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Fuzzy Retrial Queues with Priority using DSW Algorithm

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ABSTRACT

In this paper we study the priority queueing model under fuzzy environment.It optimize a fuzzy priority queueing model (preemptive priority, non-preemptive priority) in which arrival rate ,service rate,retrial rate are fuzzy numbers.Approximate method of Extension namely DSW (Dong, Shah and Wong) algorithm is used to define membership functions of the performance measures of priority queuing system . DSW algorithm is based on the α cut representation of fuzzy sets in a standard interval analysis. Numerical example is also illustrated to check the validity of the model.

Keywords: *Fuzzy set theory, Retrial queue,Priority discipline, DSW Algorithm.*

2010 Mathematics Subject Classification: 60K25, 03E72

I. INTRODUCTION

Queueing theory is a branch of applied probability theory. A queue is a waiting line of customers which demands service from a service station and it is formed when service is not provided immediately. Queueing theory was introduced by A.K Erlang [1]. The main purpose of the analysis of queueing systems is to understand the behavior of their underlying processes so that informed and intelligent decisions can be made in their organization. Most of the queueing model were studied with queueing discipline " First Come First Serve". However, situations commonly occur that an arriving customer may be distinguished according to some measure of importance. The discipline according to which the server selects the next unit and serves is known as priority discipline. When a new patient arrives into a large hospital, the severity of his problem is considered. According to this severity he is allowed into a priority queue although already there is an usual queue.Thus priority queueing model is an essential one in practical life.In priority discipline, higher priority customers are selected for service ahead of those with lower priority, regardless of their arrival into the system. If the server is free at the time of a primary arrival, the arriving customer begins to be served immediately and customer leaves the system after the service completion. If the server is busy, then the low priority customer goes to orbit and becomes a source of repeated customers. Customers from the orbit will retry to get service after some random time. If there is no higher class customers in
the queue, then only the retrial customers will be served by server.

In priority discipline there are two cases raised: (i) preemptive priority discipline (ii)Non-preemptive priority discipline. In preemptive cases the customer with the highest priority is allowed to enter service immediately even another with lower priority is already present in service when the higher customer arrives to the system.The priority discipline is said to be non-preemptive if there is no interruption and the highest priority customer just goes to the head of the queue to wait for his turn.

Aissani , Artalejo [2] analyzed the single server retrial queue subject to breakdowns.Retrial queues with breakdown and repair was investigated by Kulkarni ,Choi[3].M/G/1/r retrial queueing system with priority of primary customers is discussed by Bocharov et al[7]. Drekic and Woolford [5] studied preemptive priority queue with balking. Retrial queues and priority were discussed in detail by Falin,and Templeton[8].Fundamentals of queueing theoey was described by Gross and Harris[4]. Queueing systems analyzed in more depth by Leonard Kleinrock [9]. Nathan P.Sherman Jekrey Kharoufeh [6] have studied retrial queue with unreliable server. Shanthakumaran and Shanmugasundaram [10] described retrial Queue with Feedback on Non-Retrial Customers. In practical,the input data such as arrival rate, service rate and retrial are uncertainly known. Uncertainty is resolved by using fuzzy set theory.Hence the classical queuing model will have more application if it is expressed using fuzzy models.Fuzzy Logic was initiated in 1965 by Zadeh [11] Fuzzy queuing models have been described by such researchers like Li and Lee [12], Buckley [16], Negi and Lee [15] are analyzed fuzzy queues using Zadeh's extension principle . Kao[13] et al constructed the membership functions of the system characteristic for fuzzy queues using parametric linear programming.

Application of fuzzy logic was analyzed by Klir 17. The theory of fuzzy subset is introduced by Kaufmann[18]. Zimmermann[19] developed fuzzy set theory and applications. Multi-server fuzzy queue using DSW algorithm was discussed by Shanmugasundaram.S and Venkatesh.B [20].Recently cost analysis of priority queue investigated by Ritha W, and Lilly Robert [21].

II. DESCRIPTION OF THE MODEL

We consider a priority queueing system with single server, infinite calling population with arrival rate λ ,service rate γ and retrial rate θ . The objective of studying queueing model is to reduce the waiting time of customers in queue and also cost of the system.Here cost of the system represents long run average cost per unit time such as cost of waiting space, consumption cost of system's facility, cost of insurance, etc. To establish the priority discipline fuzzy queueing model, we must compare the average total cost of the system for the three cases. No priority discipline,preemptive priority and non-preemptive priority discipline which are denoted by C,C'and C" respectively.

III. CRISP RESULTS

No Priority Retrial Queueing Model Average Total Cost Of The System When There Is No Priority Discipline, C.

$$C = (C_1\lambda_1 + C_2\lambda_2)W$$

, Where

$$W = \left(\frac{\lambda + \theta}{(\mu - \lambda)\theta}\right)$$

(B) Preemptive Priority Retrial Queueing Model Average total cost of the system when there is Preemption priority, C'.

$$C' = C_1T_1 + C_2T_2$$

where

$$T_1 = \frac{\rho_1^2(1 + \rho_2 - \rho)}{(1 - \rho_1)^2}$$

and

$$T_2 = \rho_2 + \frac{\rho_1}{1 - \rho_1}\left(\rho_2 + \frac{\lambda_2}{\theta}\right) + \frac{\rho_2}{(1 - \rho)(1 - \rho_1)}\left[\rho_2 + \frac{\rho\rho_2}{(1 - \rho_1)}\right] + \frac{3\rho\rho_2}{2(1 - \rho_1)^2}$$

(c) Non-Preemptive Priority Retrial Queueing Model Average Total Cost Of System When There Is Non Preemptive Priority, C"

$$C'' = C_1L_1 + C_2L_2$$

where $L_1 = \frac{\rho\rho_1}{1 - \rho_1} + \rho_1$ and $L_2 = \frac{\rho\rho_2}{(1 - \rho_1)(1 - \rho)} + \frac{\lambda_2\rho}{\theta(1 - \rho)} + \rho_2$

Comparison of the three total costs shows which of priority discipline minimizes the average total cost function of the system.

IV. FUZZY RETRIAL QUEUES WITH PRIORITY DISCIPLINE

Fuzzy retrial queues with priority discipline are described by fuzzy set theory. This paper develops fuzzy retrial queueing models with priority discipline in which the input source arrival rate, service rate and retrial rate are uncertain parameters. Approximate methods of extension are propagating fuzziness for continuous valued mapping determined the membership functions for the output variable.DSW algorithm [14] is one of the approximate methods which makes use of intervals at various a-cut levels in defining membership functions. It was the full a-cut intervals in a standard interval analylsis. The DSW algorithm greatly simplifies manipulation of the extension principle for continuous valued fuzzy variables, such as fuzzy numbers defined on the real line.

V. INTERVAL ANALYSIS ARITHMETIC

Let I_1 and I_2 be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds. Define a general arithmetic property with the symbol *, where $* = [+, -, \times, \div]$ symbolically the operation.
I1 * I2 = [a, b] * [c, d]represents another interval. The interval calculation depends on the magnitudes and signs

of the elements a, b, c and d.

$[a,b]+[c,d]=[a+c,b+d]$

$[a,b]-[c,d]=[a-d,b-c]$

$[a,b]\times[c,d]=[min\ (ac,ad,bc,bd),\ max\ (ac,ad,bc,bd)]$

$[a,b]\div[c,d]=[a,b]\times[\frac{1}{d},\frac{1}{c})$

where ac, ad, bc, bd are arithmetic products and $\frac{1}{d}$ and $\frac{1}{c}$ are quotients.

VI. DSW ALGORITHM

Any continuous membership function can be represented by a continuous sweep of α -cut interm from $\alpha = 0$ to $\alpha = 1$. It uses the full a-cut intervals in a standard interval analysis. The DSW algorithm [14] consists of the following steps: (i) Select a α -cut value where $0 \le \alpha \le 1$.

(ii) Find the intervals in the input membership functions that correspond to this α .

(iii) Using standard binary interval operations, compute the interval for the output membership function for the selected α -cut level.

(iv) Repeat steps (i) to (iii) for different values of α to complete a α -cut representation of the solution.

VII. SOLUTION PROCEDURE

Decisions relating the priority discipline for a retrial queueing system are mainly based on a cost function.

$$C=\sum_{i=1}^{n} C_i L_i$$

where C_i is the unit cost of system for units in class i and L_i is the average length in the system for unit of class i.

Let us consider a retrial queueing model with two unit classes arrive at α_1 of arrivals belong to one of the classes, and α_2 are in the other class. The average arrival rate at the system follows a Poisson process, and is approximately known and is given by the trapezoidal fuzzy number $\tilde{\lambda}$. The service rate from a single server is the same for both unit classes follows an exponential pattern and is distributed according to the trapezoidal fuzzy number $\tilde{\gamma}$ and the retrial of the low priority customers follows an exponential pattern and is given by the trapezoidal fuzzy number $\tilde{\theta}$.

The membership function of arrival rate, service rate and retrial rate, are denoted as $\mu_{\tilde{\lambda}}, \mu_{\tilde{\gamma}}, \mu_{\tilde{\theta}}$ respectively. Then we have the following fuzzy sets.

$$\tilde{\lambda}=\{x,\mu_{\tilde{\lambda}}(x), x\varepsilon X\}$$

$$\tilde{\gamma}=\{s,\mu_{\tilde{\gamma}}(s), s\varepsilon S\}$$

$$\tilde{\theta}=\{r,\mu_{\tilde{\theta}}(r), r\varepsilon R\}$$

where X,Y,R are crisp universal sets of arrival rate,service rate,retrial rate respectively.

The membership function of arrival rate,service rate,retrial rate are given as follows. $\mu_{\tilde{\lambda}}(x)=$

$$\begin{cases} \dfrac{x - a_1}{b_1 - a_1}, & if \quad a_1 \le x \le b_1 \;; \\ 1, & if \;\; b_1 \le x \le c_1 \;; \\ \dfrac{d_1 - x}{d_1 - c_1}, & if \;\; c_1 \le x \le d_1 \;. \end{cases}$$

$$\mu_{\tilde{\gamma}}(s) = \begin{cases} \dfrac{s - a_2}{b_2 - a_2}, & if \quad a_2 \le s \le b_2 \;; \\ 1, & if \;\; b_2 \le s \le c_2 \;; \\ \dfrac{d_2 - s}{d_2 - c_2}, & if \;\; c_2 \le s \le d_2 \;. \end{cases}$$

$$\mu_{\tilde{\theta}}(r) = \begin{cases} \dfrac{r - a_3}{b_3 - a_3}, & if \quad a_3 \le r \le b_3 \;; \\ 1, & if \;\; b_3 \le r \le c_3 \;; \\ \dfrac{d_3 - r}{d_3 - c_3}, & if \;\; c_3 \le r \le d_3 \;. \end{cases}$$

The possible distribution of unit cost of the system for unit in the same class is established by a trapezoidal fuzzy number $\tilde{C}_A, \tilde{C}_B$ with membership function.

$$\mu_{\tilde{c}_A} = \begin{cases} \dfrac{C_A - a_4}{b_4 - a_4}, & if \quad a_4 \le C_A \le b_4 \;; \\ 1, & if \;\; b_4 \le C_A \le c_4 \;; \\ \dfrac{d_4 - C_A}{d_4 - c_4}, & if \;\; c_4 \le C_A \le d_4 \;. \end{cases}$$

$$\mu_{\tilde{c}_B}(x) = \begin{cases} \dfrac{C_B - a_5}{b_5 - a_5}, & if \quad a_5 \le C_B \le b_5 \;; \\ 1, & if \;\; b_5 \le C_B \le c_5 \;; \\ \dfrac{d_5 - C_B}{d_5 - c_5}, & if \;\; c_5 \le C_B \le d_5 \;. \end{cases}$$

we choose three values of α viz, 0, 0.5 and 1. For instance when α = 0, we obtain 5 intervals as follows. $\tilde{\lambda}_0 = [a_1, d_1]; \tilde{\gamma}_0 = [a_2, d_2]; \tilde{\theta}_0 = [a_3, d_3]; \tilde{C}_{A,0} = [a_4, d_4]; \tilde{C}_{B,0} = [a_5, d_5]$ Similarly when, α = 0.5, 1. we obtain 10 intervals and it is denoted by $\tilde{\lambda}_{0.5}, \tilde{\gamma}_{0.5}, \tilde{\theta}_{0.5}, \tilde{C}_{A,0.5}, \tilde{C}_{B,0.5}, \tilde{\lambda}_1, \tilde{\gamma}_1, \tilde{\theta}_1, \tilde{C}_{A,1}, \tilde{C}_{B,1}$. The average total cost of the system in three situation (i) No priority discipline (ii) Preemptive priority discipline (ii) Non-preemptive priority discipline are calculated for different α level values. Interval arithmetic is used for computational efficiency.

(i) Average total cost of the system when there is no priority discipline.

$$\tilde{C}_0 = [\tilde{C}_{A,0}\tilde{\lambda}_{1,0} + \tilde{C}_{B,0}\tilde{\lambda}_{2,0}][\frac{\tilde{\lambda}_0 + \tilde{\theta}_0}{(\tilde{\gamma}_0 - \tilde{\lambda}_0)\tilde{\theta}_0}]$$

$$\tilde{C}_{0.5} = [\tilde{C}_{A,0.5}\tilde{\lambda}_{1,0.5} + \tilde{C}_{B,0.5}\tilde{\lambda}_{2,0.5}][\frac{\tilde{\lambda}_{0.5} + \tilde{\theta}_{0.5}}{(\tilde{\gamma}_{0.5} - \tilde{\lambda}_{0.5})\tilde{\theta}_{0.5}}]$$

$$\tilde{C}_1 = [\tilde{C}_{A,1}\tilde{\lambda}_{1,1} + \tilde{C}_{B,1}\tilde{\lambda}_{2,1}][\frac{\tilde{\lambda}_1 + \tilde{\theta}_1}{(\tilde{\gamma}_1 - \tilde{\lambda}_1)\tilde{\theta}_1}]$$

(ii) Average total cost of the system when there is preemptive discipline.

$$\tilde{C}_0' = \tilde{C}_{A,0}\frac{\frac{\tilde{\lambda}_{1,0}^2}{\gamma_0^2}(1+\frac{\tilde{\lambda}_{2,0}}{\tilde{\gamma}_0}-\frac{\tilde{\lambda}_0}{\tilde{\gamma}_0})}{(1-\frac{\tilde{\lambda}_{1,0}}{\tilde{\gamma}_0})^2} + \tilde{C}_{B,0}[\frac{\tilde{\lambda}_{2,0}}{\tilde{\theta}_0} + \frac{\frac{\tilde{\lambda}_{1,0}}{\tilde{\gamma}_0}[\frac{\tilde{\lambda}_{2,0}}{\tilde{\gamma}_0}+\frac{\tilde{\lambda}_{2,0}}{\tilde{\theta}_0}]}{[1-\frac{\tilde{\lambda}_{1,0}}{\tilde{\gamma}_0}]} + \frac{\frac{\tilde{\lambda}_{2,0}}{\tilde{\gamma}_0}}{(1-\frac{\tilde{\lambda}_0}{\tilde{\gamma}_0})(1-\frac{\tilde{\lambda}_{1,0}}{\tilde{\gamma}_0})}[\frac{\tilde{\lambda}_{2,0}}{\tilde{\theta}_0} + \frac{(\frac{\tilde{\lambda}_{2,0}}{\tilde{\gamma}_0})(\frac{\tilde{\lambda}_0}{\tilde{\gamma}_0})}{(1-\frac{\tilde{\lambda}_{1,0}}{\tilde{\gamma}_0})}] + \frac{3(\frac{\tilde{\lambda}_0}{\tilde{\gamma}_0})(\frac{\tilde{\lambda}_{2,0}}{\tilde{\gamma}_0})}{2[1-\frac{\tilde{\lambda}_{1,0}}{\tilde{\gamma}_0}]^2}]$$

$$\tilde{C}_{0.5}' = \tilde{C}_{A,0.5}\frac{\frac{\tilde{\lambda}_{1,0.5}^2}{\gamma_{0.5}^2}(1+\frac{\tilde{\lambda}_{2,0.5}}{\tilde{\gamma}_{0.5}}-\frac{\tilde{\lambda}_{0.5}}{\tilde{\gamma}_{0.5}})}{(1-\frac{\tilde{\lambda}_{1,0.5}}{\tilde{\gamma}_{0.5}})^2} + \tilde{C}_{B,0.5}[\frac{\tilde{\lambda}_{2,0.5}}{\tilde{\theta}_{0.5}} + \frac{\frac{\tilde{\lambda}_{1,0.5}}{\tilde{\gamma}_{0.5}}[\frac{\tilde{\lambda}_{2,0.5}}{\tilde{\gamma}_{0.5}}+\frac{\tilde{\lambda}_{2,0.5}}{\tilde{\theta}_{0.5}}]}{[1-\frac{\tilde{\lambda}_{1,0.5}}{\tilde{\gamma}_{0.5}}]} + \frac{\frac{\tilde{\lambda}_{2,0.5}}{\tilde{\gamma}_{0.5}}}{(1-\frac{\tilde{\lambda}_{0.5}}{\tilde{\gamma}_{0.5}})(1-\frac{\tilde{\lambda}_{1,0.5}}{\tilde{\gamma}_{0.5}})}[\frac{\tilde{\lambda}_{2,0.5}}{\tilde{\theta}_{0.5}} + \frac{(\frac{\tilde{\lambda}_{2,0.5}}{\tilde{\gamma}_{0.5}})(\frac{\tilde{\lambda}_{0.5}}{\tilde{\gamma}_{0.5}})}{(1-\frac{\tilde{\lambda}_{1,0}}{\tilde{\gamma}_0})}] + \frac{3(\frac{\tilde{\lambda}_0}{\tilde{\gamma}_{0.5}})(\frac{\tilde{\lambda}_{2,0.5}}{\tilde{\gamma}_{0.5}})}{2[1-\frac{\tilde{\lambda}_{1,0.5}}{\tilde{\gamma}_{0.5}}]^2}]$$

$$\tilde{C}_1' = \tilde{C}_{A,1}\frac{\frac{\tilde{\lambda}_{1,1}^2}{\gamma_1^2}(1+\frac{\tilde{\lambda}_{2,1}}{\tilde{\gamma}_1}-\frac{\tilde{\lambda}_1}{\tilde{\gamma}_1})}{(1-\frac{\tilde{\lambda}_{1,1}}{\tilde{\gamma}_1})^2} + \tilde{C}_{B,1}[\frac{\tilde{\lambda}_{2,1}}{\tilde{\theta}_1} + \frac{\frac{\tilde{\lambda}_{1,1}}{\tilde{\gamma}_1}[\frac{\tilde{\lambda}_{2,1}}{\tilde{\gamma}_1}+\frac{\tilde{\lambda}_{2,1}}{\tilde{\theta}_1}]}{[1-\frac{\tilde{\lambda}_{1,1}}{\tilde{\gamma}_1}]} + \frac{\frac{\tilde{\lambda}_{2,1}}{\tilde{\gamma}_1}}{(1-\frac{\tilde{\lambda}_1}{\tilde{\gamma}_1})(1-\frac{\tilde{\lambda}_{1,1}}{\tilde{\gamma}_1})}[\frac{\tilde{\lambda}_{2,1}}{\tilde{\theta}_1} + \frac{(\frac{\tilde{\lambda}_{2,1}}{\tilde{\gamma}_1})(\frac{\tilde{\lambda}_1}{\tilde{\gamma}_1})}{(1-\frac{\tilde{\lambda}_{1,1}}{\tilde{\gamma}_1})}] + \frac{3(\frac{\tilde{\lambda}_1}{\tilde{\gamma}_1})(\frac{\tilde{\lambda}_{2,1}}{\tilde{\gamma}_1})}{2[1-\frac{\tilde{\lambda}_{1,1}}{\tilde{\gamma}_1}]^2}]$$

$$\tilde{C}_0'' = \tilde{C}_{A,0}[\frac{\frac{\tilde{\lambda}_{1,0}}{\tilde{\gamma}_0}\frac{\tilde{\lambda}_0}{\tilde{\gamma}_0}}{[1-\frac{\tilde{\lambda}_{1,0}}{\tilde{\gamma}_0}]} + \frac{\tilde{\lambda}_{1,0}}{\tilde{\gamma}_0}] + \tilde{C}_{B,0}[\frac{(\frac{\tilde{\lambda}_{2,0}}{\tilde{\gamma}_0})(\frac{\tilde{\lambda}_0}{\tilde{\gamma}_0})}{[1-\frac{\tilde{\lambda}_{1,0}}{\tilde{\gamma}_0}][1-\frac{\tilde{\lambda}_0}{\tilde{\gamma}_0}]} + (\frac{\tilde{\lambda}_{2,0}}{\tilde{\theta}_0})\frac{(\frac{\tilde{\lambda}_0}{\tilde{\gamma}_0})}{[1-\frac{\tilde{\lambda}_0}{\tilde{\gamma}_0}]} + (\frac{\tilde{\lambda}_{2,0}}{\tilde{\gamma}_0})]$$

$$\tilde{C}_{0.5}'' = \tilde{C}_{A,0.5}[\frac{\frac{\tilde{\lambda}_{1,0.5}}{\tilde{\gamma}_{0.5}}\frac{\tilde{\lambda}_{0.5}}{\tilde{\gamma}_{0.5}}}{[1-\frac{\tilde{\lambda}_{1,0.5}}{\tilde{\gamma}_{0.5}}]} + \frac{\tilde{\lambda}_{1,0.5}}{\tilde{\gamma}_{0.5}}] + \tilde{C}_{B,0.5}[\frac{(\frac{\tilde{\lambda}_{2,0.5}}{\tilde{\gamma}_{0.5}})(\frac{\tilde{\lambda}_{0.5}}{\tilde{\gamma}_{0.5}})}{[1-\frac{\tilde{\lambda}_{1,0.5}}{\tilde{\gamma}_{0.5}}][1-\frac{\tilde{\lambda}_{0.5}}{\tilde{\gamma}_{0.5}}]} + (\frac{\tilde{\lambda}_{2,0.5}}{\tilde{\theta}_{0.5}})\frac{(\frac{\tilde{\lambda}_{0.5}}{\tilde{\gamma}_{0.5}})}{[1-\frac{\tilde{\lambda}_{0.5}}{\tilde{\gamma}_{0.5}}]} + (\frac{\tilde{\lambda}_{2,0.5}}{\tilde{\gamma}_{0.5}})]$$

$$\tilde{C}_1'' = \tilde{C}_{A,1}[\frac{\frac{\tilde{\lambda}_{1,1}}{\tilde{\gamma}_1}\frac{\tilde{\lambda}_1}{\tilde{\gamma}_1}}{[1-\frac{\tilde{\lambda}_{1,1}}{\tilde{\gamma}_1}]} + \frac{\tilde{\lambda}_{1,1}}{\tilde{\gamma}_1}] + \tilde{C}_{B,1}[\frac{(\frac{\tilde{\lambda}_{2,1}}{\tilde{\gamma}_1})(\frac{\tilde{\lambda}_1}{\tilde{\gamma}_1})}{[1-\frac{\tilde{\lambda}_{1,1}}{\tilde{\gamma}_1}][1-\frac{\tilde{\lambda}_1}{\tilde{\gamma}_1}]} + (\frac{\tilde{\lambda}_{2,1}}{\tilde{\theta}_1})\frac{(\frac{\tilde{\lambda}_1}{\tilde{\gamma}_1})}{[1-\frac{\tilde{\lambda}_1}{\tilde{\gamma}_1}]} + (\frac{\tilde{\lambda}_{2,1}}{\tilde{\gamma}_1})]$$

VIII. NUMERICAL EXAMPLE

Consider a telephone switching system in which calls arrive in two classes. With utilization of 15 % and 85 % calls arrive at this system in accordance with a poisson process, the service times and retrial times follow an exponential distribution. The arrival rate,service rate and retrial rate are trapezoidal fuzzy numbers given by $\lambda = [26\ \ 30\ \ 32\ \ 34]$, $\gamma = [38\ \ 40\ \ 42\ \ 44]$ and $\theta = [22\ \ 24\ \ 26\ \ 28]$ per minute respectively. The possibility distribution of unit cost of inactivity of two classes are trapezoidal fuzzy number C_A =[15 17 19 20], C_B =[2 3 5 6] respectively. The system manager The system manager wants to evaluate the total cost of the system when there is no priority discipline,preemptive priority discipline, non-preemptive priority discipline in the retrial queue.

No Priority discipline:

: $C_0 = [2.329,\ \ 87.35]$, $C_{0.5} = [3.4227,\ \ 12.0433]$, $C_1 = [10.593,\ \ 42.8511]$

Preemptive Priority discipline:

$C_0' = [9.358,\ \ 192.945]$, $C_{0.5}' = [15.50,\ \ 108.22]$, $C_1' = [26.438,\ \ 65.888]$

Non-Preemptive Priority discipline:

$C_0'' = [(5.236,\ \ 117.857]$, $C_{0.5}'' = [4.46,\ \ 77.26]$, $C_1'' = [12.82,\ \ 35.10]$

IX. CONCLUSION

Comparison of the three total costs shows which of the priority disciplines minimizes the average total cost function of system. Even though they are overlapping fuzzy numbers,so minimum average total cost of system is achieved with the non preemptive priority discipline. The method proposed enables reasonable solution for each case,with different level of possibility. This approach provides more information to help design fuzzy priority discipline queuing system.

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