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Energy-Mass-Size balance model for dynamic control of a wet open circuit grinding mill: Part I - Model development

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Abstract

Majority of traditional mill simulation models have been successfully applied in mill circuit design and optimization. However, only limited success has been reported with respect to dynamic mill control. This is partly attributable to the fact that these models are based on steady state analysis and fail to account for energy balance inside the mill. In this paper, an integrated model has been developed based on energy-mass-size balance for dynamic simulation of open circuit wet grinding mills under MATLAB environment. The model combines information from energy-mass balance, material breakage mechanisms, fundamental material properties and the milling conditions in a simple and clear representation of the physics and thermodynamics of the wet milling process, thereby addressing the limitations of traditional model formulations. Simulation test results depicting the dynamic response of the model are presented in Part II. Successful validation of this modelling approach would go alongside improving the current methods of continuous control of grinding mills.

Keywords: *Energy balance, Modelling, Simulation, Mixing, Grinding mill, Mill power, Mill temperature*

I. Introduction

Effective mill control is believed to be a key element in the drive towards lower energy and material costs in grinding circuits but such a control system would require good understanding of the dynamic behaviour of mills when the operating variables are changed and the interactions between external process variables (open to manipulations) and performance variables of the process. In deed grinding mills in mineral processing plants never really operate under steady state conditions as variations in feed size distributions, feed rate, feed slurry density and ore characteristics continuously upset the smooth running process. Dynamic simulation (on-line or off-line) can help to gain better understanding of the dynamic behaviour of the mill by assessing the "what if" situations. Hence, prior insights into the behaviour of the milling circuit can be made available such as response to grinding process disturbances during transitions between various steady-states. Also, the complex interactions within the mill circuit can be explored and evaluated with minimum cost and changes can then be made to the circuit to realise high efficiency and throughput. The key elements required for accurate dynamic simulation are good dynamic models, precise measurements of process variables and accurate estimation procedures for process parameters [1]. A range of simulation models exist for mill simulation (JKSimMet, METSIM, MODSIM etc) and have been widely used for mill circuit design and optimization with varying levels of success. However these models do not provide for analysis of energy balance inside the mill and are mainly based on steady state analysis. Energy balance is a key aspect in control and optimization of a comminution process apart from mass and size balance. Energy balance data which is related to the mill temperature profile is believed to be a key indicator of the process dynamics as shown in the work by Van Drunick and Moys [2]. Interpretation of the temperature signature may yield some useful information of the mill operation characteristics such as variations in feed rate, slurry properties and holdup mass inside the mill which are directly linked to grinding efficiency. Having the knowledge of these dynamics enables effective control of the grinding circuit which in turn results in stable operation with optimum production capacity and energy efficiency. This research broadly aims to develop dynamic models based on energy, mass and size balance for simulation of open circuit wet ball mills under MATLAB environment. It integrates the energy balance with the population balance model in a simple and clear representation of the physics and thermodynamics of the wet ball milling process. Easily measurable parameters such as mill temperature and product size distribution would serve as control parameters. The success of this technique would go alongside improving the current methods of continuous control of grinding mills.

II. Development of dynamic model

A dynamic model is developed for a wet overflow ball mill based on a set of mass and energy balances to simulate the mill. The energy balance relies on temperature and mass flow data. The key control parameters to be measured are mill temperature (feed and discharge streams), mill power draw and the mass flow rate in the feed stream. Manipulated variables are feed size distribution, feed % solids, feed dilution water and ore hardness. Figure 1 is a schematic representation of the energy and mass flow streams around the mill operating in an open circuit configuration.

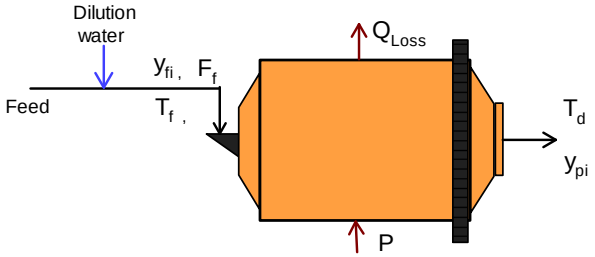

Figure 1: Representation of mass and energy streams around the mill and the modelling structure in MATLAB

Where, P is the power input (kW) to the mill (corrected for motor losses), F_f is the feed flow rate into the mill (fresh feed + dilution water), T_f is the temperature of the mill feed, T_d is the temperature of mill discharge, Q_{Loss} is the rate of energy loss to the environment (kJ/s), y_{fi} is the mass fraction of mill feed solids in size class i while y_{pi} is the mass fraction of mill product solids in size class i.

The mill is depicted as system of N perfectly mixed cells of equal volume, *V* with a net volumetric flow rate, F_f and recirculation rate, F_b. The Figure 2 is the conceptual framework of the model. The total flow rate through stages 1 and N is $F_f + F_b$ while the flow rate through stages 2 to N-1 is $F_f + 2F_b$. The set of equations which describes the dynamics of the system can be obtained by performing a material balance. The temperature of the mill discharge is assumed to be equal to the temperature in the last mixer and so is the solids concentration in the slurry.

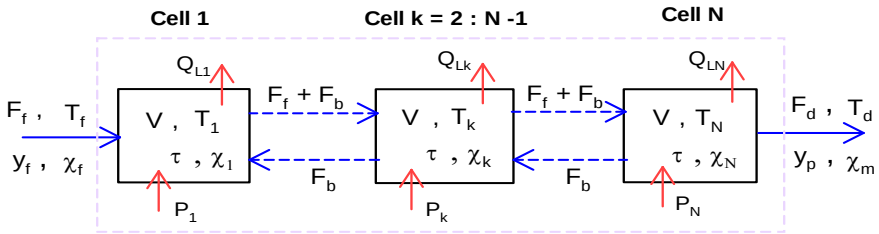

Figure 2: Depiction of in-mill mass and energy streams represented by N equally sized and fully mixed segments with back-mixing.

The model inputs are the breakage and selection function parameters, mill feed rate, mill dilution water, feed solids concentration, feed size distribution, ore characteristics, mill rotational speed and mill fill level. The outputs (target variables) are the mill discharge temperature, mill power and the product size distribution.

2.1 Energy balance

From the fundamental laws of physics, the energy profile within a body depends upon the rate of energy input including internal generation, its capacity to store some of the energy and the rate of energy transfer to the surrounding environment i.e. Energy input = Energy output + Energy accumulation. Considering a wet ball mill operating in a continuous mode, then this equation can be expanded as follows: E_{in} *(Power, mill feed) =* E_{out} *(mill discharge) +* $E_{absorbed}$ *(particle fracture) +* E_{loss} *(Sound, Vibration, Evaporation, conduction, convection, radiation) + accumulation.*

$$Q_f(t) + P(t) = Q_d(t) + Q_p(t) + Q_{Loss}(t) + \frac{dE(t)}{dt} \quad (1)$$

Where the subscripts *f and d* denote feed and discharge respectively while the symbol *P* represents the mill power. The term Q_p represents the actual energy absorbed per unit time by the particles to cause fracture, which is a function of material properties and mill conditions [3]. Typically this is so small relative to the total energy input to the mill that it cannot be reliably measured. Since the model will be tested and validated against data obtained on a single mill, some simplifying assumptions will be considered in the energy balance. The assumptions are as follows:

- That all the net power supplied to the mill, is transmitted to the mill charge as frictional energy where part of it is converted to heat and lost to the environment through mill discharge product and vapour with the rest being lost in form of sound and vibration.
- That the mill contents are well mixed both in the radial and axial directions.
- That the slurry and grinding media in the mill are at thermal equilibrium and that the reference enthalpy of water and solids is zero at 0^0C.

The overall dynamic energy balance around the mill can then be presented as in equation 2 where Q_{Loss} represents the energy loss per unit time which comprises losses through conduction, convection, vapour, sound and vibration while M is the slurry holdup mass inside the mill approximated as, $(M = \tau \times F_m)$.

$$\frac{d\left[C_m M(t)T(t)\right]}{dt} = C_f T_f F_f(t) - C_d T(t) F_d(t) + P(t) - Q_p(t) - Q_{Loss}(t) \quad (2)$$

The value of C (specific heat capacity of slurry) varies with the proportion of solids (χ) in the slurry and can be estimated by equation 3 in which the subscripts *s, w, f, m* denote solids, water, feed and mill respectively.

$$\left.\begin{aligned} C_m &= \chi_m C_s + (1-\chi_m) C_{wm} \\ C_f &= \chi_f C_s + (1-\chi_f) C_{wf} \end{aligned}\right\} \quad (3)$$

The rate of energy loss through sound and vibration is typically small relative to the net energy input into the mill, hence it could be neglected. Also, for mills fitted with rubber liners, energy loss through convection and conduction is minimal and hence can be ignored (note that measuring liner temperature could equally be a challenging task). A significant proportion of the energy loss to the environment will occur through evaporation (i.e. heat loss through vapour at the feed and discharge openings) which has to be accounted for. Therefore besides the temperature measurements, the prediction accuracy of this model will depend on correct estimation of Q_{Loss} and Q_p where latter represents the actual energy expended per unit time to cause particle fracture (i.e. to create new surfaces), which is proportional to the new surface area created. Using standard bond work index, Q_p is estimated to lie between 1- 3% of the net energy input to the mill [4, 5]. Therefore, in this study, Q_p is modeled as a function of the net mill power draw in the form $Q_p = \lambda_p P$ where λ_p is a constant that is dependent on ore characteristics (hardness and size) and could be estimated by the semi-empirical relation $\lambda_p = \kappa \upsilon^{3.5} / \pi$ in which, υ is the Poisson ratio of the ore material while κ is the size coefficient (= 6 for spherical particles).

The rate of energy loss from the mill through water vapour is a function of in-mill temperature, the humidity of the air overlaying the mill load and the slurry-air interfacial area where the latter is related to the size of the slurry pool. Thus, Q_{Loss} can be estimated by the following relation.

$$Q_{Loss}(t) = K_v \lambda_L \rho_a L_m D_m (1-J)\left[q_s - q_a\right]/\tau \quad ; \quad for \quad J \le J_{max} \quad (4)$$

Where λ_L (kJ/kg) is the latent heat of vapourization [= 2501 – 2.361(T-273)], ρ_a is the density of overlaying air, L_m is the mill length, J is the load volumetric filling (J_{max} is dependent on the size of the discharge trunnion), D_m is the mill diameter while K_v is a parameter whose value is to be estimated and is related to the volume of vapour generated. The parameters q_s and q_a represent the specific humidity of air overlaying the load at saturated and unsaturated states respectively which are determined by the following expressions in which, ε (= 0.622) is the ratio of molecular weight of water vapour to that of dry air and RH is the relative humidity (%) at temperature, T [6].

$$\begin{aligned} &q_s = \varepsilon p_{sat} / \left\{ p_{atm} - (1-\varepsilon) p_{sat} \right\} \quad ; \quad q_a = \varepsilon p_v / p_{(dryair)} \quad ; \quad p_v = p_{sat} RH / 100 \\ &where \quad p_{sat} = 0.6108 \exp\left[17.27(T-273)/(T-35.7)\right] \quad in \ kPa, \end{aligned} \quad (5)$$

To further assess the impact of change in the proportion of solids in slurry on the in-mill temperature, the residence time τ has been correlated to the feed solids concentration in the form, ($\tau = k_1 \chi_f - k_2 J + k_3 / F_f$) [7]. Here, k_1, K_2, K_3 are empirical constants to be determined by regression while *J* is the fractional filling of the mill with charge. Now, performing an energy balance around the mill, we obtain the following equations that describe the temperature variation with time for the n mixers representing the mill. The symbols ℓ and $\bar{\tau}$ represent the length and residence time of a single mixer respectively while φ_b is the back-mixing coefficient $\left\{= F_b \Big/ \left(F_f + F_b\right)\right\}$ which is related to the axial dispersion coefficient (see Figure 2).

Mixer 1:
$$\frac{dT_1(t)}{dt} = \frac{1}{\bar{\tau} C_1}\left\{\begin{array}{l} C_f T_f(t) + \dfrac{C_2 \varphi_b T_2(t) - C_1 T_1(t)}{(1-\varphi_b)} + \dfrac{(1-\lambda_p) P_1(t)}{F_f(t)} \ldots \\ - \dfrac{K_v \rho_a \lambda_L \ell D_m (1-J)\left(q_{s_1} - q_{a_1}\right)}{\bar{\tau} F_f(t)} \end{array}\right\} \quad (6a)$$

Mixer k = 2 to N-1:
$$\frac{dT_k(t)}{dt} = \frac{1}{\bar{\tau} C_k}\left\{\begin{array}{l} \dfrac{C_k T_k(t) + C_{k+1}\varphi_b T_{k+1}(t) - C_k(1+\varphi_b) T_k(t)}{(1-\varphi_b)} + \dfrac{(1-\lambda_p) P_k(t)}{F_f(t)} \ldots \\ - \dfrac{K_v \rho_a \lambda_L \ell D_m (1-J)\left(q_{s_k} - q_{a_k}\right)}{\bar{\tau} F_f(t)} \end{array}\right\} \quad (6b)$$

Mixer N:
$$\frac{dT_n(t)}{dt} = \frac{1}{\bar{\tau} C_n}\left\{\begin{array}{l} \dfrac{C_{n-1} T_{n-1}(t) - C_n T_n(t)}{(1-\varphi_b)} + \dfrac{(1-\lambda_p) P_n(t)}{F_f(t)} \ldots \\ - \dfrac{K_v \rho_a \lambda_L \ell D_m (1-J)\left(q_{s_n} - q_{a_n}\right)}{\bar{\tau} F_f(t)} \end{array}\right\} \quad (6c)$$

The values of the back-mixing coefficient (φ_b) lie between 0 (no mixing) and 1 (perfectly mixed state). This parameter is a strong function of solids fraction in slurry and could be estimated by the relation, ($\varphi_b = k_A + k_B / \chi_f$) proposed by Makokha and Moys [7], where k_A = -0.114 and k_B = 0.591 are obtained by regression.

2.2 Mill power

The mill power is expected to vary with changes in mill holdup mass, feed size distribution and slurry properties. The equation used here to predict the power drawn by the mill is based on modification of Hogg and Fuerstenau power model. The model gives the relationship of mill power draw with respect to mill dimensions and operating conditions and treats the contribution of grinding media, ore and slurry as independent components in charge. The gross mill power is expressed as,

$$P_L = K_p D^{3.5} (L/D) N_c \rho_{ap} \left(J - 1.065 J^2\right) Sin\psi \quad (7)$$

Where K_p is a fitting parameter that corrects for the inaccuracies in estimation of the dynamic load shape as well as the effect of slurry pool volume on mill power. The symbols L and D represent mill dimensions in meters, J is the mill filling as a fraction of the total mill volume, N_c is the mill rotational speed as a fraction of the critical speed, ψ is the load angle of repose (in radians) while ρ_{ap} is the apparent charge density (media and ore) given by the following equation [9].

$$\rho_{ap} = J_b(1-\varepsilon_v)\rho_b + (J-J_b)\rho_{ore}(1-\varepsilon_v) + \rho_{sl}U\varepsilon_v J \quad (8)$$

Where J_b is the balls filling as a fraction of mill volume, ε_v is the voidage within the load while U is the powder / Slurry filling in the load interstices. The load angle of repose, ψ (radians) is dependent on the load dynamic behaviour and is mathematically computed from the load toe angle, θ_T (radians) and shoulder angle, θ_S (radians) as, $\psi = \left[(\theta_T + \theta_S)/2\right] - \pi$. The load toe and shoulder angles can be easily measured by online instruments, but for purpose of modelling, the load angles (θ_T and θ_S) could be estimated by the following relations that are slightly modified from those presented by Apelt [8] and Morrell [9].

$$\left.\begin{aligned}\theta_T &= \frac{\pi}{2} + a_1\chi_m(1-a_2J)\left(1-\exp^{-19.42N_c}\right)\\ \theta_S &= 2\pi - \theta_T(b_1 - b_2\chi_m N_c)/J\end{aligned}\right\} \quad (9)$$

In which, a_1, a_2, b_1, b_2 are parameters determined by regression analysis. In this study, data collected from an industrial ball was used to estimate the parameters and the optimal values obtained are: $a_1 = 1.42;\ a_2 = 1.13;\ b_1 = 0.145;\ b_2 = 0.025$.

2.3 Mass-Size balance

The size-mass balance is developed with reference to the mill system in Figure 1. Applying the principle of mass conservation to the milling process, the rate of accumulation of material of size i = rate of entry of material of size i from the feed + entry of material of size i from breakage of larger sizes - material destroyed in size i by fracture – disappearance of material of size i through discharge. This can be mathematically represented by the following steady state equation.

$$\chi_m F_m y_{pi} = \chi_f F_m y_{fi} - \chi_f F_d y_{pi} + \chi_m M\left(\sum_{j=1;i>1}^{i-1} b_{ij}S_j l_j(t) - S_i l_i\right) \quad (10)$$

$$where: b_{i,j} = B_{i,j} - B_{i+1,j},\ n > i > j\ ; \qquad b_{n,j} = B_{n,j},\quad j = n$$

The parameters *b* and *S* are the breakage distribution function and the selection function respectively (that can be obtained using equations proposed by Austin *et al* [10], while F_m, χ_m and χ_f denote the mass flow rate through the mill, the slurry solids fraction inside the mill and in the mill feed respectively. The rest of the parameters retain their earlier definitions. For a well-mixed mill, the size of particles inside the mill would be equal to the mill product size ($l_i = y_{pi}$) and the average residence time, $\tau = M/F_m$.

Austin *et al* [10] presented an equation for determining the selection function which characterizes particles only by their sizes. However, It is well recognized that the mechanical properties of brittle materials such as mineral bearing ore rocks strongly depend on deformation rate and strain rate. Therefore in population balance modelling of particle breakage process, the selection function should allow for particles to be characterized simultaneously by their size and fracture energy. The function should comprise the probability of a particle being selected for breakage and the probability of the energies generated by the impacts being sufficient to break the particles.

According to the work by Crespo [11], the probability of the energy applied being sufficient to break the particle is related to the fraction of the absorbed impact energies in a given time interval that are higher than the fracture energy. Similar observations were made by Tugcan and Rajamani [12] in a separate study using ultra fast load cell data who further indicated the dependence of this probability on particle size and material specific properties. For instance, a material with a higher hardness index would store more strain energy during deformation hence depending on the level of applied strain, it may require repeated impacts to fracture a hard material. This observation is supported by the data from Discrete Element Method (DEM) presented by Rajamani *et al* [13] and Datta *et al* [14], which established that the impact frequency inside the mill depends on the level of energy applied. In a continuous ball mill, the applied impact energy (E_p) could be equated to the ratio of the actual energy expended to cause particle fracture (Q_p) to the mass feed rate of material into the mill as follows:

$$E_p \approx Q_P/F_m = \lambda_p P/F_m \quad (11)$$

Where P is the net mill power (kW), F_m is the mass flow rate through the mill (kg/s) while the parameter λ_p (discussed earlier in section 2) represents the proportion of energy input to the mill that is actually utilized in breakage of the particles which lies between 1 – 3% [4, 5].

Due to lack of appropriate data on fracture energies of various mineral ore rocks, here we suggest the use of hardness number (ξ), which is an easily measurable variable to simulate the energy-dependent rate of particle fracture on single impact. Based on the Mohs' hardness scale (http://www.geology.com), the values of ore hardness number (ξ) range from 1 to 10 corresponding respectively to the softest and hardest ores. The hardness number could be standardized to a hardness index $\{\xi^* = 1/\log(2\xi+1)\}$ with values of < 1, >1 or 1 corresponding to softer, harder and normal ores respectively. Using this simplifying assumption, then the dynamic mass-size balance model can be re-written as follows:

$$\frac{d\left[M(t)y_{pi}(t)\right]}{dt} = F_{fs}(t)y_{fi}(t) - F_{ds}(t)y_{pi}(t) + \xi^*\left\{\sum_{\substack{j=1\\ i>1}}^{i-1} b_{ij}S_j M(t)y_{pj}(t) - S_i M(t)y_{pi}(t)\right\} \quad \text{..... (12)}$$

The vectors, $y_{pi}(t) = [y_{p1}(t), y_{p2}(t), \ldots\ldots y_{pn}(t)]^T$ and $y_{fi} = [y_{f1}, y_{f2}, \ldots\ldots y_{fm}]^T$ represent the mass fraction of mill product solids and mill feed solids in discrete size classes respectively while F_{fs} $(= \chi_f F_f)$ and F_{ds} $= (\chi_m F_d)$ are the mass flow rates of the solids in the feed and discharge streams respectively. For overflow mills, the type used in this study, the volume of slurry present in the mill is reasonably constant over a wide range of operating conditions, hence the volumetric feed rate to the mill equals volumetric discharge rate at all times. Utilizing this fact, the variation in proportion of solids in slurry, defined here as the mass of solids per unit mass of slurry in the mill, can be described by the following equation.

$$\frac{d\chi_m(t)}{dt} = \frac{1}{\tau}(\chi_f - \chi_m) \quad (13)$$

The variation of water holdup inside the mill that results from changes in the water flow rates in the feed and discharge streams as the slurry solids concentrations changes could be represented as follows:

$$\frac{dm_w(t)}{dt} = F_w + F_f(\chi_m - \chi_f) \quad (14)$$

Similarly to the energy balance, the size-mass balance is performed with reference to the mill system presented in Figure 2. The superscripts in the brackets denote the mixing cell number i.e. k = 1 to n where n is the total number of mixing cells considered.

Mixer 1:

$$\frac{d\left[m_1(t)y_{pi}^{(1)}(t)\right]}{dt} = F_f(t)\left\{\chi_f y_{fi}(t) + \frac{\chi_2\varphi_b y_{pi}^{(2)}(t)}{(1-\varphi_b)} - \frac{\chi_1 y_{pi}^{(1)}(t)}{(1-\varphi_b)}\right\} + m_1(t)\xi^*(t)\left\{\sum_{\substack{j=1\\ i>1}}^{i-1} b_{ij}S_j y_{pj}^{(1)}(t) - S_i y_{pi}^{(1)}(t)\right\}$$

(15a)

Mixer k = 2 to n-1:

$$\frac{d\left[m_k(t)y_{pi}^{(k)}(t)\right]}{dt} = \frac{F_f(t)}{(1-\varphi_b)}\left\{\chi_{k-1}y_{pi}^{(k-1)}(t) + \chi_{k+1}\varphi_b y_{pi}^{(k+1)}(t) - \chi_k(1+\varphi_b)y_{pi}^{(k)}(t)\right\} \ldots$$

$$+ m_k(t)\xi^*(t)\left\{\sum_{\substack{j=1\\ i>1}}^{i-1} b_{ij}S_i y_{pj}^{(k)}(t) - S_i y_{pi}^{(k)}(t)\right\}$$

(15b)

Mixer n:

$$\frac{d\left[m_n(t)\,y_{pi}^{(n)}(t)\right]}{dt} = \frac{F_f(t)}{(1-\varphi_b)}\left\{\chi_{n-1}y_{pi}^{(n-1)}(t) - \chi_n y_{pi}^{(n)}(t)\right\} + m_n(t)\,\xi^*(t)\left\{\sum_{\substack{j=1\\ i>1}}^{i-1} b_{ij}S_j y_{pj}^{(n)}(t) - S_i y_{pi}^{(n)}(t)\right\}$$

(15c)

The mass holdup of solids in each individual mixer at any time instant is dependent on the level of solids concentration in the mixer, the mass feed rate and the residence time of the individual mixer as follows:

$$m_1(t) = \bar{\tau}\,\chi_1 F_f(t)\;;\quad m_k(t) = \bar{\tau}\,\chi_k F_f(t)\;;\quad m_n(t) = \bar{\tau}\,\chi_n F_f(t)\quad\Big\} \tag{16}$$

It is noteworthy to mention that while all the n mixers may be equal in volume, due to slight variations in slurry solids concentration between the mixers, the mass of solids m_1, m_k and m_n may not be equal. Performing the solids balance around the n mixers, we obtain the following equations representing the dynamic variation of solids concentration in each respective mixer.

Mixer 1:
$$\frac{d\chi_1(t)}{dt} = \frac{1}{\bar{\tau}}\left[\chi_f + \frac{\chi_2\varphi_b - \chi_1}{1-\varphi_b}\right]$$

(17a)

Mixer k = 2 to N-1:
$$\frac{d\chi_k(t)}{dt} = \frac{1}{\bar{\tau}}\left[\frac{\chi_{k-1} + \chi_{k+1}\varphi_b - (1+\varphi_b)\chi_k}{1-\varphi_b}\right]$$

(17b)

Mixer N:
$$\frac{d\chi_n(t)}{dt} = \frac{1}{\bar{\tau}}\left(\frac{\chi_{n-1} - \chi_n}{1-\varphi_b}\right)$$

(17c)

The task of mass and energy balancing was accomplished numerically using MATLAB ode45 solver.

III. Conclusions

A dynamic energy-mass-size balance model has been developed that can be utilized for dynamic simulation of a wet open circuit grinding mill. This model may serve as predictive tools for providing insights of the process and thus it would lay a ground on which sound control schemes can be created for improved mill product quality and process performance. Simulation test results using this model are presented in Part II of this research.

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