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TYPE-2 FUZZY LINEAR PROGRAMMING PROBLEMS WITH PERFECTLY NORMAL INTERVAL TYPE-2 RESOURCE

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ABSTRACT

In this paper, the Perfectly normal Interval Type-2 Fuzzy Linear Programming (PnIT2FLP) model is considered. This model is reduced to crisp linear programming model. This transformation is performed by a proposed ranking method. Based on the proposed fuzzy ranking method and arithmetic operation, the solution of Perfectly normal Interval Type-2 Fuzzy Linear Programming model is obtained by the solutions of linear programming model with help of MATLAB. Finally, the method is illustrated by numerical examples.

Keywords: *Fuzzy sets, Normal, Type-2 fuzzy sets, Interval type-2 fuzzy sets, Type-2 trapezoidal fuzzy numbers, Linear programming, Fuzzy linear programming.*

I. Introduction

Mathematical model is a technique for determining an approach to achieve the best outcome with specific objective(s) (objective function(s)) from the list of requirements (constraints) which are represented by mathematical relations or equations. Mathematical model plays an important role in decision-making. Depending on the nature of equation involved in the problem, a mathematical model is called a linear or nonlinear programming problem. Linear Programming Problem is a special type of Mathematical model in which objective function(s) is linear and the constraints are linear equalities or linear inequalities or combination of both. It can be applied successfully when the objective function and constraints are in deterministic and well defined. But unfortunately, objective function and constraints are in non-deterministic and uncertain in nature. There exist various types of uncertainties in nature, such as randomness of occurrence of events, imprecision and ambiguity of the data and linguistic vagueness, etc., These types of uncertainties are categorized as stochastic uncertainty and fuzziness by Zimmerman[1]

The Linear Programming problems with stochastic uncertainty can be modeled and solved by stochastic mathematical programming techniques. The Stochastic programming (Probabilistic programming) deals with circumstances where some or all of the parameters of the optimization problem are described by stochastic (or probabilistic or random) variables rather than by deterministic quantities. It were developed and solved by Beale[2], Dantzig[3], Charnes[4]. Depending on the nature of equation involved (in terms of random variables) in the problem, a stochastic optimization problem is called a stochastic linear or nonlinear programming problem. And the Linear Programming problems with fuzziness can be modeled and solved by fuzzy mathematical programming techniques. The Fuzzy mathematical programming is effective when the information is vague or imprecise/ambiguous. It can be classified by Inuiguchi[5] as follows:

- Fuzzy mathematical programming with vagueness,
- Fuzzy mathematical programming with ambiguity,
- Fuzzy mathematical programming with vagueness and ambiguity.

The first category was initially developed by Bellman and Zadeh [6]. It deals with the flexibility on objective functions and constraints. These types of fuzzy mathematical programming are also called flexible programming. The second category treats ambiguous coefficient of objective functions and constraints. Dubois and Prade[7] have suggested a method for solving the system of linear equations with ambiguous co-efficient using fuzzy mathematical programming techniques. Since then, many approaches to such kinds of problems were developed. Dubois and Prade[8] introduced four inequality indices between fuzzy numbers using the notion of fuzzy coefficient by applying possibility theory. Hence, this type of fuzzy mathematical programming is usually called, the possibilistic programming.

The third type of fuzzy mathematical programming is the combination of vagueness and ambiguity. This type was first formulated by Negoita et al.[9]. In this type, the vague decision maker's preference is represented by a fuzzy satisfactory region and a fuzzy function value is required to include in the given fuzzy satisfactory region. This type of fuzzy mathematical programming is also called robust programming. Generally, a linear programming problem with fuzzy coeficients or fuzzy variables or combination of both are called fuzzy linear programming problem.

Zimmerman[1],[10] introduced the first formulation of fuzzy linear programming to address the impreciseness of the parameters in linear programming problems with fuzzy constraints and objective functions. Figueroa followed Zimmermann proposal and he proposed a general method to handle uncertainty to the Right Hand Side parameters of a Linear Programming model by means of Interval Type-2 Fuzzy Sets (IT2FS) and trapezoidal membership functions and it solved[11].

In a review of lot of articles published on this subject, it was found that no one provides a general method to handle uncertainty to the Right-Hand-Side (RHS) parameters of a Linear Programming model by means of Perfectly normal Interval Type-2 Fuzzy Sets (PnIT2FSs) and trapezoidal membership functions. In this paper, we present a Linear Programming model with Perfectly normal Interval Type-2 Fuzzy Sets Right Hand Side (PnIT2FS RHS) parameters and solved it based on the ranking values and the arithmetic operations of PnIT2FSs. First, we present the arithmetic operations between PnIT2FSs. Then, we present a fuzzy ranking method to calculate the ranking values of PnIT2FSs. Based on the proposed fuzzy ranking method and the proposed arithmetic operations between PnIT2FSs, we present a new method to handle Type-2 fuzzy linear programming model. The rest of this paper is organized as follows. In Section 2, we briefly review the definitions of type-2 fuzzy sets. In Section 3, we present the Perfectly normal Interval Type-2 Fuzzy sets(PnIT2FSs) and arithmetic operations between sets. In Section 4, we present a method for ranking PnIT2FSs. In Section 5, we present a Type-2 fuzzy linear programming model with PnIT2FS RHS parameters and handling Perfectly normal Interval Type-2 Fuzzy Linear Programming(PnIT2FLP) model based on the proposed ranking method and the arithmetic operations. In Section 6, we use an example to illustrate the proposed method. The conclusions are discussed in Section 7.

II. Preliminaries

In 1965, Zadeh [12] introduced the concept of fuzzy set as a mathematical way of representing impreciseness in real world problems.

Definition 2.1: [13]

Let X be a non-empty set. A fuzzy set $\tilde{A}$ in X is characterized by its membership function $\mu_{\tilde{A}} : X \to [0,1]$ and $\mu_{\tilde{A}}(x)$ is interpreted as the degree of membership of element x in fuzzy set $\tilde{A}$ for each $x \in X$. It is clear that $\tilde{A}$ is completely determined by the set of tuples

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) \middle| x \in X \right\}.$$

Definition 2.2:[14]

A Type-2 fuzzy set $\tilde{A}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{A}}(x,u)$ as follows: $\tilde{A} = \left\{ \left((x,u), \mu_{\tilde{A}}(x,u) \right) \middle| \forall x \in X, \forall u \in J_x \subseteq [0,1], 0 \le \mu_{\tilde{A}}(x,u) \le 1 \right\}$ where $J_x \subseteq [0,1]$ is the primary membership function at x, and $\int_{u \in J_x} \mu_{\tilde{A}}(x,u)/u$ indicates the second membership at x. For discrete situations, $\int$ is replaced by $\sum$.

Definition 2.3:[14][15]

Let $\tilde{A}$ be a type-2 fuzzy set in the universe of discourse X represented by a type-2 membership function $\mu_{\tilde{A}}(x,u)$. If all $\mu_{\tilde{A}}(x,u) = 1$, then $\tilde{A}$ is called an IT2FS. An IT2FS can be regarded as a special case of the type-2 fuzzy set, which is defined as

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1/(x,u) = \int_{x \in X} \left[\int_{u \in J_x} 1/u \right] / x$$

where x is the primary variable, $J_x \subseteq [0,1]$ is the primary membership of x, u is the secondary variable, and $\int_{u \in J_x} 1/u$ is the secondary membership function at x.

It is obvious that the IT2FS $\tilde{A}$ defined on X is completely determined by the primary membership which is called the footprint of uncertainty, and the footprint of uncertainty can be expressed as follows:

$$FOU(\tilde{A}) = \bigcup_{x \in X} J_x = \bigcup_{x \in X} \{(x,u) | u \in J_x \subseteq [0,1]\}$$

Definition 2.4[16][17]

Let $\tilde{A}$ be an IT2FS, uncertainty in the primary membership of a type-2 fuzzy set consists of a bounded region called the footprint of uncertainty, which is the union of all primary membership. Footprint of uncertainty is characterized by upper membership function and lower membership function. Both of the membership functions are type-1 fuzzy sets. Upper membership function is denoted by $\overline{\mu}_{\tilde{A}}$ and lower membership function is denoted by $\underline{\mu}_{\tilde{A}}$ respectively.

Definition 2.5:[16]

An interval type-2 fuzzy number is called trapezoidal interval type-2 fuzzy number where the upper membership function and lower membership function are both trapezoidal fuzzy numbers, i.e.,

$$\tilde{A} = (A^L, A^U) = \left(\left(a_1^L, a_2^L, a_3^L, a_4^L ; H_1(A^L), H_2(A^L) \right), \left(a_1^U, a_2^U, a_3^U, a_4^U ; H_1(A^U), H_2(A^U) \right) \right),$$

where $H_j(A^L)$ and $H_j(A^U), (j = 1,2)$ denote membership values of the corresponding elements a_{j+1}^L and $a_{j+1}^U, (j = 1,2,3)$, respectively.

Definition 2.6[18]

The upper membership function and lower membership function of an IT2FSs are type-1 membership function, respectively.

Definition 2.7[19]

A IT2FS, $\tilde{A}$, is said to be perfectly normal if both its upper and lower membership function are normal. It is denoted by PnIT2FS: ie. $\sup \overline{\mu}_{\tilde{A}}(x) = \sup \underline{\mu}_{\tilde{A}} = 1$.

III. Perfectly Normal IT2TrFN

Definition 3.1

Let $\tilde{A} = [A^L, A^U]$ be PnIT2FS on X. Let $A^U = (a_2^L, a_3^L, \alpha_L, \beta_L)$ and $A^L = (a_2^U, a_3^U, \alpha_U, \beta_U)$ be the lower and upper trapezoidal fuzzy number, respectively, with respect to $\tilde{A}$ defined on the universe of discourse

X, where $a_2^L \le a_3^L, a_2^U \le a_3^U, \alpha_L, \alpha_U \ge 0$ and $\beta_L, \beta_U \ge 0$.$\left[a_2^L, a_3^L\right]$ is the core of $\tilde{A}^L$, and $\alpha_L \ge 0$, $\beta_L \ge 0$ are the left-hand and right-hand spreads and $\left[a_2^U, a_3^U\right]$ is the core of $\tilde{A}^U$ and $\alpha_U \ge 0$, $\beta_U \ge 0$ are the left-hand and right-hand spreads. The membership functions of x in $\tilde{A}^L$ and $\tilde{A}^U$ are expressed as follows:

$$\underline{\mu}_{\tilde{A}}(x) = \begin{cases} \left(x - a_2^L + \alpha_L\right)/\alpha_L, & a_2^L - \alpha_L \le x \le a_2^L, \\ 1, & a_2^L \le x \le a_3^L, \\ -\left(x - a_3^L - \beta_L\right)/\beta_L, & a_3^L \le x \le a_3^L + \beta_L, \\ 0, & otherwise. \end{cases}$$

$$\overline{\mu}_{\tilde{A}}(x) = \begin{cases} \left(x - a_2^U + \alpha_U\right)/\alpha_U, & a_2^U - \alpha_U \le x \le a_2^U, \\ 1, & a_2^U \le x \le a_3^U, \\ -\left(x - a_3^U - \beta_U\right)/\beta_U, & a_3^U \le x \le a_3^U + \beta_U, \\ 0, & otherwise. \end{cases}$$

$\underline{\mu}_{\tilde{A}}$ and $\overline{\mu}_{\tilde{A}}$ are lower and upper bounds, respectively of $\tilde{A}$ (see Figure). Then, $\tilde{A}$ is a Perfectly normal Interval Type-2 Trapezoidal Fuzzy Number (PnIT2TrFN) on X and is represented by the following: $\tilde{A} = \left[A^L, A^U\right] = \left(\left(a_2^L, a_3^L, \alpha_L, \beta_L\right), \left(a_2^U, a_3^U, \alpha_U, \beta_U\right)\right)$. Obviously, If $a_2^L = a_3^L$, $a_2^U = a_3^U$ the PnIT2TrFN reduce to the perfectly normal interval type-2 triangular fuzzy number (PnIT2TFN). If $A^L = A^U$, then the PnIT2TrFN $\tilde{A}$ becomes a type-1 trapezoidal fuzzy number[18].

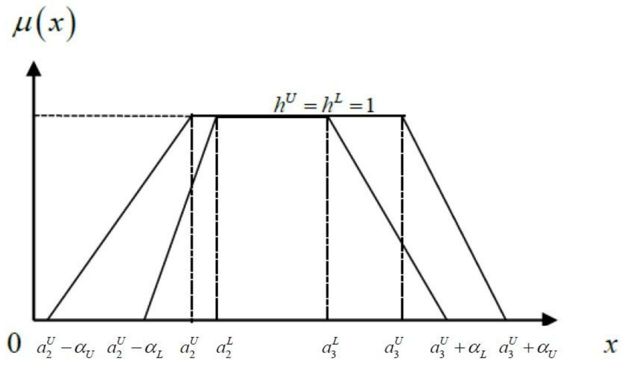

Figure 1: The lower trapezoidal membership function $\tilde{A}^L$ and the upper trapezoidal membership function $\tilde{A}^U$ of the PnIT2FS $\tilde{A}$.

Definition 3.2 (Primary $\alpha - cut$ of an PnIT2FS)

The primary $\alpha - cut$ of an PnIT2FS is ${}^{\alpha}\tilde{A} = \left\{(x,u) \middle| J_x \ge \alpha, u \in [0,1]\right\}$ which is bounded by two regions

$${}^{\alpha}\underline{\mu}_{\tilde{A}}(x) = \left\{\left(x, \underline{\mu}_{\tilde{A}}(x)\right) \middle| \underline{\mu}_{\tilde{A}}(x) \ge \alpha, \forall \alpha \in [0,1]\right\}$$

and

$${}^{\alpha}\overline{\mu}_{\tilde{A}}(x) = \left\{\left(x, \overline{\mu}_{\tilde{A}}(x)\right) \middle| \overline{\mu}_{\tilde{A}}(x) \ge \alpha, \forall \alpha \in [0,1]\right\}.$$

Definition 3.3 (Crisp bounds of PnIT2FN)

The crisp boundary of the primary $\alpha - cut$ of the PnIT2FN $\tilde{A} = \left(A^L, A^U\right)$ is closed interval ${}^{\alpha^{cb}}\tilde{A}$ shall be obtained as follows: The A^L and A^U are the lower and upper interval valued bounds of $\tilde{A}$. Also the boundary of A_α^L and A_α^U can be defined as the boundaries of the $\alpha - cuts$ of each interval type-1 fuzzy set,

$$A_\alpha^L = \left[\inf_x {}^\alpha \underline{\mu}_{\tilde{A}}(x), \sup_x {}^\alpha \underline{\mu}_{\tilde{A}}(x)\right] = \left[a_l^L, a_u^L\right]$$

$$A_\alpha^U = \left[\inf_x {}^\alpha \overline{\mu}_{\tilde{A}}(x), \sup_x {}^\alpha \overline{\mu}_{\tilde{A}}(x)\right] = \left[a_l^U, a_u^U\right]$$

$${}^{\alpha^{cb}}\tilde{A} = \left[\inf_x \left\{{}^\alpha \mu_{\tilde{A}}(x,u)\right\}, \sup_x \left\{{}^\alpha \mu_{\tilde{A}}(x,u)\right\}\right] = \left[\left[a_l^U, a_l^L\right], \left[a_u^L, a_u^U\right]\right] = \left[\left[{}^\alpha \overline{a}^L, {}^\alpha \underline{a}^L\right], \left[{}^\alpha \underline{a}^R, {}^\alpha \overline{a}^R\right]\right]$$

which is equivalent to say ${}^{\alpha^{cb}}\mu_{\tilde{A}} \in \left[\left[{}^\alpha \overline{a}^L, {}^\alpha \underline{a}^L\right], \left[{}^\alpha \underline{a}^R, {}^\alpha \overline{a}^R\right]\right]$.

Evidently, For PnIT2TrFN From the figure 3 , ${}^\alpha \overline{a}^L \le {}^\alpha \underline{a}^L \le {}^\alpha \underline{a}^R \le {}^\alpha \overline{a}^R$.

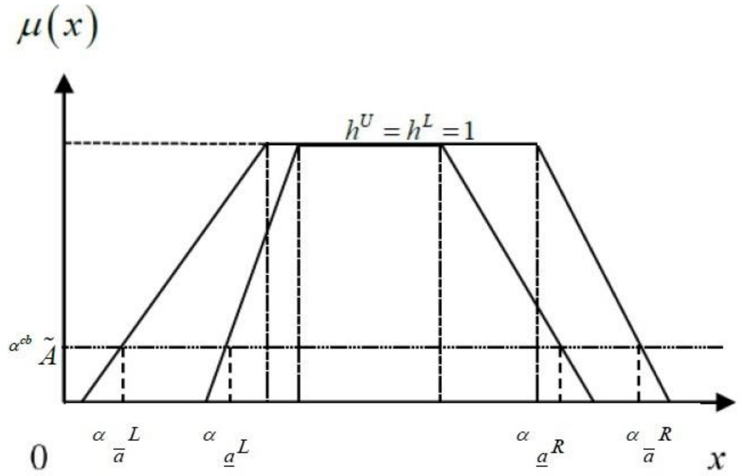

Figure 2: Crisp bounds of PnIT2TrFN

a. Arithmetic Operations On PnIT2TrFN

Definition 3.4.

If $\tilde{A} = \left(\left(a_2^L, a_3^L, \alpha_L, \beta_L\right), \left(a_2^U, a_3^U, \alpha_U, \beta_U\right)\right)$ and $\tilde{B} = \left(\left(b_2^L, b_3^L, \gamma_L, \theta_L\right), \left(b_2^U, b_3^U, \gamma_U, \theta_U\right)\right)$ are PnIT2TrFNs, then $\tilde{C} = \tilde{A} + \tilde{B}$ is also a PnIT2TrFN and defined by

$$\tilde{C} = \left(\left(a_2^L + b_2^L, a_3^L + b_3^L, \alpha_L + \gamma_L, \beta_L + \theta_L\right), \left(a_2^U + b_2^U, a_3^U + b_3^U, \alpha_U + \gamma_U, \beta_U + \theta_U\right)\right)$$

Definition 3.5.

If $\tilde{A} = \left(\left(a_2^L, a_3^L, \alpha_L, \beta_L\right), \left(a_2^U, a_3^U, \alpha_U, \beta_U\right)\right)$ and $\tilde{B} = \left(\left(b_2^L, b_3^L, \gamma_L, \theta_L\right), \left(b_2^U, b_3^U, \gamma_U, \theta_U\right)\right)$ are PnIT2TrFNs, then $\tilde{C} = \tilde{A} - \tilde{B}$ is also a PnIT2TrFN and defined by

$$\tilde{C} = \left(\left[a_2^L - b_3^U, a_3^L - b_2^U, \alpha_L + \theta_U, \beta_L + \gamma_U\right], \left[a_2^U - b_3^L, a_3^U - b_2^L, \alpha_U + \theta_L, \beta_U + \gamma_L\right]\right)$$

Definition 3.6.

Let $\lambda \in \mathbb{R}$. If $\tilde{A} = \left(\left(a_2^L, a_3^L, \alpha_L, \beta_L\right), \left(a_2^U, a_3^U, \alpha_U, \beta_U\right)\right)$ is a PnIT2TrFN, then $\tilde{C} = \lambda \tilde{A}$ is also a PnIT2TrFN and is given by

$$\tilde{C} = \lambda \tilde{A} = \begin{cases} \left(\left(\lambda a_2^L, \lambda a_3^L, \lambda \alpha_L, \lambda \beta_L\right), \left(\lambda a_2^U, \lambda a_3^U, \lambda \alpha_U, \lambda \beta_U\right)\right); if \quad \lambda \ge 0 \\ \left(\left(\lambda a_3^U, \lambda a_2^U, |\lambda| \beta_U, |\lambda| \alpha_U\right), \left(\lambda a_3^L, \lambda a_2^L, |\lambda| \beta_L, |\lambda| \alpha_L\right)\right); if \quad \lambda < 0 \end{cases}$$

IV. The proposed method for ranking of PnIT2TrFN

Several methods for solving type-2 fuzzy linear programming problems can be seen in Figueroa Garcia [11],[20], Stephen Dinagar and Anbalagan[19], Nurnadiah Zamri et. al[21],Jin et.al[22] and others. One of the most convenient of these methods is based on the concept of comparison of fuzzy numbers by use of ranking functions. Ranking functions of $F(\mathbb{R})$ is to define a ranking function $R : F(\mathbb{R}) \rightarrow \mathbb{R}$ which maps each fuzzy number into the real line, where a natural order exists. We define orders on $F(\mathbb{R})$ as follows:

- $\tilde{A} \underset{R}{\le} \tilde{B} \Leftrightarrow R(\tilde{A}) \le R(\tilde{B})$,
- $\tilde{A} \underset{R}{<} \tilde{B} \Leftrightarrow R(\tilde{A}) < R(\tilde{B})$,
- $\tilde{A} \underset{R}{=} \tilde{B} \Leftrightarrow R(\tilde{A}) = R(\tilde{B})$, where $\tilde{A}$ and $\tilde{B}$ are in $F(R)$. Also we write $\tilde{A} \underset{R}{\le} \tilde{B} \Leftrightarrow \tilde{B} \underset{R}{\ge} \tilde{A}$.

Lemma 4.1

Let R be any linear ranking function. Then

- $\tilde{A} \underset{R}{\ge} \tilde{B} \Leftrightarrow \tilde{A} - \tilde{B} \underset{R}{\ge} 0 \Leftrightarrow -\tilde{B} \underset{R}{\ge} -\tilde{A}$.
- If $\tilde{A} \underset{R}{\ge} \tilde{B}$ and $\tilde{C} \underset{R}{\ge} \tilde{D}$, then $\tilde{A} + \tilde{C} \underset{R}{\ge} \tilde{B} + \tilde{D}$.

Definition 4.1 :

Let $\tilde{A}$ be a PnIT2TrFN: $\tilde{A} = \left[A^L, A^U\right] = \left(\left(a_2^L, a_3^L, \alpha_L, \beta_L\right), \left(a_2^U, a_3^U, \alpha_U, \beta_U\right)\right)$ then the ranking value $R(\tilde{A})$ is given as

$$R(\tilde{A}) = \frac{R(\tilde{A}^L) + R(\tilde{A}^U)}{4}$$

. Then, the ranking value $R(\tilde{A}^L)$ of the lower trapezoidal membership function $\tilde{A}^L$ of the perfectly normal interval type-2 fuzzy set is calculated as follows:

$$R(\tilde{A}^L) = \sum_{I=1}^{3} M_i(\tilde{A}^L) - \frac{1}{4}\sum_{j=1}^{4} N_j(\tilde{A}^L),$$

where $M_1\left(\tilde{A}^L\right)=a_2^L-\frac{\alpha_L}{2}, M_2\left(\tilde{A}^L\right)=\frac{\left(a_2^L+a_3^L\right)}{2}, M_3\left(\tilde{A}^L\right)=a_3^L+\frac{\beta_L}{2}, N_1\left(\tilde{A}^L\right)=\frac{\alpha_L}{2}$,
$N_2\left(\tilde{A}^L\right)=\frac{a_2^L-a_3^L}{2}, N_3\left(\tilde{A}^L\right)=\frac{\beta_L}{2}$ and

$$N_4\left(\tilde{A}^L\right)=\frac{1}{2}\left[\left(\left(a_2^L-\alpha_L\right)-S\right)^2+\left(a_2^L-S\right)^2+\left(a_3^L-S\right)^2+\left(\left(a_3^U+\beta_U\right)-S\right)^2\right]^{\frac{1}{2}},$$

where $S=\frac{1}{4}\left(\left(a_2^L-\alpha_L\right)+a_2^L+a_3^L+\left(a_3^L+\beta_L\right)\right)$.\newline In the same way, the ranking value $R\left(\tilde{A}^U\right)$ of the upper trapezoidal membership function of the PnIT2FS is calculated as follows:

$$R\left(\tilde{A}^U\right)=\sum_{I=1}^{3} M_i\left(\tilde{A}^U\right)-\frac{1}{4}\sum_{j=1}^{4} N_j\left(\tilde{A}^U\right),$$

where

$M_1\left(\tilde{A}^U\right)=a_2^U-\frac{\alpha_U}{2}, M_2\left(\tilde{A}^U\right)=\frac{\left(a_2^U+a_3^U\right)}{2}, M_3\left(\tilde{A}^U\right)=a_3^U+\frac{\beta_U}{2}, N_1\left(\tilde{A}^U\right)=\frac{\alpha_U}{2}$,
$N_2\left(\tilde{A}^U\right)=\frac{a_2^U-a_3^U}{2}, N_3\left(\tilde{A}^U\right)=\frac{\beta_U}{2}$ and

$$N_4\left(\tilde{A}^U\right)=\frac{1}{2}\left[\left(\left(a_2^U-\alpha_U\right)-S\right)^2+\left(a_2^U-S\right)^2+\left(a_3^U-S\right)^2+\left(\left(a_3^U+\beta_U\right)-S\right)^2\right]^{\frac{1}{2}},$$

where $S=\frac{1}{4}\left(\left(a_2^U-\alpha_U\right)+a_2^U+a_3^U+\left(a_3^U+\beta_U\right)\right)$.

Remark 1. Note that
$R\left(\tilde{A}\right)=\left(\left(a_2^L,a_3^L,\alpha_L,\beta_L\right),\left(a_2^U,a_3^U,\alpha_U,\beta_U\right)\right)=\left((0,0,0,0),(0,0,0,0)\right)=0$,
$R\left(\tilde{A}\right)=\left(\left(a_2^L,a_3^L,\alpha_L,\beta_L\right),\left(a_2^U,a_3^U,\alpha_U,\beta_U\right)\right)=\left((1,1,0,0),(1,1,0,0)\right)=1$ and $R\left(k\tilde{A}\right)=kR\left(\tilde{A}\right)$ and
$R\left(\tilde{A}+k\tilde{B}\right)=R\left(\tilde{A}\right)+kR\left(\tilde{B}\right)$.

V. Type-2 Fuzzy Linear Programming with PnIT2 RHS parameters

In this section, we propose a Type-2 fuzzy linear programming model with Right-Hand-Side (resources) are PnIT2FS.

$$\underset{x\in X}{Opt} Z=cx$$
$$S.t. Ax \preceq \tilde{b}, x\geq 0$$

Where $A=\left(a_{ij}\right)_{m\times n}\in \square^{m\times n}$, $c=\left(c_1,c_2,\ldots,c_n\right)$, $x=\left(x_1,x_2,\ldots,x_n\right)^T\in \square^n$ and $\tilde{b}=\left(b_1,b_2,\ldots b_m\right)^T$, are PnIT2FS. The Type-2 fuzzy order $\preceq$ and $\succeq$ are exist .

Definition 5.1.
Consider a set of right-hand-side(resources) parameters of a Fuzzy linear programming problem define as an PnIT2FS $\tilde{b}$ defined on the closed interval

$$\tilde{b}_i\in\left[\left[{}^{\alpha}\overline{b_i}^L,{}^{\alpha}\underline{b_i}^L\right],\left[{}^{\alpha}\underline{b_i}^R,{}^{\alpha}\overline{b_i}^R\right]\right]\in\square$$

and $i\in\square_n$. The membership function which represents the fuzzy space $Supp\left(\tilde{b}_i\right)$ is

$$\tilde{b}_i = \int_{b_i \in \mathbb{R}} \left[\int_{u \in J_{b_i}} 1/u \right] / b_i, i \in \mathbb{R}_n, J_{b_i} \subseteq [0,1].$$

Here, $\tilde{b}$ is bounded by both lower and upper primary membership function, namely

$$^{\alpha}\underline{\mu}_{b_i} = \left\{ (b_i, u) \middle| \underline{\mu}_{b_i} \geq \alpha \right\},$$

with parameter $^{\alpha}\underline{b}_i^{L}$ & $^{\alpha}\underline{b}_i^{R}$ and

$$^{\alpha}\overline{\mu}_{b_i} = \left\{ (b_i, u) \middle| \overline{\mu}_{b_i} \geq \alpha \right\}$$

with parameter $^{\alpha}\overline{b}_i^{L}$ & $^{\alpha}\overline{b}_i^{R}$.

5.1 Type-2 Fuzzy Linear Programming model with PnIT2TrFNs.

Let us consider the following Type-2 Fuzzy Linear Programming model with right-hand-side (resources) as PnIT2TrFNs:

$$Opt Z = \sum_{j=1}^{n} c_j x_j$$

$$S.t \sum_{j=1}^{n} a_{ij} x_j \leq \tilde{b}_i, i = 1, 2, \ldots, m$$

$$x_j \geq 0, \; j = 1, 2, \ldots, n.$$

(5.1)

where $\tilde{b}_i = \left((\tilde{b}_i)^L, (\tilde{b}_i)^U \right) = \left(\left((b_i)_2^L, (b_i)_3^L, (\gamma_i)_L, (\theta_i)_L \right), \left((b_i)_2^U, (b_i)_3^U, (\gamma_i)_U, (\theta_i)_U \right) \right)$ are PnIT2TrFNs, a_{ij}, c_i are crisp coefficients, and x_i are the decision variable.

5.2 The proposed algorithm.

The procedure for handling the proposed Type-2 Fuzzy Linear Programming model with PnIT2 Right-Hand-Side parameters to determine the optimal solution based on the proposed fuzzy ranking method and arithmetic operation is summarized in the following steps:

Steps:1. Formulate the chosen problem into the following Type-2 Fuzzy Linear Programming model,

$$\underset{x \in X}{Opt} Z = cx$$

$$S.t. Ax \leq \tilde{b}, x \geq 0$$

where $A = (a_{ij})_{m \times n} \in \mathbb{R}^{m \times n}$, $c = (c_1, c_2, \ldots, c_n)$, $x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$ and $\tilde{b} = (b_1, b_2, \ldots b_m)^T$ is an PnIT2TrFNs

Steps:2 Using the ranking function (Definition 4.1)The fuzzy linear programming problem transform in to crisp linear programming problem.i.e,

$$R(\tilde{b}_i) = R\left((\tilde{b}_i)^L, (\tilde{b}_i)^U \right) = R\left(\left((b_i)_2^L, (b_i)_3^L, (\gamma_i)_L, (\theta_i)_L \right), \left((b_i)_2^U, (b_i)_3^U, (\gamma_i)_U, (\theta_i)_U \right) \right)$$

,the Type-2 Fuzzy Linear Programming problem, obtained in Step:1, can be written as:

$$\underset{x \in X}{Opt} Z = cx$$

$$S.t. Ax \leq R(\tilde{b}), x \geq 0$$

Step : 3. Solve the crisp linear programming problem using simplex method with help of MATLAB, obtained in Step 2.

Remark : The proposed method is named as “robust Conclusion of Recourse Management" method for solving Type-2 fuzzy linear programming model. Is abbreviated as “robust CRM”.

VI. Numerical Illustration

An application of the proposed method is introduced with an example, where all its resources are defined as PnIT2TrFNs. The optimal solution will be obtained in-terms of crisp for the following problem. Consider the agricultural problem, a farmer who has at his disposal given amounts of land, manpower and water resources for his agriculture purpose, he used and produced two type of crops. Each ton of crop1 produced yields a certain known profit but requires that certain known amount of the three resources be used up. Each ton of crop2 produced also yields a known profit and involves using known amount of the three resources. An obvious question which arises is: how much of each of crops should the farmer produce in order to maximize his profit. While the three available resources are uncertain? The available data are tabulated in Table 1

Table 1:

	Unit of resource used up per ton grown of:		
Resource	*Crop*1	*Crop*2	Total availability of resource(fuzzy)
Land	2	1	8
Manpower	1	1	5
Water	1	2	8
Profit Per ton grown	2	3	-

To solve the considered problem, First, we formulate the problem as an Type-2 fuzzy linear programming model as follows: let, x_1 and x_2 designate these quantities. x_1 and x_2 are referred as the decision variables become the purpose in solving the problem is to decide what values these variables should taken. Consider the objective of profit maximization. If x_1 tons of crop1 are grown, the profit accruing from crop1 and grown will be $2x_1$. Similarly, if x_2 tons of crop2 are grown, a profit of $3x_2$ will result. Thus the total profit gained by growing x_1 tons of crop1 together with x_2 tons of crop2 will be $2x_1 + 3x_2$. Given that profit must be maximized, the objective may be written mathematically as:

$$MaxZ = 2x_1 + 3x_2$$

Consider now the first constraint, that no more land (arable land of area) than is available may be used. If x_1 tons of crop1 are grown, the total amount of land used for crop1 is $2x_1$, similarly, if x_2 tons of crop2 are grown, the total amount of land used for crop2 is $1x_2$. Thus the total amount of land used by growing x_1 tons of crop1 together with x_2 tons of crop2 is $2x_1 + x_2$ which must not exceed total arable lands of area are close to 8 unit. Thus mathematical, x_1 and x_2 must be such as:

$$2x_1 + x_2 \leq \tilde{8}$$

Similarly, consideration of the constraints on manpower and water yields the mathematical conditions:

$$x_1 + x_2 \leq \tilde{5}$$

$$x_1 + 2x_2 \leq \tilde{8}$$

Finlay, there is the obvious stipulation that negative amounts of crops cannot be produced i.e.

$$x_1, x_2 \geq 0 .$$

The preceding of the five mathematical expression together completely describe optimization problem to be solved as follows:

$$
\begin{aligned}
& Max\ Z = 2x_1 + 3x_2 \\
& S.t\ \ 2x_1 + x_2 \le \tilde{b}_1 \\
& x_1 + x_2 \le \tilde{b}_2 x_1 + 2x_2 \le \tilde{b}_3 \\
& x_j \ge 0,\ j = 1, 2.
\end{aligned}
\qquad (6.1)
$$

where

$\tilde{b}_1 = \left[\left(7.85, 7.875, 0.25, 0.25\right), \left(8.15, 8.175, 0.25, 0.25\right)\right]$,

$\tilde{b}_2 = \left[\left(3.5, 3.75, 0.75, 0.75\right), \left(6.5, 6.75, 0.75, 0.75\right)\right]$,

$\tilde{b}_3 = \left[\left(7.85, 7.875, 0.33, 0.33\right), \left(8.15, 8.175, 0.33, 0.33\right)\right], c_1 = 4$, $c_2 = 7$, $c_3 = 6, c_4 = 5$ and $c_5 = 4$. By using the ranking function (Definition4.1),The fuzzy linear programming problem (6.1) transformed in to crisp linear programming problem as follows:

$$
\begin{aligned}
& Max\ Z = 2x_1 + 3x_2 \\
& S.t\ \ 2x_1 + x_2 \le 8.09 \\
& x_1 + x_2 \le 5.26 \\
& x_1 + 2x_2 \le 8.12 \\
& x_j \ge 0,\ j = 1, 2.
\end{aligned}
\qquad (6.2)
$$

Solve the crisp linear programming problem equation (6.2) using simplex method with help of MATLAB. we can get the optimal solution Maximum profit is 13.38 and with $x_1 = 12/5, x_2 = 143/50$.

VII. Conclusion

In this paper, Fuzzy Linear Programming model with Perfectly Normal Interval Type-2 Resource was studied. Further a Perfectly normal Interval Type-2 Fuzzy Linear Programming model was converted into an equivalent crisp linear programming model using the concept of ranking function and a new ranking function, named as “robust CRM”, was proposed for converting the Perfectly normal Interval Type-2 Fuzzy Linear Programming. The resultant linear programming problem was solved by simplex method with help of MATLAB. The discussed method was illustrated through an example. In future proposed method can be extended to solve problems like Type-2 fuzzy linear programming problem with symmetric type-2 triangular or symmetric type-2 trapezoidal membership function.

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