ISSN (e): 2250 – 3005 || Volume, 06 || Issue, 03||March – 2016 ||
International Journal of Computational Engineering Research (IJCER)

On intuitionistic fuzzy β generalized closed sets

Saranya M[1], Jayanthi D[2]
[1] MSc Mathematics, Avinashilingam University, Coimbatore, Tamil Nadu, India
[2]Assistant Professor of Mathematics, Avinashilingam University, Coimbatore, Tamil Nadu, India

ABSTRACT
In this paper, we have introduced the notion of intuitionistic fuzzy β generalized closed sets, and investigated some of their properties and characterizations

KEYWORDS: *Intuitionistic fuzzy topology, intuitionistic fuzzy β closed sets, intuitionistic fuzzy β generalized closed sets.*

I. Introduction

The concept of fuzzy sets was introduced byZadeh [12] and later Atanasov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper, we have introduced the notion of intuitionistic fuzzy β generalized closed sets, and investigated some of their properties and characterizations.

II. Preliminaries

Definition 2.1: [1]An *intuitionistic fuzzy set* (IFS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \longrightarrow [0,1]$ and $\nu_A : X \longrightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively, and$0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X.
An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$instead of denoting$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2: [1] Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,
(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$,
(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0\sim = \langle x, 0, 1\rangle$ and $1\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3: [3]An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFSsin X satisfying the following axioms:

(i) $0\sim, 1\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X,τ) is called *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X,τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4:[5] An IFS A = $\langle x, \mu_A, \nu_A \rangle$ in an IFTS (X,τ) is said to be an

(i) intuitionistic fuzzy β closed set (IFβCS for short) if int(cl(int(A))) $\subseteq$ A,

(ii) intuitionistic fuzzy β open set (IFβOS for short) if A $\subseteq$ cl(int(cl(A))).

Definition 2.5: [6]Let A be an IFS in an IFTS (X,τ). Then the β-interior and β-closure of A are defined as

βint(A) = $\cup$ {G / G is an IFβOS in X and G $\subseteq$ A}.

βcl(A) = $\cap$ {K / K is an IFβCS in X and A $\subseteq$ K}.

Note that for any IFS A in (X,τ), we have βcl(A^c) = (βint(A))^c andβint(A^c) = (βcl(A))^c.

Result 2.6: Let A be an IFS in (X,τ), then

(i) βcl(A)$\supseteq$ A $\cup$ int(cl(int(A)))

(ii) βint(A) $\subseteq$ A $\cap$ cl(int(cl(A)))

Proof: (i)Now int(cl(int(A))) $\subseteq$ int(cl(int(βcl(A))) $\subseteq$ βcl(A), since A $\subseteq$ βcl(A) and βcl(A) is an IFβCS. Therefore A $\cup$int(cl(int(A))) $\subseteq$ βcl(A).
(ii) can be proved easily by taking complement in (i).

III. Intuitionistic fuzzyβgeneralized closed sets

In this section we have introduced intuitionistic fuzzy β generalized closed sets and studied some of their properties.

Definition 3.1: An IFS A in an IFTS (X,τ) is said to be an *intuitionistic fuzzy β generalized closed set* (IFβGCS for short) if βcl(A) $\subseteq$ U whenever A$\subseteq$ U and U is an IFβOS in (X,τ).
The complement A^c of an IFβGCS A in an IFTS (X,τ) is called an intuitionistic fuzzy β generalized open set (IFβGOS in short) in X.

The family of all IFβGCSs of an IFTS (X,τ) is denoted by IFβGC(X).

Example 3.2: Let X = {a, b} and G = $\langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then τ = {0~, G, 1~} is an IFT on X. Let A = $\langle x, (0.4_a, 0.3_b) (0.6_a, 0.7_b)\rangle$ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in[0,1]$, $\mu_b \in [0,1]$, $\nu_a \in[0,1]$, $\nu_b\in [0,1]$ / $0\leq \mu_a+\nu_a\leq1$ and $0 \leq \mu_b+\nu_b\leq 1$}.
We have A $\subseteq$ G. Asβcl(A) = A,βcl(A) $\subseteq$ G, where G is an IFβOS in X. This implies that A is an IFβGCS in X.

Theorem 3.3: EveryIFCSin (X, τ) is an IFβGCS in (X,τ) but not conversely.
Proof: Let A be an IFCS. Thereforecl(A) = A. Let A $\subseteq$ U and U be an IFβOS. Sinceβcl(A) $\subseteq$ cl(A) =A $\subseteq$ U, we have βcl(A) $\subseteq$ U. Hence A is an IFβGCS in (X, τ).

Example 3.4: Let X = {a, b} and G = $\langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then τ = {0~, G, 1~} is an IFT onX. Let A = $\langle x, (0.4_a, 0.3_b) (0.6_a, 0.7_b)\rangle$ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in[0,1]$, $\mu_b \in [0,1]$, $\nu_a \in[0,1]$, $\nu_b\in [0,1]$ / $0\leq \mu_a+\nu_a\leq1$ and $0 \leq \mu_b+\nu_b\leq 1$}.
We have A $\subseteq$ G. As βcl(A) = A,βcl(A) $\subseteq$ G, where G is an IFβOS in X. This implies that A is an IFβGCS in X, but not an IFCS, sincecl(A) = $G^c \neq$ A.

Theorem 3.5: EveryIFRCSin (X,τ) is an IFβGCS in (X,τ) but not conversely.
Proof: Let A be an IFRCS[10]. Since every IFRCS is an IFCS [9], by theorem 3.3, A is an IFβGCS.

Example 3.6: Let X = {a, b} and G = $\langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$.Then τ = {0~, G, 1~} is an IFT onX. Let A = $\langle x, (0.4_a, 0.3_b) (0.6_a, 0.7_b)\rangle$ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in[0,1]$, $\mu_b \in [0,1]$, $\nu_a \in[0,1]$, $\nu_b\in [0,1]$ / $0\leq \mu_a+\nu_a\leq1$ and $0 \leq \mu_b+\nu_b\leq 1$}.
We have A $\subseteq$ G. As βcl(A) = A,βcl(A) $\subseteq$ G, where G is an IFβOS inX. This implies that A is an IFβGCS in X, but not an IFRCS, sincecl(int(A)) = cl(0~) = 0~ $\neq$ A.

Theorem 3.7: EveryIFSCSin (X,τ) is an IFβGCS in (X,τ) but not conversely.
Proof: Assume A is an IFSCS [5]. Let A $\subseteq$ U and U be an IFβOS. Sinceβcl(A) $\subseteq$scl(A) = A and A $\subseteq$ U, by hypothesis, we haveβcl(A) $\subseteq$ U. Hence A is an IFβGCS.

Example 3.8: Let X = {a, b} and G = ⟨x, $(0.5_a, 0.4_b)$, $(0.5_a, 0.6_b)$⟩.Then τ = {0∼, G, 1∼} is an IFT on X. Let A = ⟨x, $(0.4_a, 0.3_b)$ $(0.6_a, 0.7_b)$⟩ be an IFS in X.

Then, IFβC(X) = {0∼, 1∼, $\mu_a \epsilon[0,1]$, $\mu_b \epsilon$ [0,1], $\nu_a \epsilon[0,1]$, $\nu_b \epsilon$ [0,1] / $0\leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}. We have A ⊆ G.As βcl(A) = A,βcl(A) ⊆ G, where G is an IFβOS in X. This implies that A is an IFβGCS in X, but not an IFSCS, since int(cl(A)) = int(G^c) = G ⊈ A.

Theorem 3.9: EveryIFαCSin (X,τ) is an IFβGCS in (X,τ) but not conversely.
Proof: Assume A is an IFαCS [5]. Let A ⊆ U and U be an IFβOS. Sinceβcl(A) ⊆ αcl(A) =A and A ⊆ U, by hypothesis, we haveβcl(A) ⊆ U. Hence A is an IFβGCS.

Example 3.10: Let X = {a, b} and G = ⟨x, $(0.5_a, 0.4_b)$, $(0.5_a, 0.6_b)$⟩. Thenτ = {0∼, G, 1∼} is an IFT on X. Let A = ⟨x, $(0.4_a, 0.3_b)$ $(0.6_a, 0.7_b)$⟩ be an IFS in X.

Then, IFβC(X) = {0∼, 1∼, $\mu_a \epsilon[0,1]$, $\mu_b \epsilon$ [0,1], $\nu_a \epsilon[0,1]$, $\nu_b \epsilon$ [0,1] / $0\leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}. We have A ⊆ G. As βcl(A) = A,βcl(A) ⊆ G, where G is an IFβOS in X. This implies that A is an IFβGCS in X, but not an IFαCS, since cl(int(cl(A))) = cl(int(G^c)) = cl(G) = G^c⊈A.

Theorem 3.11: EveryIFPCSin (X,τ) is an IFβGCS in (X,τ) but not conversely.
Proof: Assume A is an IFPCS [5]. LetA ⊆ U and U be an IFβOS.Sinceβcl(A) ⊆pcl(A) = Aand A⊆ U, by hypothesis, we have βcl(A) ⊆ U. Hence A is an IFβGCS.

Example 3.12: Let X = {a, b} and G = ⟨x, $(0.5_a, 0.6_b)$, $(0.5_a, 0.4_b)$⟩, Then τ = {0∼, G, 1∼} is an IFT on X. Let A = ⟨x, $(0.5_a, 0.7_b)$ $(0.5_a, 0.3_b)$⟩ be an IFS in X.

Then, IFβC(X) = {0∼, 1∼, $\mu_a \epsilon[0,1]$, $\mu_b \epsilon$ [0,1], $\nu_a \epsilon[0,1]$, $\nu_b \epsilon$ [0,1] /μ_b <0.6 whenever $\mu_a \geq 0.5$,μ_a <0.5 whenever $\mu_b \geq 0.6$,$0\leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.
NowA⊆ 1∼. As βcl(A) = 1∼ ⊆ 1∼, we have A is an IFβGCS in X, but not an IFPCS since cl(int(A)) = cl(G) =1∼ ⊈ A.

Remark 3.13: Every IFGCS and every IFβGCS are independent to each other.

Example 3.14: Let X = {a, b} and G_1 = ⟨x, $(0.5_a, 0.5_b)$, $(0.5_a, 0.5_b)$⟩ and G_2 = ⟨x,$(0.3_a, 0.1_b)$,$(0.7_a, 0.8_b)$⟩. Then τ = {0∼, G_1, G_2 1∼} is an IFT on X. Let A = ⟨x, $(0.4_a, 0.3_b)$ $(0.6_a, 0.7_b)$⟩ be an IFS in X. Then A ⊆ G_1 andcl(A) = G_1^c⊆G_1.Therefore A is an IFGCS in X.

Now IFβC(X) = {0∼, 1∼, $\mu_a \epsilon[0,1]$, $\mu_b \epsilon$ [0,1], $\nu_a \epsilon[0,1]$, $\nu_b \epsilon$ [0,1] / either $\mu_a \geq 0.5$ and $\mu_b \geq 0.5$ orμ_a< 0.3 and μ_b<0.1, $0\leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.
Since A ⊆ G_1 where G_1 is an IFβOS in X but βcl(A) =⟨x, $(0.5_a, 0.5_b)$, $(0.5_a, 0.5_b)$⟩ ⊈ A, A is not an IFβGCS.

Example 3.15: Let X = {a, b} and G = ⟨x, $(0.5_a, 0.4_b)$, $(0.5_a, 0.6_b)$⟩. Then τ = {0∼, G, 1∼} is an IFT on X. Let A = ⟨x, $(0.4_a, 0.3_b)$ $(0.6_a, 0.7_b)$⟩ be an IFS in X.

Then, IFβC(X) = {0∼, 1∼, $\mu_a \epsilon[0,1]$, $\mu_b \epsilon$ [0,1], $\nu_a \epsilon[0,1]$, $\nu_b \epsilon$ [0,1] /$0\leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}. We have A ⊆ G. As βcl(A) = A,βcl(A) ⊆ G, where G is an IFβOS in X. This implies that A is an IFβGCS in X, but not an IFGCS in X, sincecl(A) = G^c⊈ G.

Theorem 3.16: EveryIFβCSin (X,τ) is an IFβGCS in (X,τ) but not conversely.
Proof: Assume A is an IFβCS [5] then βcl(A) = A. Let A ⊆ U and U be an IFβOS. Thenβcl(A) ⊆ U, by hypothesis. Therefore A is an IFβGCS.

Example 3.17: Let X = {a, b} and G = ⟨x, $(0.5_a, 0.7_b)$, $(0.5_a, 0.3_b)$⟩, thenτ = {0∼, G, 1∼} is an IFT on X. Let A = ⟨x, $(0.5_a, 0.8_b)$ $(0.5_a, 0.2_b)$⟩ be an IFS in X.

Then, IFβC(X) = {0∼, 1∼, μ_a $\epsilon[0,1]$, $\mu_b \epsilon$ [0,1], ν_a $\epsilon[0,1]$, $\nu_b \epsilon$ [0,1] / provided μ_b <0.7 whenever$\mu_a \geq 0.5$, μ_a <0.5 whenever $\mu_b \geq 0.7$, $0\leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.
Now A ⊆ 1∼ and βcl(A) = 1∼ ⊆ 1 ∼. This implies that A is an IFβGCS in X,but not an IFβCS,sinceint(cl(int(A))) = int(cl(G)) = int(1 ∼) = 1 ∼ ⊈ A.

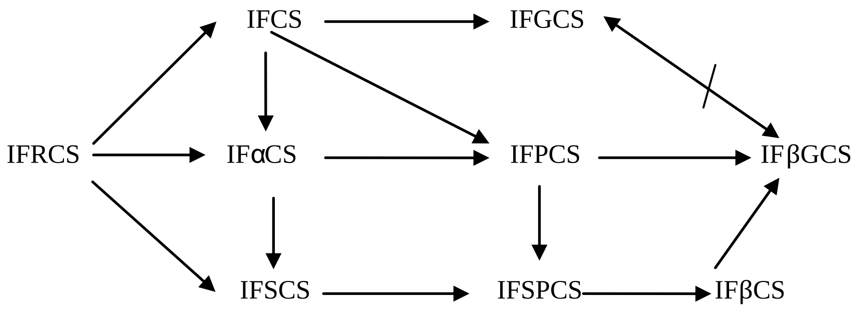
Theorem 3.18: Every IFSPCS in (X,τ) is an IFβGCS in (X,τ) but not conversely.
Proof: Assume A is an IFSPCS[11]. Since every IFSPCS is an IFβCS [7], by theorem 3.16, A is an IFβGCS.

Example 3.19: Let X = {a, b} and G = ⟨x, $(0.5_a, 0.4_b)$, $(0.5_a, 0.6_b)$⟩. Then τ = {0∼, G, 1∼} is an IFT on X. Let A = ⟨x, $(0.4_a, 0.3_b)$ $(0.6_a, 0.7_b)$⟩ be an IFS in X.

Then, IFβC(X) = {0∼, 1∼, $\mu_a \epsilon[0,1]$, $\mu_b \epsilon [0,1]$, $\nu_a \epsilon[0,1]$, $\nu_b \epsilon [0,1]$ /$0\leq \mu_a+\nu_a\leq1$ and $0 \leq \mu_b+\nu_b\leq 1$}. Here A is an IFβCS in X. As int(cl(int(A))) = 0∼ ⊆ A. Therefore A is an IFβGCS in X.

Since IFPC(X) = {0∼, 1∼, $\mu_a \epsilon[0,1]$, $\mu_b \epsilon [0,1]$, $\nu_a \epsilon[0,1]$, $\nu_b \epsilon [0,1]$ /either $\mu_b\geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a\geq 0.5$, $0\leq \mu_a+\nu_a\leq1$ and $0 \leq \mu_b+\nu_b\leq 1$}.
But A is not an IFSPCS in X, as we cannot find any IFPCS B such that int(B) ⊆ A ⊆ B in X.

In the following diagram, we have provided relations between various types of intuitionistic fuzzy closedness.



The reverse implications are not true in general in the above diagram.

Remark 3.20: The union of any two IFβGCS is not an IFβGCS in general as seen from the following example.

Example 3.21: Let X = {a, b} and τ = {0∼, G_1, G_2, 1∼} where G_1 = ⟨x, $(0.7_a, 0.8_b)$, $(0.3_a, 0.2_b)$⟩ and G_2 = ⟨x, $(0.6_a, 0.7_b)$, $(0.4_a, 0.3_b)$⟩. Then the IFSs A = ⟨x, $(0.6_a, 0.5_b)$ $(0.4_a, 0.3_b)$⟩ and B = ⟨x, $(0.4_a, 0.8_b)$, $(0.4_a, 0.2_b)$⟩ are IFβGCSs in (X,τ) but A ∪ B is not an IFβGCS in (X,τ).

Then IFβC(X) = {0∼, 1∼, $\mu_a \epsilon[0,1]$, $\mu_b \epsilon [0,1]$, $\nu_a \epsilon[0,1]$, $\nu_b \epsilon [0,1]$ / provided $\mu_b <0.7$ whenever $\mu_a\geq0.6$, $\mu_a <0.6$ whenever $\mu_b \geq0.7$, $0\leq \mu_a+\nu_a\leq1$ and $0 \leq \mu_b+\nu_b\leq 1$}.
As βcl(A) = A, we have A is an IFβGCS in X and βcl(B) = B, we have B is an IFβGCS in X. Now A ∪ B = ⟨x, $(0.6_a, 0.8_b)$, $(0.4_a, 0.2_b)$⟩ ⊆ G_1, where G_1 is an IFβOS, but βcl(A ∪ B) = 1∼ ⊈ G_1.

Theorem 3.22: Let (X,τ) be an IFTS. Then for every A ϵ IFβGC(X) and for every B ϵ IFS(X), A ⊆ B ⊆ βcl(A) ⇒ B ϵ IFβGC(X).
Proof: Let B ⊆ U and U be an IFβOS. Then since, A ⊆ B, A ⊆ U. By hypothesis, B ⊆ βcl(A). Therefore βcl(B) ⊆ βcl(βcl(A)) = βcl(A) ⊆ U, since A is an IFβGCS. Hence B ϵ IFβGC(X).

Theorem 3.23: An IFS A of an IFTS (X,τ) is an IFβGCS if and only if $A\,{}_q^c\,F \Rightarrow \beta cl(A)\,{}_q^c\,F$ for every IFβCS F of X.
Proof: (Necessity): Let F be an IFβCS and $A\,{}_q^c\,F$, then A ⊆ F^c [9], where F^c is an IFβOS. Then βcl(A) ⊆ F^c, by hypothesis. Hence again [9] $\beta cl(A)\,{}_q^c\,F$.
Sufficiency: Let U be an IFβOS such that A ⊆ U. Then U^c is an IFβCS and A ⊆ $(U^c)^c$. By hypothesis, $A\,{}_q^c\,U^c \Rightarrow \beta cl(A)\,{}_q^c\,U^c$. Hence by [9], βcl(A) ⊆ $(U^c)^c$ = U. Therefore βcl(A) ⊆ U. Hence A is an IFβGCS.

Theorem 3.24: Let (X,τ) be an IFTS. Then every IFS in (X,τ) is an IFβGCS if and only if IFβO(X) = IFβC(X).
Proof : (Necessity): Suppose that every IFS in (X,τ) is an IFβGCS. Let U ϵ IFβO(X), and by hypothesis, βcl(U) ⊆ U ⊆ βcl(U). This implies βcl(U) = U. Therefore U ϵ IFβC(X). Hence IFβO(X) ⊆ IFβC(X). Let Aϵ IFβC(X), then $A^c \epsilon$ IFβO(X) ⊆ IFβC(X). That is, $A^c \epsilon$ IFβC(X). Therefore AϵIFβO(X). Hence IFβC(X) ⊆ IFβO(X). Thus IFβO(X) = IFβC(X).
Sufficiency: Suppose that IFβO(X) = IFβC(X). Let A ⊆ U and U be an IFβOS. By hypothesis βcl(A) ⊆ βcl(U) =U, since U ϵ IFβC(X). Therefore A is an IFβGCS in X.

Theorem 3.25: If A is an IFβOS and an IFβGCS in (X,τ) then A is an IFβCS in (X,τ).
Proof: Since $A \subseteq A$ and A is an IFβOS, by hypothesis, $\beta cl(A) \subseteq A$. But $A \subseteq \beta cl(A)$. Therefore $\beta cl(A) = A$. Hence A is an IFβCS.

Theorem 3.26: Let A be an IFβGCS in (X,τ) and $p_{(\alpha,\beta)}$ be an IFP in X such that $int(p_{(\alpha,\beta)})_q \beta cl(A)$, then $int(cl(int(p_{(\alpha,\beta)})))\ _q$ A.
Proof: Let A be an IFβGCSand let $(int(p_{(\alpha,\beta)}))_q \beta cl(A)$.
Suppose $int(cl(int(p_{(\alpha,\beta)})))\ _q{}^c$ A, since by [9] $A \subseteq [int(cl(int(p_{(\alpha,\beta)})))]^c$.This implies $[int(cl(int(p_{(\alpha,\beta)})))]^c$ is an IFβOS. Then by hypothesis,
$\beta cl(A) \subseteq [int(cl(int(p_{(\alpha,\beta)})))]^c$
$= cl(int(cl[(p_{(\alpha,\beta)})]^c$.
$\subseteq cl(cl[(p_{(\alpha,\beta)})]^c$.
$= cl[(p_{(\alpha,\beta)})]^c$.
$= (int(p_{(\alpha,\beta)}))^c$. This impliesint$(p_{(\alpha,\beta)})_q{}^c \beta cl(A)$,which is a contradiction to the hypothesis. Hence $int(cl(int(p_{(\alpha,\beta)})))\ _q$ A.

Theorem 3.27: Let $F \subseteq A \subseteq X$ where A is an IFβOS and an IFβGCS in X. Then F is an IFβGCS in A if and only if F is an IFβGCS in X.

Proof: Necessity: Let U be an IFβOS in X and $F \subseteq U$. Also let F be an IFβGCS in A. Then clearly $F \subseteq A \cap U$ and $A \cap U$ is an IFβOS in A. Hence the β closure of F in A, $\beta cl_A(F) \subseteq A \cap U$. By theorem 3.25, A is an IFβCS. Therefore $\beta cl(A) = A$ and the β closure of F in X, $\beta cl(F) \subseteq \beta cl(F) \cap \beta cl(A) = \beta cl(F) \cap A = \beta cl_A(F) \subseteq A \cap U \subseteq U$. That is, $\beta cl(F) \subseteq U$ whenever $F \subseteq U$. Hence F is an IFβGCS in X.

Sufficiency: Let V be an IFβOS in A such that $F \subseteq V$. Since A is an IFβOS in X, V is an IFβOS in X. Therefore $\beta cl(F) \subseteq V$, since F is an IFβGCS in X. Thus $\beta cl_A(F) = \beta cl(F) \cap A \subseteq V \cap A \subseteq V$. Hence F is an IF$\beta$GCS in A.

Theorem 3.28: For an IFS A, the following conditions are equivalent:

(i) A is an IFOS and an IFβGCS

(ii) A is an IFROS

Proof: (i) $\Rightarrow$ (ii) Let A be an IFOS and an IFβGCS. Then $\beta cl(A) \subseteq A$ and $A \subseteq \beta cl(A)$ this implies that $\beta cl(A) = A$. Therefore A is an IFβCS, since $int(cl(int(A))) \subseteq A$. Since A is an IFOS, $int(A) = A$. Therefore $int(cl(A)) \subseteq A$. Since A is an IFOS, it is an IFPOS. Hence $A \subseteq int(cl(A))$. Therfore $A = int(cl(A))$. Hence A is an IFROS.

(ii) $\Rightarrow$ (i) Let A be an IFROS. Therefore $A = int(cl(A))$. Since every IFROS in an IFOS and $A \subseteq A$. This implies $int(cl(A)) \subseteq A$. That is $int(cl(int(A))) \subseteq A$. Therfore A is an IF$\beta$CS. Hence A is an IF$\beta$GCS.

Theorem 3.29: For an IFOS A in (X,τ), the following conditions are equivalent.

(i) A is an IFCS

(ii) A is an IFβGCS and an IFQ-set

Proof: (i) $\Rightarrow$ (ii) Since A is an IFCS, it is an IFβGCS. Now $int(cl(A)) = int(A) = A = cl(A) = cl(int(A))$, by hypothesis. Hence A is an IFQ-set[8].

(ii) $\Rightarrow$ (i) Since A is an IFOS and an IFβGCS, by theorem 3.28, A is an IFROS. ThereforeA $= int(cl(A)) = cl(int(A)) = cl(A)$, by hypothesis. A is an IFCS.

Theorem3.30: Let (X,τ) be an IFTS, then for every $A \in IFSPC(X)$ andfor every B in X, $int(A) \subseteq B \subseteq A \Rightarrow B \in IF\beta GC(X)$.

Proof: Let A be an IFSPCS in X. Then there exists an IFPCS, (say) C such that $int(C) \subseteq A \subseteq C$. By hypothesis, $B \subseteq A$. Therefore $B \subseteq C$. Since $int(C) \subseteq A$, $int(C) \subseteq int(A)$ and $int(C) \subseteq B$, by hypothesis. Thus $int(C) \subseteq B \subseteq C$ and by [5], $B \in IFSPC(X)$. Hence by Theorem 3.18,$B \in IF\beta GC(X)$.

References

[1] K. Atanassov. Intuitionistic Fuzzy SetsFuzzy Sets and Systems,1986, 87-96.
[2] C. Chang. Fuzzy Topological SpacesJ. Math. Anal. Appl., 1968, 182-190.
[3] D. Coker. An Introduction to Intuitionistic Fuzzy Topological Space,Fuzzy Sets and Systems,1997, 81-89.
[4] D. Coker and M.Demirci. On Intuitionistic Fuzzy Points,Notes on Intuitionistic Fuzzy Sets1(1995), 79-84.
[5] H. Gurcay, D. Coker andEs. A.Haydar.On Fuzzy Continuity in Intuitionistic Fuzzy Topological Spaces, The J. Fuzzy Mathematics, 1997,365-378.
[6] D. Jayanthi.Generalized β Closed Sets in Intuitionistic Fuzzy Topological Spaces,International Journal of Advance Foundation and Research in Science & Engineering (IJAFRSE) Volume1, Issue 7, December 2014.
[7] D. Jayanthi.Relation BetweenSemipre Closed Sets andBeta Closed sets in Intuitionistic Fuzzy Topological Spaces, Notes on Intuitionistic Fuzzy Sets,Vol. 19, 2013, 53-56.
[8] R. Santhi, and D. Jayanthi. Generalized Semi- Pre Connectedness in Intuitionistic Fuzzy Topological Spaces,Annals of Fuzzy Mathematics and Informatics, 3, 2012, 243-253.
[9] S.S. Thakurand Rekha Chaturvedi. Regular Generalized Closed Sets in Intuitionistic Fuzzy Topological Spaces, Universitatea Din Bacau, Studii SiCercetariStiintifice,Seria: Mathematica, 2006, 257-272.
[10] S.S. Thakur and S. Singh.On Fuzzy Semi-pre Open Sets and Fuzzy Semi-pre Continuity, Fuzzy Sets and Systems, 1998, 383-391.
[11] Young Bae Jun &seok – Zun Song. Intuitionistic Fuzzy Semi-pre Open Sets and Intuitionistic Semi-pre Continuous Mappings, jour. of Appl. Math & computing, *2005, 465-474.*
[12] L.A. Zadeh.Fuzzy Sets, Information and control,1965, 338-353.