(44),Wewillgetthemaximum amountofdemandbackloggedpercycleis

$$B(T)=\frac{D}{\delta}\left(1-e^{-\delta(T-t_w)}\right) \quad (49)$$

Thereforetheorderquantityoverthereplenishmentcyclecanbedeter minedas

$$Q_f = S_f + B(T) = W - 1 + \frac{\beta}{\lambda+\beta} e^{\lambda t_d} + e^{\lambda t_w} + \frac{\lambda}{\lambda+\beta} e^{(\lambda+\beta)t_r - \beta t_d} + \frac{D}{\delta}\left(1 - e^{-\delta(T-t_w)}\right) \quad (50)$$

Thetotalcostspercycleofthe followingelements

(a) Orderingcostpercycle$=A$

(b) HoldingcostpercycleinRW

$$HC_{RW} = F\left[\int_0^{t_d} I_r(t)dt + \int_{t_d}^{t_r} I_r(t)dt\right]$$

$$= F\left[(S_L - W + 1)t_d - \frac{1}{\lambda}\left(e^{\lambda t_d} - 1\right)\right]$$

$$- \left[\frac{1}{\lambda+\beta} e^{\lambda(t_r - t_d)} + \frac{\lambda}{\beta(\lambda+\beta)} e^{\lambda t_r + \beta t_d}\right]$$

(c) Inventoryholdingcostin OW

$$HC_{OW} = H\left[\int^{t_d} w\,dt + \int^{t_r} I_0(t)dt + \int^{t_w} I_0(t)dt\right]$$

$$HC_{OW} = H\left[Wt_d - \frac{1}{\lambda+\alpha} e^{\lambda(t_w - t_r)} + \frac{\lambda}{\alpha} e^{\lambda t_w + \alpha t_r} - \frac{W}{\alpha} e^{\alpha(t_d - t_r)}\right]$$

(d) TheBackloggedCostpercycleis$= s\int_{t_w}^{T} B(t)dt$

$$SC = \frac{sD}{\delta}\left[\frac{1}{\delta} - \left\{\frac{1}{\delta} + T - t_w\right\} e^{-\delta(T-t_w)}\right]$$

(e) TheOpportunityCostduetoSalespercycleis

$$= c_1 \int_{t_w}^{T} D\left\{1 - e^{-\delta(T-t)}\right\} dt$$

$$= c_1 D\left[T - t_w - \frac{1}{\delta}\left\{1 - e^{-\delta(T-t_w)}\right\}\right]$$

(f) TheDeteriorationCostpercycle

$$=c\left[\beta\int_{t_d}^{t_r} I_r(t)dt+\alpha\int_{tr}^{t_w} I_0(t)dt\right]$$

$$=-c\left[\frac{\beta}{\lambda+\beta}\left\{e^{\lambda(t_r-t_d)}+\frac{\lambda}{\beta}e^{\lambda t_r-\beta t_d)}\right\}\right.$$

$$\left.+\frac{\alpha}{\lambda}\left\{e^{\lambda(t_w-t_r)}+\frac{\lambda}{\alpha}e^{\lambda t_w-\beta t_r)}\right\}\right]$$

Now,theTotalrelevantcostisgivenby

$$TCL1(t_r,T)=\frac{1}{T}\left[OC+HC+OC+DC+BC\right]$$

$$=\frac{1}{T}\left[A+F\left[(S_L-W+1)t_d-\frac{1}{\lambda}\left(e^{\lambda t_d}-1\right)\right]\right.$$

$$-\left[\frac{1}{\lambda+\beta}e^{\lambda(t_r-t_d)}+\frac{\lambda}{\beta(\lambda+\beta)}e^{\lambda t_r+\beta t_d}\right]$$

$$+H\left[Wt_d - \frac{1}{\lambda+\alpha}\left\{e^{\lambda(t_w-t_r)} + \frac{\lambda}{\alpha}e^{\lambda t_w+\alpha t_r)}\right\}\right.$$

$$\left. -\frac{W}{\alpha}e^{\alpha(t_d-t_r)}\right] + \frac{SD}{\delta}\left[\frac{1}{\delta} - \left\{\frac{1}{\delta}+T-t_w\right\}e^{-\delta(T-t_w)}\right]$$

$$+cD_1\left[T-t_w - \frac{1}{\delta}\left\{1-e^{-\delta(T-t_w)}\right\}\right]$$

$$+(-c)\left[\frac{\beta}{\lambda+\beta}\left\{e^{\lambda(t-t)_d} + \frac{\lambda}{\beta}e^{\lambda t-\beta t)_d}\right\}\right.$$

$$\left. + \frac{\alpha}{\lambda+\alpha}\left\{e^{\lambda(t_w-t_r)} + \frac{\lambda}{\alpha}e^{\lambda t_w-\beta t_r)}\right\}\right] \quad (51)$$

Case4:When $t_d > t_r$

In this case, time during which no deterioration occurs is greater than the time during which inventory in RW becomes zero and the behavior of the model over the whole cycle $[0,T]$ has been graphically represented as in Figure4.

Case4:When$t_d > t_r$

Inthiscase,timeduringwhichnodeteriorationoccursisgreater than the time during which inventory in RW becomeszeroandthebehaviorofthemodeloverthewholecycle$[0,T]$ hasbeengraphicallyrepresentedasinFigure4.

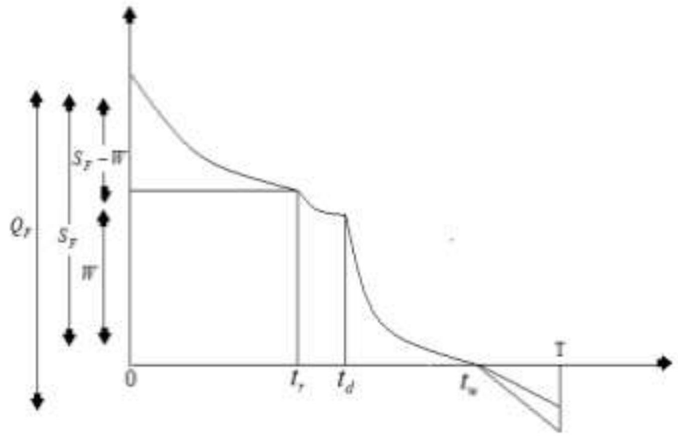


Figure-4: Two-warehouseinventorysystem,when$t_d>t_r$

Therefore,thedifferentialequationsthatdescribetheinventorylevelin theRWandOWattime$t$overtheperiod$(0,T)$are givenby:

$$\frac{dI_r(t)}{dt}=-\lambda e^{\lambda t},\quad 0\le t\le t_r \qquad (52)$$

$$\frac{dI_0(t)}{dt}=-\lambda e^{\lambda t},\quad t_r\le t\le t_d \qquad (53)$$

$$\frac{dI_0(t)}{dt}+\alpha I_0(t)=-\lambda e^{\lambda t},\quad t_d\le t\le t_w \qquad (54)$$

$$\frac{dB(t)}{dt}=De^{-\delta(T-t)} \quad ,t_w \leq t\leq T \tag{55}$$

Thesolutionoftheabovedifferentialequation Withthe boundarycondition $I_r(t_r)=0, I_r(t_r)=0, I_o(t_r)=W$, $I_0(t_w)=0, B(t_w)=0$

$$I(t)=e^{\lambda t_r}-e^{\lambda t}, 0\leq t\leq t_r \tag{56}$$

$$I_0(t)=W+e^{\lambda t_r}-e^{\lambda t}, t_r\leq t\leq t_d \tag{57}$$

$$I_0(t)=\frac{\lambda}{\lambda+\alpha}e^{(\lambda+\alpha)t_w-\alpha t}-e^{\lambda t}, t_d \leq t\leq t_w \tag{58}$$

$$B(t)=\frac{D}{\delta}\left\{e^{-\delta(T-t)}-e^{-\delta(T-t_w)}\right\}, t_w \leq t\leq T \tag{59}$$

Thenumberoflostsalesattime t is

$$L(t)=\int_{t_w}^{T}D\left(1-e^{-\delta(T-t)}\right)dt, \quad t_w\leq t\leq T$$

$$=D\left[(t-t_w)-\frac{1}{\delta}\left\{e^{-\delta(T-t)}-e^{-\delta(T-t_w)}\right\}\right] \tag{60}$$

Consideringthecontinuityofeq $I_0(t)$ at $t=t_r$, itfollowsfrom uation(57)and(58),weget

$$W+e^{\lambda t_r}-e^{\lambda t_d}=\frac{\lambda}{\lambda+\alpha}\left[e^{(\lambda+\alpha)t_w-\alpha t_d}-e^{\lambda t_d}\right] \tag{61}$$

$$t_w=\frac{\alpha}{\lambda+\alpha}t_d+\frac{1}{\lambda+\alpha}\ln\left[\frac{(\lambda+\alpha)}{\lambda}(W+e^{\lambda t_r})-\frac{\alpha}{\lambda}e^{\lambda t_d}\right] \tag{62}$$

Nowat $t=0, I_r(t)=S_L-w$ bythefollowingequation (56), Wewillgetthemaximuminventorylevelis

$$S_l=W+e^{\lambda t_r}-1$$

$$S_l=W-1+e^{\lambda t_r} \tag{63}$$

Putting $t=T$, in(59), Wewillgetthemaximumamountof demandbackloggedis

$$B(t)=\frac{D}{\delta}\left\{1-e^{-\delta(T-t_w)}\right\} \tag{64}$$

Thereforetheorderquantityoverthereplenishmentcyclecanbedeter minedas

$$Q_f = S_f + B(T) = W - 1 + e^{\lambda t_r} + \frac{D}{\delta}\left(1 - e^{-\delta(T-t_w)}\right) \quad (65)$$

Thetotalcostspercycleofthe followingelements

(a)Orderingcostpercycle=A

(b) Holding cost percycleinRWis$HC_{RW} = F\left[\int_0^{t_r} I_r(t)dt\right]$

$$= F\left[e^{\lambda t_r} t_r - \frac{1}{\lambda}e^{\lambda t_r} + 1\right] = F\left[1 + e^{\lambda t_r}\left(t_r - \frac{1}{\lambda}\right)\right]$$

(c)InventoryholdingcostinOW=

$$HC_{OW} = H\left[\int_0^{t_r} w\,dt + \int_{t_r}^{t_d} I_0(t)dt + \int_{t_d}^{t_w} I_0(t)dt\right]$$

$$HC_{OW} = H\left[Wt_d + e^{\lambda t_r}(t_d - t_r) - \frac{1}{\lambda}e^{\lambda(t_d - t_r)} - \frac{1}{\lambda+\alpha}\left\{\frac{1}{\alpha}e^{\lambda t_w + \alpha t_d} + \frac{1}{\lambda}e^{\lambda(t_w - t_d)}\right\}\right]$$

(d)Thebackloggedcost=$s\int_{t_w}^{T} B(t)dt$

$$SC = \frac{sD}{\delta}\left[\frac{1}{\delta} - \left\{\frac{1}{\delta} + T - t_w\right\}e^{-\delta(T-t_w)}\right]$$

(e) Theopportunitycost=$c_1\int_{t_w}^{T} D\left\{1 - e^{-\delta(T-t)}\right\}dt$

$$= c_1 D\left[T - t_w - \frac{1}{\delta}\left\{1 - e^{-\delta(T-t_w)}\right\}\right]$$

(f) Thedeteriorationcost=$c\alpha\int_{t_d}^{t_w} I_0(t)dt$

$$= -c\alpha\frac{\lambda}{\lambda+\alpha}\left[\frac{1}{\alpha}e^{\lambda t_w - \alpha t_d} + \frac{1}{\lambda}e^{\lambda(t_w - t_d)}\right]$$

Now,theTotalrelevantcostisgivenby

$$TCL2(t_r, T) = \frac{1}{T}\left[OC + HC + OC + DC + BC\right]$$

$$TCL2(t_r,T)=\frac{1}{T}\Bigg[A+\frac{F}{\lambda t_r}\left[1+e^{\lambda t_r}\left(t_r-\frac{1}{\lambda}\right)\right]$$

$$+H\Bigg[Wt_d+e^{\lambda(t_d-t_r)}(t_d-t_r)-\frac{1}{\lambda}e^{\lambda(t_d-t_r)}$$

$$-\frac{\lambda}{\lambda+\alpha}\left\{\frac{1}{\alpha}e^{\lambda t_w+\alpha t_d}+\frac{1}{\lambda}e^{\lambda(t_w-t_d)}\right\}\Bigg]$$

$$+\frac{SD}{\delta}\left[\frac{1}{\delta}-\left\{\frac{1}{\delta}+T-t_w\right\}e^{-\delta(T-t_w)}\right]$$

$$+c_1D\left[T-t_w-\frac{1}{\delta}\left\{1-e^{-\delta(T-t_w)}\right\}\right]$$

$$-c\alpha\frac{\lambda}{\lambda+\alpha}\left[\frac{1}{\alpha}e^{\lambda t_w-\alpha t_d}+\frac{1}{\lambda}e^{\lambda(t_w-t_d)}\right]\Bigg] \quad (66)$$

SotheTotalrelevantcostis

$$TCF(t_r,T)=\begin{cases}TCF1(t_r,T); & if\ t_d\le t_w\\ TCF2(t_r,T); & if\ t_d\ge t_w\end{cases} \quad (67)$$

$$TCL(t_r,T)=\begin{cases}TCL1(t_r,T); & if\ t_d\le t_r\\ TCL2(t_r,T); & if\ t_d\ge t_r\end{cases} \quad (68)$$

Where

$$\frac{\partial TCF1(t,T)}{\partial t_r}=\Bigg(\frac{0.5(-3+xy^{0.06})}{y^{1.03}}-\frac{x}{y^{0.97}}$$

$$+\left(\frac{0.06}{y^{1.06}}+(-0.06+0.06x)y^{-1.06+0.06x}\right)$$

$$+0.5\left(-1.(-3+x)y^{-1+0.03(-3+x)}\right)$$

$$-0.0291262\left(-1.(-3+x.)y^{-1+0.03(-3+x)}\right.$$

$$\left.+0.03(3+x)y^{-1+0.03(-3+x)}\right)$$

$$+0.12\left(-1.(-3+x)y^{-1+0.03(-3+x)}-1.xy^{-1.03+0.06x}\right.$$

$$\left.-1.(3+x)y^{-1+0.03(3+x)}-\frac{0.5(1.-y^{0.06x})}{y^{1.03}}\right)$$

$$+0.75\Bigg(\frac{2.}{y^{0.97}}-0.0291262\left(-1.(-3+x)y^{-1+0.03(-3+x)}\right.$$

$$\left.+0.03(3+x)y^{-1+0.03(3+x)}\right)$$

$$+200\left(1+10.y^{0.1(-x+y)}\right)\left(-\frac{0.1(-x+y)}{y}-0.1Log[y]\right)\Bigg)$$

$$+2000.\left(-y^{-0.1(-x+y)}-y^{-0.1(-x+y)}(10.-x+y)\right)$$

$$\left(-\frac{0.1(-x+y)}{y}-0.1Log[y]\right) \quad (a)$$

$$\frac{\partial TCF2(t_r,T)}{\partial T}=\Big(-16.6667\,y^{0.03} - 0.0509709\big(-1.y^{-0.03(-3+x)}Log[y] + 0.03y^{-0.06+0.06x}Log[y] + 0.5\big(-1.y^{0.03(-3+x)}Log[y] - 1.y^{0.03(-3+x)}Log[y]\big)\big) + 0.12\big(-1.y^{-0.03+0.06x}Log[y] - 1.y^{0.03(-3+x)}Log[Y] - 1.y^{0.03(-3+x)}Log[y] - 1.y^{0.03(3+x)}Log[y]\big) + 0.03y^{0.03(3+x)}Log[Y]\big) + 200(-1+1.y^{0.1(-x+y)}Log[y]\big) + 2000.\big(y^{-0.1(-x+y)} - 0.1y^{-0.1(-x+y)}(10.-x+y)Log[y]\big)\Big) \quad \text{(b)}$$

$$\frac{\partial TCL1(t_r,T)}{\partial t_r} = -cHy^{-1+c\lambda} - \frac{m\beta\left((-b+x)y^{-1+(-b+x)\lambda}\lambda + \frac{y^{-1+b\beta+x\lambda}\lambda(b\beta+x\lambda)}{\beta}\right)}{\beta+\lambda} + \Bigg(F-(b-c)y^{-1+(b-c)\lambda} - \frac{(b-c)y^{-1+(-b+x)\lambda}\lambda}{\beta+\lambda} - \frac{y^{-1+b\beta+x\lambda}\lambda(b\beta+x\lambda)}{\beta(\beta+\lambda)} + (b-c)\left(by^{-1+b\lambda}\lambda - cy^{-1+c\lambda}\lambda + \frac{by^{-1+b\lambda}\beta\lambda}{\beta+\lambda}\right) + \frac{y^{-1-b\beta+x(\beta+\lambda)}\lambda(-b\beta+x(\beta+\lambda))}{\beta+\lambda}\Bigg)$$

$$+\frac{Ds\left(-y^{-(-x+y)\delta} - y^{-(-x+y)\delta}\left(-x+y+\frac{1}{\delta}\right)\left(-\frac{(-x+y)\delta}{y} - \delta Log[y]\right)\right)}{\delta} + Dr\left(1+\frac{-y^{-(-x+y)\delta}\left(-\frac{(-x+y)\delta}{y} - \delta Log[y]\right)}{\delta}\right) \quad \text{(c)}$$

$$\frac{\partial TCL2(t_r,T)}{\partial T} = Dr\left(-1+y^{-(-x+y)\delta}Log[y]\right) + \frac{Ds\left(y^{-(-x+y)\delta} - y^{-(-x+y)\delta}\left(-x+y+\frac{1}{\delta}\right)\delta Log[y]\right)}{\delta}$$

$$-\frac{m\beta\left(y^{(bx)}\lambda Log[y]+\frac{y^{b\beta+x\lambda}\lambda^2 Log[y]}{\beta}\right)}{\beta+\lambda}$$

$$+F\left((b-c)y^{-b\beta+x(\beta+\lambda)}\lambda Log[y]-\frac{y^{(-b+x)\lambda}\lambda Log[y]}{\beta+\lambda}-\frac{y^{b\beta+x\lambda}\lambda^2 Log[y]}{\beta(\beta+\lambda)}\right) \quad \text{(d)}$$

$$\frac{\partial^2 TCL1(t,T)}{\partial t^2}=-0.5(-2+x)y^{-1+0.03(-2+x)}-16.6667(0.06+0.03x)y^{-0.94+0.03x}+4\left(0.0608738y^{1.09}+1.(3-x)y^{-1+0.03(3-x)}+0.015(-2+x)y^{-1+0.03(-2+x)}+1.(0.06+0.03x)y^{-0.94+0.03x}\right)$$

$$+0.5\left(-\frac{2.}{y^{0.94}}+2\left(\frac{0.03}{y^{0.94}}+0.05(-0.06+0.06x)y^{-1.06+0.06x}\right)\right)+0.75\left(-200(4-x)y^{3-x}-0.970874\left(0.03(3-x)y^{-1+0.03(3-x)}+0.03(0.09+x)y^{-0.91+x}\right)\right)$$

$$+200\left(1+10.y^{0.1(-3+y)}\left(-\frac{0.1(-3+y)}{y}-0.1Log[y]\right)\right)+2000.\left(-y^{-0.1(-3+y)}-y^{-0.1(-3+y)}(7.+y)\left(-\frac{0.1(-3+y)}{y}-0.1Log[y]\right)\right) \quad \text{(e)}$$

$$\frac{\partial^2 TCL2(t,T)}{\partial T^2}=-0.5y^{0.06+0.03x}Log[y]+0.03y^{-0.06+0.06x}Log[y]-0.5y^{0.03(-2+x)}Log[y]+4\left(-1.y^{0.03(3-x)}Log[y]+0.03y^{0.06+0.03x}Log[y]+0.015y^{0.03(-2+x)}Log[y]\right)+0.75\left(200y^{4-x}Log[y]-0.970873786407767\left(-0.03y^{0.03(3-x)}Log[y]+0.03y^{0.09+x}Log[y]\right)\right) \quad \text{(f)}$$

$$\frac{\partial^2 TCF1(t,T)}{\partial t_r^2} = 0.015(-33.3333+x)xy^{-1+0.03x}$$
$$-0.116505\left(\frac{1.}{y^{0.97}}+2.09y^{1.09}\right)$$
$$+0.75\left(1.(2-x)y^{-1+0.03(2-x)}+0.03(2-x)xy^{-1+0.03x}\right.$$
$$\left.-0.0291262\left(\frac{1.}{y^{0.97}}+2.09y^{1.09}\right)\right)$$
$$+2\left(1+10.y^{-}\left(-\frac{0.1(-3+y)y}{}-0.1Log[y]\right)\right)$$
$$+2000.-y^{-0.1(-3+y)}-y^{-0.1(-3+y)}(7.+y)$$
$$\left(-\frac{0.1(-3+y)}{y}-0.1Log[y]\right) \quad (g)$$

$$\frac{\partial^2 TCF2(t_r,T)}{\partial T^2} = 0.75\left(-y^{0.03x}+1.y^{0.03(2-x)}Log[y]\right.$$
$$\left.+0.03(2-x)y^{0.03x}Log[y]\right)+0.5\left(y^{0.03x}\right.$$
$$\left.+0.03(-33.3333+x)y^{0.03x}Log[y]\right) \quad (h)$$

Optimality:TheNecessaryconditionforoptimalityis

$$\frac{\partial TCL(t_r,T)}{\partial t_r}=0, \quad \frac{\partial TCL(t_r,T)}{\partial T}=0$$
$$\frac{\partial^2 TCL(t_r,T)}{\partial t_r^2}>0, \quad \frac{\partial^2 TCL(t_r,T)}{\partial T^2}>0$$

and

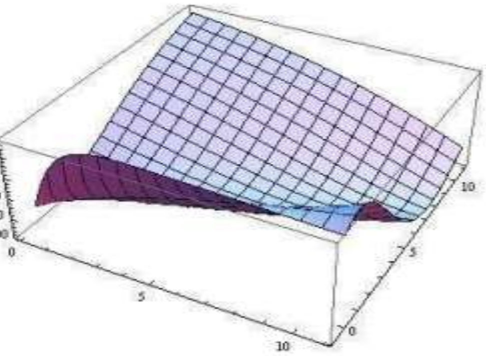
$$\frac{\partial^2 TCL}{\partial t_r^2}\times\frac{\partial^2 TCL}{\partial T^2}-\frac{\partial^2 TCL}{\partial T\partial t_r}\times\frac{\partial^2 TCL}{\partial t_r\partial T}>0$$

$$\frac{\partial TCF(t_r,T)}{\partial t_r}=0, \quad \frac{\partial TCF(t_r,T)}{\partial T}=0$$
$$\frac{\partial^2 TCF(t_r,T)}{\partial t_r^2}>0, \quad \frac{\partial^2 TCF(t_r,T)}{\partial T^2}>0$$

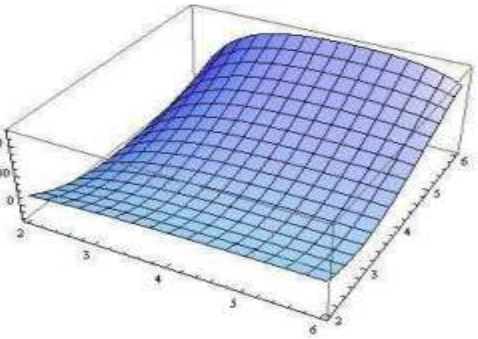
and

$$\frac{\partial^2 TCF}{\partial t_r^2}\times\frac{\partial^2 TCF}{\partial T^2}-\frac{\partial^2 TCF}{\partial T\partial t_r}\times\frac{\partial^2 TCF}{\partial t_r\partial T}>0$$

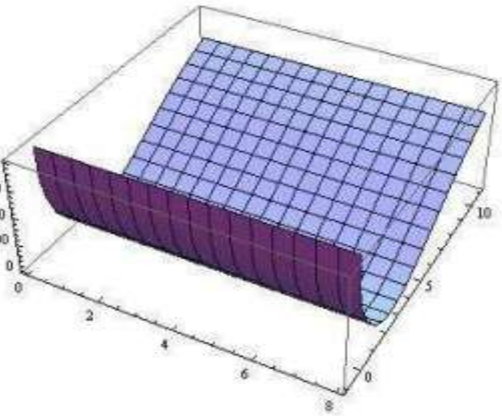
Now,theTotalrelevantcost pertimeunittimeisgivenby



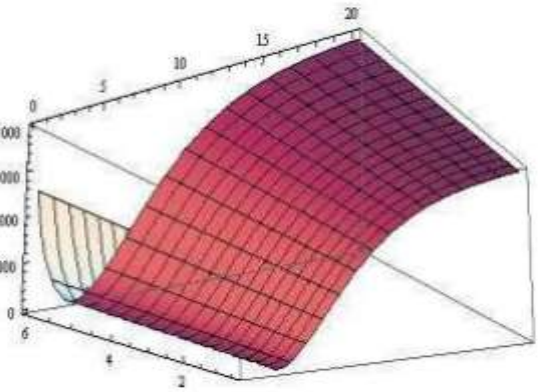
Case-1: $t_d < t_w$



Case-3: $t_d < t_r$



Case-2: $t_d > t_w$



Case-4: $t_d > t_r$

NUMERICALANDSENSITIVITYANALYSIS

To illustratetheresult,letusconsideraninventorysystemwith

%Chang	ChangesVa	Different fromOriginalValue	Ratio= Different/Originalues	% ChangeswSignificant
+50	3481.68	+4.95	+0.0015	0.15% Up
+25	3479.76	+3.03	+0.0009	0.09% Up
-25	3475.31	-1042	-0.0004	0.04%Dow
-50	3473.59	-3.14	-0.0008	0.08%Dow

TCL1[0.03] 2657.22

%Changes	Changes Value	Different fromOriginalValues	Ratio=Different/ OriginalValues	% ChangeswithSignificant
+50	3377.46	-5.61	-0.002	0.2% Down
+25	3380.05	-3.02	-0.001	0.1% Down
-25	3384.31	+1.24	+0.0004	0.04% Up
-50	3385.77	+2.70	+0.0008	0.08% Up

% Changes	Changes Value	Different fromOriginalValues	Ratio=Different/Original Values	% ChangeswithSignificant
+50	2613.36	-43.80	-0.016	1.6% Down
+25	2630.80	-26.42	-0.010	1% Down
-25	2669.28	+12.06	+0.0045	0.45% Up
-50	2790.39	+133.17	+0.050	5% Up

TCL2[0.03] 3197.35

%Changes	Changes Value	DifferentfromOriginalValues	Ratio=Different/OriginalValues	% ChangeswithSignificant
+50	3210.19	+12.84	+0.004	0.4% Up
+25	3205.16	+7.81	+0.002	0.2% Up
-25	3183.98	-13.37	-0.004	0.4% Down
-50	3156.71	-40.64	-0.013	1.3% Down

TCF1[0.03] 3383.07

%Changes	Changes Value	Differen tfromOr iginal Values	Ratio=Differe ntOriginal Values	% Changes withSignificant
+50	3379.28	-3.79	-0.001	0.1% Down
+25	3380.74	-2.33	-0.0007	0.07%Down
-25	3387.46	+4.39	+0.0014	0.14% Up
-50	3397.85	+14.78	+0.0043	0.43% Up

TCF2[□ □ 0.03]□ 3476.73

%Changes	Changes Value	Differen tfromOr iginal Values	Ratio=Differe ntOriginal Values	% Changes withSignificant
+50	3482.51	+5.78	+0.0017	0.17% Up
+25	3480.51	+3.47	+0.0009	0.09% Up
-25	3470.94	-5.79	-0.0017	0.17%Down
-50	3459.36	-17.43	-0.005	0.5%Down

TCL1[□ □ 0.03]□ 2657.22

%Changes	Changes Value	Differen tfromOr iginal Values	Ratio=Differe ntOriginal Values	% Changes withSignificant
+50	2668.72	+11.50	+0.0043	0.43%Down
+25	2664.10	+6.88	+0.0022	0.22%Down
-25	2645.84	-11.38	-0.0043	0.43% Up
-50	2623.28	-33.94	-0.0130	1.3% Up

TCL2[□ □ 0.03]□ 319.735

TCF1[D□ 100]□ 3383.07

% Changes	ChangesV alue	Differentfro m OriginalVal ues	Ratio=Differ enOriginal Values	%Changesw iSignificant
+50	4766.02	+1082.95	+0.320	32% Up
+25	4074.55	+691.148	+0.204	20.4%Up
-25	2691.59	-691.148	-0.204	20.4%Down
-50	2000.12	-1082.95	-0.320	32% Do wn

TCF2[D□ 100]□ 3476.73

% Changes	ChangesV alue	Differentfro m OriginalVal	Ratio=Differ enOriginal Values	%Changesw iSignificant

		ues		
+50	4859.68	+1382.95	+0.3977	39.77%Up
+25	4168.20	+691.47	+0.199	19.9%Up
-25	2785.25	-691.47	-0.199	19.4% Down
-50	2093.76	-1382.95	-0.3977	39.77%Down

TCL1[D□ 100]□ 2657.22

% Change	ChangesValue	Different froOriginal Values	Ratio=Different/Original Values	% Changes witSignificant
+50	4040.17	+1382.95	+0.520	52.0%Up
+25	3348.69	+690.74	+0.529	25.9%Up
-25	1965.74	-691.48	-0.261	26.1%Down
-50	1274.26	-1382.96	-0.520	52.0%Down

TCL2[D□ 100]□ 3197.35

% Changes	ChangesValue	Different fromOriginal Values	Ratio=DifferenOriginal Values	%ChangeswiSignificant
+50	4620.55	+1423.20	+0.445	44.5%Up
+25	3908.95	+711.60	+0.223	22.3%Up
-25	2485.75	-711.60	-0.223	22.3%Down
-50	1774.15	-1423.20	-0.445	44.5%Down

TCF1[W□ 200]□ 3383.07

%Changes	Changes Value	DifferentfromOriginal Values	Ratio=DifferentOriginal Values	% Changes withSignificant
+50	3197.35			
+25	3197.35			
-25	3197.35	NotChanged		
-50	3197.35			

% Changes	Changes Value	Different fromOriginal Values	Ratio=Different/iginal Values	%ChangeswiSignificant
+50	3633.07	+250.00	+0.074	7.4% Up

+25	3508.07	+125.00	+0.037	3.7% Up
-25	3258.07	-125.00	-0.037	3.7%Down
-50	3133.07	-250.00	-0.074	7.4%Down

TCF2[W□ 200]□ 3476.73

% Changes	ChangesValue	DifferentfromOriginal Values	Ratio=Differen t/iginal Values	% Changes withSignificant
+50	3801.73	+325.00	+0.093	9.3% Up
+25	3639.23	+162.5	+0.047	4.7% Up
-25	3314.23	-162.5	-0.047	4.7%Down
-50	3151.73	-325.00	-0.093	9.3%Down

TCL1[W□ 200]□ 2657.22

% Changes	ChangesValue	DifferentfromOriginal Values	Ratio=Differen t/iginal Values	% Changes withSignificant
+50	2507.22	-150.00	-0.056	5.6%Down
+25	2582.22	-75.00	-0.029	2.9%Down
-25	2732.22	+75.00	+0.029	2.9% Up
-50	2807.22	+150.00	+0.056	5.6% Up

TCL2[W□ 200]□ 3197.35

% Changes	ChangesValue	DifferentfromOriginal Values	Ratio=Differen t/iginal Values	% Changes withSignificant
+50	3347.35	+150.00	+0.047	4.7% Up
+25	3272.35	+75.00	+0.024	2.4% Up
-25	3122.35	-75.00	-0.024	2.4%Down
-50	3047.35	-150.00	-0.047	4.7%Down

TCF1[A□ 100]□ 3383.07

% Changes	ChangesValue	Differentfrom OriginalValues	Ratio=DifferenOriginal Values	% Changes withSignificant
+50	3433.07	+50	+0.015	1.5% Up
+25	3408.07	+25	+0.008	0.8% Up
-25	3358.07	-25	-0.008	0.8%Down
-50	3333.07	-50	-0.015	1.5%Down

TCF2[A□ 100]□ 3476.73

% Changes	Changes Value	Different fromOriginal Values	Ratio=Differen Original Values	% Changes withSignificant
+50	3526.73	+50	+0.014	1.4% Up
+25	3501.73	+25	+0.007	0.7% Up
-25	3451.73	-25	-0.007	0.7%Down
-50	3426.73	-50	-0.014	1.4%Down

TCL1[A□100]□ 2657.22

% Changes	ChangesValue	Different fromOriginal Values	Ratio=Differnt/Original Valu	% Changes withSignificant
+50	2702.22	+50	+0.019	1.9% Up
+25	2682.22	+25	+0.01	1% Up
-25	2632.22	-25	-0.01	1% Down
-50	2607.22	-50	-0.019	1.9%Down

TCL2[A□ 100]□ 3197.35

% Changes	ChangesValue	Different fromOriginal Values	Ratio=Differnt/Original Valu	% Changes withSignificant
+50	3247.35	+50	+0.0156	1.56%Up
+25	3222.35	+25	+0.0078	0.78%Up
-25	3172.35	-25	-0.0078	0.78%Down
-50	3147.35	-50	-0.0156	1.56%Down

TCF1[F□ 0.5]□ 3383.07

% Changes	ChangesValue	Different fromOriginal Values	Ratio=DifferenOriginal Values	% Changes withSignificant
+50	5024.60	+1641.53	+0.485	48.5%Up
+25	4203.84	+820.77	+0.243	24.3%Up
-25	2562.30	-820.77	-0.243	24.3%Down
-50	1741.53	-1641.53	-0.485	48.5%Down

TCF2[F□ 0.5]□ 3476.73

% Changes	ChangesValue	Different fromOriginal Values	Ratio=DifferentOriginal Values	% Changes withSignificant

+50	3559.77	+83.04	+0.024	2.4% Up
+25	3518.25	+41.52	+0.012	1.2% Up
-25	3435.21	-41.52	-0.012	1.2%Down
-50	3393.69	-83.04	-0.024	2.4%Down

TCL1[F□ 0.5]□ 2657.22

% Changes	ChangesValue	Different fromOriginal Values	Ratio=DifferenOriginal Values	% Changes withSignificant
+50	2657.06	-0.16	-0.00006	0.006%Down
+25	2657.14	-0.08	-0.00003	0.003%Down
-25	2657.29	+0.07	+0.00003	0.003%Up
-50	2657.37	+0.15	+0.00006	0.006%Up

TCL2[F□ 0.5]□ 3197.35

% Changes	Changes Value	Different fromOriginal Values	Ratio=Different/OriginalValues	% Changes withSignificant
+50	3189.01	-8.34	-0.0026	0.26%Down
+25	3193.18	-4.17	-0.0013	0.13%Down
-25	3201.52	+4.17	+0.0013	0.13% Up
-50	3205.69	+8.34	+0.0026	0.26% Up

TCF1[H□ 0.75]□ 3383.07

% Changes	ChangesValue	Differentfrom OriginalValues	Ratio=DifferenOriginal Values	% Changes withSignificant
+50	3547.70	+164.63	+0.0486	4.86%Up
+25	3465.39	+82.32	+0.0243	2.43%Up
-25	3300.75	-82.32	-0.0243	2.43%Down
-50	3218.44	-164.63	-0.0486	4.86%Down

TCF2[H□ 0.75]□ 3476.73

% Changes	ChangesValue	Different fromOriginal Values	Ratio=DifferenOriginal Values	% Changes withSignificant
+50	3701.19	+224.46	+0.0645	6.45%Up
+25	3587.46	+112.23	+0.0322	3.22%Up
-25	3364.50	-112.23	-0.0322	3.22%Down
-50	3252.26	-221.46	-0.0645	6.45%Down

TCL1[H□ 0.75]□ 2657.22

% Changes	ChangesValue	Different fromOriginal Values	Ratio=DifferentOriginal Values	% Changes withSignificant
+50	2506.15	-151.07	-0.0568	5.68%Down
+25	2582.69	-74.53	-0.0284	2.84%Down
-25	2732.75	+74.53	+0.0284	2.84%Up
-50	2808.28	+151.07	+0.0568	5.68%Up

TCL2[H□ 0.75]□ 3197.35

% Changes	Changes Value	Different fromOriginal Values	Ratio=Differen Original Values	% Changes withSignificant
+50	3334.40	+137.05	+0.0428	4.28%Up
+25	3264.96	+67.61	+0.0214	2.14%Up
-25	3128.82	-67.61	-0.0214	2.14%Down
-50	3060.29	-137.05	-0.0428	4.28%Down

TCF1[□ □ 0.1]□3383.07

% Changes	ChangesValue	Different fromOriginal Values	Ratio=DifferenOriginal Values	% Changes wiSignificant
+50	2580.35	-802.82	-0.2408	24.08%Down
+25	2903..51	-479.56	-0.1477	14.77%Down
-25	4373.63	+790.56	+0.2372	23.72%Up
-50	5738.45	+2355.88	+0.6873	68.73%Up

TCF2[□□ 0.1]□ 3476.73

% Changes	ChangesValue	Different fromOriginal Values	Ratio=DifferenOriginal Values	%ChangeswiSignificant
+50	2673.91	-802.82	-0.2308	23.08%Down
+25	2997.17	-479.56	-0.1377	13.77%Down
-25	4267.29	+790.56	+0.2272	22.72%Up
-50	5832.61	+2355.88	+0.6773	67.73%Up

TCL1[□ □ 0.1]□ 2657.22

% Changes	ChangesValue	Different fromOriginal Values	Ratio=DifferenOriginal Values	%ChangeswiSignificant

+50	1854.40	-802.82	-0.3021	30.21%Down
+25	2177.65	-479.57	-0.1804	18.04%Down
-25	3447.78	+790.56	+0.1800	18.00%Up
-50	5013.10	+2355.88	+0.8866	88.66%Up

TCL2[□ □ 0.1]□ 319.735

% Changes	ChangesValue	Different fromOriginal Values	Ratio=DifferenOriginal Values	%ChangeswiSignificant
+50	2333.47	-863.58	-0.2701	27.01%Down
+25	2686.25	-510.80	-0.1595	15.95%Down
-25	4021.63	+824.58	+0.2577	25.77%Up
-50	5623.62	+2426.57	+0.7588	75.88%Up

TCF1[□ □ 1]□ 3383.07

% Changes	ChangesValue	Differentfrom OriginalValues	Ratio=DifferentOriginal Values	% Changes withSignificant
+50	3382.88	-0.19	-0.000056	0.0056%Down
+25	3382.96	-0.11	-0.000042	0.0042%Down
-25	3383.24	+0.17	+0.000052	0.0052%Up
-50	3383.57	+0.50	+0.000148	0.0148%Up

TCF2[□□ 1]□ 3476.73

% Changes	ChangesValue	Differentfrom OriginalValues	Ratio=Different/iginal Values	% Changes withSignificant
+50	3476.73	NoChanges		
+25	3476.73			
-25	3476.73			
-50	3476.73			

TCL1[□ □ 1]□ 2657.22

TCL1[s□ 2]□ 2657.22

% Changes	ChangesValue	DifferentfromOriginalValues	Ratio=DifferentOriginal Values	% Changes withSignificant

+50	4080.42	+1423.2	+0.5355	53.55%Up
+25	3368.82	+711.60	+0.2678	26.78%Up
-25	1945.62	-711.60	-0.2678	26.78%Down
-50	1234.02	-1423.2	-0.5355	53.55%Down

TCL2[s□ 2]□ 3197.35

% Changes	Changes Value	Different from Original Values	Ratio=Different/ iginal Values	% Changes withSignificant
+50	2456.95	-200.27	-0.0753	7.53%Down
+25	2578.51	-78.71	-0.0296	2.96%Down
-25	2691.21	+33.99	+0.0128	1.28%Up
-50	2656.33	-0.89	-0.0003	0.03%Down
% Changes	ChangesV alue	Differentfr omOrigina lValues	Ratio=Different Original Values	% Changes withSignificant
+50	4620.55	+1423.20	+0.4451	44.51%Up
+25	3908.95	+711.60	+0.2225	22.25%Up
-25	2485.75	-711.20	-0.2225	22.25%Down
-50	1774.15	-1423.20	-0.4451	44.51%Down

TCL2[□ □ 1]□ 319735

TCF1[c_1

% Changes	Changes Value	Different from Original Values	Ratio=Different Original Values	% Changes withSignificant
+50	3342.82	-40.25	-0.0119	1.19%Down
+25	3362.95	-20.12	-0.0059	0.59%Down
-25	3403.19	+20.12	+0.0059	0.59% Up
-50	3423.32	+40.25	+0.0119	1.19% Up

% Changes	Changes Value	Different fromOri ginal Values	Ratio=Different/ iginal Values	% Changes withSignificant
+50	3193.46	-3.39	-0.00106	0.106%Down
+25	3196.17	-0.40	-0.00028	0.028%Down
-25	3196.87	-0.48	-0.00015	0.015%Down
-50	3197.80	+0.45	+0.00014	0.014%Up

□ 2]□ 338307

TCF1[s□ 2]□ 3383.07
TCF2[c_1

% Changes	Changes Value	Different fromOri ginal Values	Ratio=Different Original Values	% Changes withSignificant
+50	3436.48	-40.25	-0.0116	1.16%Down
+25	3456.60	-20.12	-0.0058	0.58%Down
-25	3496.85	+20.12	+0.0058	0.58% Up
-50	3516.97	+40.25	+0.0116	1.16% Up

□ 2]□ 3476.73

% Changes	Changes Value	Different fromOri ginal Values	Ratio=Different Original Values	% Changes withSignificant
+50	4806.27	+1423.20	+0.4207	42.07%Up
+25	4094.67	+711.60	+0.2103	21.03%Up
-25	2671.47	-711.60	-0.2103	21.03%Down
-50	1959.87	-1423.20	-4207	42.07%Down

TCF2[s□ 2]□ 3476.73

% Changes	Changes Value	Different fromOri ginalVal ues	Ratio=Different Original Values	% Changes withSignificant
+50	4899.93	+1423.20	+0.4093	40.93%Up
+25	4188.33	+711.60	+0.2046	20.46%Up
-25	2765.13	-711.20	-0.2046	20.46%Down
-50	2053.53	-1423.20	-0.4093	40.93%Down

TCL1[c_1
□ 2]□ 2657.22

% Changes	ChangesValue	Differentfrom Original Values	Ratio=DifferentOriginal Values	% Changes withSignificant
+50	2616.97	-40.25	-0.01514	1.51%Down
+25	2637.09	-20.12	-0.00757	0.75%Down
-25	2677.34	+20.12	+0.00757	0.75% Up
-50	2697.46	+40.25	+0.01514	1.51%Up

TCL2[c_1□ 2]□ 3197.35

% Changes	ChangesValue	DifferentfromOriginal Values	Ratio=DifferentOriginal Values	% Changes withSignificant
+50	3196.95	-0.40	-0.000125	0.0125%Down
+25	3197.15	-0.20	-0.000063	0.0063%Down
-25	3197.55	+0.20	+0.000063	0.0063%Up
-50	3197.75	+0.40	+0.000125	0.0125%Up

TCF1[c□ 4]□ 3383.07

% Changes	Changes Value	DifferentfromOriginalValues	Ratio=Different/Oginal Values	% Changes withSignificant
+50	3378.78	-4.29	-0.00126	0.126%Down
+25	3380.93	-2.14	-0.00063	0.063%Down
-25	3385.21	+2.14	+0.00063	0.063%Up
-50	3387.36	+4.29	+0.00126	0.126%Up

TCF2[c□ 4]□ 3476.73

% Changes	ChangesValue	DifferentfromOriginal Values	Ratio=Different/Oginal Values	% Changes withSignificant
+50	3474.64	-2.09	-0.00060	0.06%Down
+25	3475.68	-1.05	-0.00030	0.03%Down
-25	3477.77	+1.05	+0.00030	0.03% Up
-50	3478.82	+2.09	+0.00060	0.06% Up

TCL1[c□ 4]□ 2657.22

% Changes	ChangesValue	DifferentfromOriginal Values	Ratio=Different/Oginal Values	% Changes withSignificant
+50	2723.04	+65.82	+0.02477	2.47% Up
+25	2690.13	+32.91	+0.01238	1.23% Up
-25	2629.31	-32.91	-0.01238	1.23%Down
-50	2591.39	-65.82	-0.02477	2.47%Down

TCL2[c□ 4]□ 3197.35

% Changes	Changes Value	Different fromOriginal Values	Ratio=Different/Oginal Values	% Changes withSignificant
+50	3194.11	-3.24	-0.0010	0.1% Up
+25	3195.53	-1.82	-0.0006	0.06% Up
-25	3198.36	+1.01	+0.0003	0.03%Down

-50	3199.78	+2.43	+0.0008	0.08%Down

TCF1[t_d□ 2]□ 3383.07

% Changes	ChangesValue	DifferentfromOriginal Values	Ratio=DifferentOriginal Values	% Changes withSignificant
+50	5291.76	+1908.69	+0.5642	56.42%Up
+25	4315.14	+932.07	+0.2754	27.54%Up
-25	2495.53	-887.54	-0.2621	26.21%Down
-50	1652.51	-1730.56	-0.5113	51.13%Down

TCF2[t_d□ 2]□ 3476.73

% Changes	ChangesValue	DifferentfromOriginal Values	Ratio=DifferentOriginal Values	% Changes withSignificant
+50	3576.45	+99.72	+0.0288	2.88% Up
+25	3526.59	+50.14	+0.0150	1.50% Up
-25	3426.86	-49.87	-0.0149	1.49%Down
-50	3376.98	-99.75	-0.0289	2.89%Down

TCL1[t_d□ 2]□ 2657.22

% Changes	ChangesValue	DifferentfromOriginal Values	Ratio=DifferentOriginal Values	% Changes withSignificant
+50	-6187.20	-8844.42	-3.328	332.8%Down
+25	934.05	-1723.17	-0.6484	64.84%Down
-25	3031.34	+374.12	+0.1407	14.07%Up
-50	3068.42	+411.20	+0.1546	15.46%Up

TCL2 [t_d□ 2]□ 3197.35

% Changes	ChangesValue	DifferentfromOriginal Values	Ratio=DifferentOriginal Values	% Changes withSignificant
+50	3340.44	+143.09	+0.0448	4.48% Up
+25	3270.05	+72.70	+0.0225	2.25% Up
-25	3123.48	-73.87	-0.0226	2.26%Down
-50	3049.02	-148.33	-0.0510	5.10%Down

TCF1[□ □0.03,□ □ 0.03,□ □ 0.1,□ □ 1]□ 3383.07

% Changes	ChangesValue	DifferentfromOriginal Values	Ratio=DifferentOriginal Values	% Changes withSignificant
+50	2571.20	-811.87	-0.2399	23.99%Down
+25	2898.13	-484.94	-0.1433	14.33%Down
-25	4182.43	+799.36	+0.2361	23.61%Up
-50	5765.16	+2382.03	+0.7041	70.41%Up

TCF2[□ □ 0.03,□ □ 0.03,□ □ 0.1,□1]□ 3476.73

% Changes	ChangesValue	Differentfrom OriginalValues	Ratio=DifferentOriginal Values	% Changes withSignificant
+50	2684.76	-791.97	-0.2277	22.77%Down
+25	3003.70	-473.03	-0.1360	13.60%Down
-25	4256.31	+799.58	+0.2298	22.98%Up
-50	5799.52	+2322.79	+0.6678	66.78%Up

TCL1[□ □ 0.03,□ □ 0.03,□ □ 0.1,□ □ 1]□ 2657

% Changes	ChangesValue	DifferentfromOriginalValues	Ratio=Different/Orinal Values	% Changes withSignificant
+50	2162.44	-494.78	-0.1862	18.62%Down
+25	2079.39	-577.83	-0.2171	21.71%Down
-25	3514.69	+857.47	+0.3225	32.25%Up
-50	5111.42	+2454.20	+0.9235	92.35%Up

TCL2[□□ 0.03,□ □ 0.03,□ □ 0.1,□ □ 1]□ 3197.35

% Changes	ChangesValue	Differentfrom Original Values	Ratio=Different/Orinal Values	% Changes withSignificant
+50	2346.31	-851.04	-0.2661	26.61%Down
+25	2692.54	-504.81	-0.1585	15.85%Down
-25	4008.58	+811.23	+0.2536	25.36%Up
-50	5582.23	+2384.88	+0.7456	74.56%Up

TCF1[D□ 100,W□ 200,A□ 100]□ 3383.07

% Changes	ChangesValue	Differentfrom OriginalValues	Ratio=Different/iginal Values	% Changes withSignificant
+50	5066.02	+1682.95	+0.4974	49.74%Up

+25	4224.55	+840.95	+0.2483	24.83%Up
-25	2541.59	-841.48	-0.2484	24.84%Down
-50	1700.12	-1682.95	-0.4974	49.74%Down

TCF2[D□ 100,W□ 200,A□ 100]□ 3476.73

%Changes	Changes Value	Different fromOriginal Values	Ratio=Different/iginal Values	% ChangeswithSignificant
+50	5234.68	+1757.95	+0.5053	50.53%Up
+25	4355.70	+878.97	+0.2525	25.25%Up
-25	2579.75	-896.98	-0.2610	26.10%Dow
-50	1718.78	-1757.95	-0.5053	50.53%Dow

TCL1[D□ 100,W□ 200,A□ 100]□ 2657.22

%Changes	ChangesValue	DifferentfromOriginal Values	Ratio=Different/iginal Values	% ChangeswithSignificant
+50	-9329.24	-11986.46	-4051	451%Dow
+25	-7758.22	-10414.44	-3.92	392%Dow
-25	-4616.17	-7273.39	-2.73	273%Dow
-50	-3045.15	-5702.37	-2.14	214%Dow

TCL2[D□ 100,W□ 200,A□ 100]□ 3197.35

% Changes	ChangesValue	DifferentfromOriginal Values	Ratio=Different/iginal Values	% Changes witSignificant
+50	5038.63	+1841.28	+0.5758	57.58%Up
+25	4189.54	+992.19	+0.3102	31.02%Up
-25	2491.34	-706.01	-0.2208	32.08%Down
-50	1642.24	-1555.11	-0.4863	48.63%Down

In TCF1, By changing the value of the parameters there is asignificant changesin optimum valuehappeningwith nochangestheTCF1takesamount3383.07%changesindifferent parameters makes effect less or more amount onTCF1givenintables.

Aschangingofparameter HinquarterlypercentilethechangesinTCF1 occurs2.430%inrightmanner.

Asparameter□ changesquarterlytheeffectonTCF113% inoppositetoright mannerfromitsoriginal value.

Asparameter□changesquarterlytheeffectonTCF1 changesatveryless0.0042%fromitsoriginalvalue.

Asparametersschangesquarterlypercentilethechangesin TCF1veryhighat21.03%fromitsoriginal value.

Asparameter$c_1$changesquarterlypercentilethechangesinTCF1changesat0.59%fromitsoriginalvalues.

Asparametercchangesquarterlypercentilethechangesin TCF1changesat0.063%fromitsoriginal values.

As parameter $t_d$changes quarterly percentile in changes

inTCF1changeshighlyinrightwayat27.54%fromitsoriginalvalues.
Asparameter changesquarterlypercentilethechangesin
TCF1changes0.1 %fromitsoriginal value.
Asparameter changesquarterlypercentilethechangesin
TCF1changesverylessat0.07%reverseorderfromitsoriginalvalues.
As parameterD changes quarterly percentile the changes inTCF1changes20.4%fromits originalvalues.

As parameter W changes quarterly percentile the changes inTCF1 changes at 3.7 % from its original value in right order.As parameterA changes quarterly percentile the changes inTCF1 changes at 0.8 % from its originalvalues in rightorder.
As parameter F changes quarterly percentile the changes inTCF1changesat24.3%inrightorderfromitsoriginal
AsparameterD ,W,A changesquarterlypercentilesimultaneously the changes in TCF2 changes at25.25%fromitsoriginalvaluesinrightorder.
In TCL1, By changing the value of the parameters there is asignificant changesin optimum valuehappeningwith nochangestheTCL1takesamount2657.22%changesin
different parameters makes effect less or more amount onTCL1givenintables.

value.
Asparameter ,
 and

changesquarterlypercentile
AschangingofparameterHinquarterlypercentilethechangesinTCL1 occurs2.84 %inreversemanner.

simultaneouslythechangesinTCF1isat14.33%fromits
originalvaluesinreverseorderfromitsoriginalvalue.
AsparameterD ,W,A changesquarterlypercentilesimultaneously the changes in TCF1 changes at 24.83 %fromitsoriginalvaluesinrightorder.
In TCF2, By changing the value of the parameters there is asignificant changesin optimum valuehappeningwith nochangestheTCF2takesamount3476.73%changesindifferent parameters makes effect less or more amount onTCF2givenintables.
AschangingofparameterHinquarterlypercentilethe
Asparameter changes quarterlytheeffectonTCL118.04%inoppositetoright mannerfromitsoriginal valuein reverseorder.
Asparameter changesquarterlytheeffectonTCL1
changesatveryless2.96%fromitsoriginalvalueinreverseorder.
Asparameterschangesquarterlypercentilethechangesin
TCL1veryhighat21.03%fromitsoriginalvalue.
Asparameter$c_1$changesquarterlypercentilethechangesinTCL1changesat0.75%fromitsoriginalvaluesinreverse

changesinTCF2occurs2.430%inright manner.
Asparameter changesquarterlytheeffectonTCF2
order.
Asparameter
cchangesquarterlypercentilethechangesin

changesat3.22%inoppositetorightmannerfromitsoriginalvalue.
 changesquarterlytheeffectonTCF2no
changesfromitsoriginalvalue.
Asparameterschangesquarterlypercentilethechangesin
TCF2 changes at very high 22.25% from its original value.Asparameter$c_1$changesquarterlypercentilethechangesin
TCF2changesat0.58 %fromitsoriginalvalues.
Asparametercchangesquarterlypercentilethechangesin
TCF2changes0.03%inreverseorderfrom itsoriginalvalue.
Asparameter$t_d$changesquarterlypercentileinchangesin
TCL1 changes 1.23% in reverse order from its original valueinrightmanner.
Asparameter$t_d$changesquarterlypercentileinchangesin
TCL1changeshighlyinrightwayat64.84%fromitsoriginalvaluesinreverse order.
Asparameter changesquarterlypercentilethechangesin

TCL1 changes 1% from its original value in reverse order.Asparameter changesquarterlypercentilethechangesin
TCL1changesverylessat0.22%reverseorderfromitsoriginalvaluesinrightmanner.
AsparameterDchangesquarterlypercentilethechangesinTCL1changes20.3%fromitsoriginalvaluesinright
TCF2changeshighly in rightway at 1.5%fromitsoriginalvalues.
manner.
Asparameter
wchangesquarterlypercentilethechangesin

Asparameter changesquarterlypercentilethechangesinTCF2changes0.09 %fromitsoriginalvalue.
Asparameter changesquarterlypercentilethechangesin
TCF2changesverylessat0.09%reverseorderfromitsoriginalvalues.
AsparameterDchangesquarterlypercentilethechangesinTCF2changes19.90 %fromitsoriginalvalues.
Asparameterwchangesquarterlypercentilethechangesin
TCF2 changes at 4.7 % from its original value in right order.As parameterA changes quarterly percentile the changes inTCF2 changes at 0.7 % from its originalvalues in rightorder.
As parameter F changes quarterly percentile the changes inTCF2changesat 1.2%inrightorder.
Asparameter , , and changes quarterly
percentile simultaneouslythechangesin TCF2isat13.60%fromitsoriginalvaluesinreverse order.
TCL1changesat2.9%fromitsoriginalvalueinreverse
order.
As parameterA changes quarterly percentile the changes inTCL1 changes at 1.0 % from its original values in rightorder.
As parameter F changes quarterly percentile the changes inTCL1 changes at 0.003 % in reverse order from its originalvalue.
Asparameter , , and changes quarterly
percentile simultaneously the changes in TCL1 is at 21.71 %from its original values in reverse order from its originalvalue.
AsparameterD ,W,A changesquarterlypercentilesimultaneously the changes in TCL1 changes at 392 % fromitsoriginal valuesinverse order.
In TCL2, By changing the value of the parameters there is asignificant changesin optimum valuehappeningwith nochangestheTCL2takesamount3197.35%changesin

different parameters makes effect less or more amount onTCL2givenintables.
AschangingofparameterHinquarterlypercentilethechangesinTCL2occurs2.14 %inrightmanner.
Asparameter changesquarterly theeffectonTCL2 15.95
% in opposite to right manner from its original value inreverseorder.
Asparameter changesquarterlytheeffectonTCL2
changesatveryless0.015%fromitsoriginalvalueinreverseorder.
Asparameterschangesquarterlypercentilethechangesin
TCL2veryhighat20.46%fromitsoriginalvalue.
Asparameter$c_1$changesquarterlypercentilethechangesin
TCL2changesat0.0063%fromitsoriginalvaluesinreverseorder.
Asparametercchangesquarterlypercentilethechangesin
TCL2 changes 0.06% in reverse order from its original valueinreversemanner.
Asparameter$t_d$changesquarterlypercentileinchangesin
TCL2changeshighlyinrightwayat2.25%fromitsoriginalvaluesinrightorder.
Asparameter changesquarterlypercentilethechangesin
TCL2 changes 0.41 %from its original value in right order.Asparameter changesquarterlypercentilethechangesin
TCL2nochangesfromitsoriginal values inrightmanner.AsparameterDchangesquarterlypercentilethechangesinTCL2changes26.1%fromitsoriginalvaluesinrightmanner.
Asparameterwchangesquarterlypercentilethechangesin
TCL2 changes at 2.4 % from its original value in right order.As parameterA changes quarterly percentile the changes inTCL2 changes at 0.78 % from its original values in rightorder.
As parameter F changes quarterly percentile the changes inTCL2 changes at 0.13 % in reverse order from its originalvalue.

Asparameter□ ,□,□ and□changes quarterly percentile simultaneously the changes in TCL2 is at 25.36 %from its original values in reverse order from its originalvalue.

AsparameterD ,W,A changesquarterlypercentilesimultaneously the changes in TCL2 changes at 32.08 %fromitsoriginalvaluesinrightorder.

CONCLUSION

In this study, a two-warehouse inventory model for non-instantaneous deteriorating items with exponential demandrate under time varying holding cost has been developed.Shortagesareallowedandcompletelybacklogged.Thedeterioration rate in OW is assumed to be higher than in RWandtheholdingcostinRWisgreaterthanthatinOWbecause of the difference in the storage facilities of bothwarehouse.Whileformulatingtheinventorymodelfordeteriorating items, usually it is assumed that the items startdeterioratingassoonastheyenterintothewarehouse.However,therearenumerousproductslikefooditems(dry fruits,foodgrainsetc.),electronicitems(refrigerator,television, etc.), that have a shelf-life and start deterioratingafteratimelagthatistermedasnon-instantaneousdeterioration.Moreover,thepricediscount,lowcoststorage, huge demand etc. and under such a situation onemay decide to procure large quantity of the items whichwould arise the problem of storing. As the capacity of ownwarehouseislimited,thereforeonehastohireanotherrentedwarehouseto storetheexcessquantity.

The proposed model can further be extended by includingsomemorerealisticfeatures,suchasinventory-level-dependent demand, price-dependent demand, inflation andpermissibledelayinpaymentsetc.

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