ISSN (e): 2250 – 3005 || Volume, 06 || Issue, 01 ||January – 2016 ||
International Journal of Computational Engineering Research (IJCER)

Existence and Stability of the Equilibrium Points in the Photogravitational Magnetic Binaries Problem When the Both Primaries Are Oblate Spheriods

Mohd Arif
Department of mathematics, Zakir Husain Delhi college New Delhi 110002, India

Abstract *: In this article we have discussed the equilibrium points in the photogravitational magnetic binaries problem when the bigger primary is a oblate spheroid and source of radiation and the small primary is a oblate body and have investigated the stability of motion around these points.*

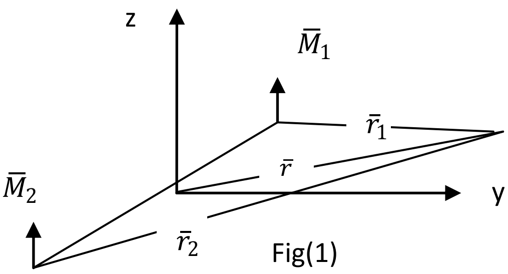
Key words : *magnetic binaries problem, source of radiation, stability, equilibrium points, restricted three body problem, oblate body.*

I. Introduction

In (1950) Radzievskii have studied the Sun-planet-particle as photogravitational restricted problem, which arise from the classical restricted three body problem when one of the primary is an intense emitter of radiation. In (1970) Chernikov and in (1980) Schuerman have studied the existence of equilibrium points of the third particle under the influence of gravitation and the radiation forces. The stability of these points was studied in the solar problem by Perezhogin in (1982). The lagrangian point and there stability in the case of photogravitational restricted problem have been studied by K.B.Bhatnagar and J. M. Chawla in (1979). Mavraganis. A. (1979) have studied the stationary solutions and their stability in the magnetic-binary problem when the primaries are oblate spheroids. Khasan (1996) studied libration solution to the photogravitational restricted three body problem by considering both of the primaries are radiating. He also investigated the stability of collinear and triangular points. In (2011) Shankaran, J.P.Sharma and B.Ishwar have been generalized the photogravitational non-planar restricted three body problem by considering the smaller primary as an oblate spheroid. The existence and stability of collinear equilibrium points in, the planar elliptical restricted three body problem under the effect of the photogravitational and oblateness primaries have been discussed by C. Ramesh kumar and A. Narayan in (2012). In (2015) Arif. Mohd. have discussed the equilibrium points and there stability in the magnetic binaries problem when the bigger primary is a source of radiation. In this article we have discussed the equilibrium points and there stability in the photogravitational magnetic binaries problem when the bigger primary is a oblate spheroid and source of radiation and the small primary is a oblate body.

II. Equation of motion

Two oblate bodies (the primaries), in which the bigger primary is a source of radiation with magnetic fields move under the influence of gravitational force and a charged particle P of charge q and mass m moves in the vicinity of these bodies. The equation of motion and the integral of relative energy in the rotating coordinate system including the effect of the gravitational forces of the primaries on the charged particle P fig(1) written as:



Fig(1)

$\ddot{x} - \dot{y}\, f = U_x$ (1)
$\ddot{y} + \dot{x}\, f = U_y$ (2)
$\dot{x}^2 + \dot{y}^2 = 2U - C$ (3)

Where

$$f = 2\,\omega - \left(\frac{q_1}{r_1^3} + \frac{q_1 I_1}{2(1-\mu)r_1^5} + \frac{\lambda}{r_2^3} + \frac{\lambda I_2}{2\mu r_2^5}\right), U_x = \frac{\partial U}{\partial x} \text{ and } U_y = \frac{\partial U}{\partial y}$$

$$U = \frac{\omega^2}{2}(x^2+y^2) + \frac{\omega q_1}{(1-\mu)}\{(x^2+y^2) - \mu x\}\left(\frac{(1-\mu)}{r_1^3} + \frac{I_1}{2r_1^5}\right) + \frac{\omega\lambda}{\mu}\{(x^2+y^2) + (1-\mu)x\}\left(\frac{\mu}{r_2^3} + \frac{I_2}{2r_2^5}\right) + \frac{q_1(1-\mu)}{r_1} + \frac{q_1 I_1}{2r_1^3} + \frac{\mu}{r_2} + \frac{I_2}{2r_2^3} \qquad (4)$$

I_i, $(i = 1,2)$ The moments of inertia of the primaries given as the difference of the axial and equatorial moments of inertia.

q_1 is the source of radiation of bigger primary.

$$\omega = 1 + \frac{3I_1}{2(1-\mu)} + \frac{3I_2}{2\mu}$$

Here we assumed

1. Primaries participate in the circular motion around their centre of mass
2. Position vector of P at any time t be $\overline{r} = (xi + yj + zk)$ referred to a rotating frame of reference O(x, y, z) which is rotating with the same angular velocity $\overline{\omega} = (0, 0, \omega)$ as those the primaries.
3. Initially the primaries lie on the x-axis.
4. The distance between the primaries as the unit of distance and the coordinate of one primary is $(\mu, 0, 0)$ then the other is $(\mu-1, 0, 0)$.
5. The sum of their masses as the unit of mass. If mass of the one primaries μ then the mass of the other is $(1-\mu)$.
6. The unit of time in such a way that the gravitational constant G has the value unity and q= mc where c is the velocity of light.

$r_1^2 = (x-\mu)^2 + y^2$, $r_2^2 = (x+1-\mu)^2 + y^2$, $\lambda = \frac{M_2}{M_1}$ (M_1, M_2 are the magnetic moments of the primaries which lies perpendicular to the plane of the motion).

q_1 is the source of radiation of bigger primary.

Therefore, instead of dealing with the full equations of planar magnetic-binaries problem, it makes more sense to work with a system of equations that describe the motion of the charged particle in the vicinity of the secondary mass, this type of system was derived by Hill in 1878. By making some assumptions and transferring the origin of the coordinate system to the second mass fig(2) the equations of motion (1) and (2), become

$$\ddot{\zeta} - \dot{\nu} f_1 = U_\zeta \qquad (5)$$

$$\ddot{\nu} + \dot{\zeta} f_1 = U_\nu \qquad (6)$$

Where

$$f_1 = 2\,\omega - \left(\frac{1}{r_1^3} + \frac{I_1}{2(1-\mu)r_1^5} + \frac{\lambda}{r_2^3} + \frac{\lambda I_2}{2\mu r_2^5}\right), U_\zeta = \frac{\partial U}{\partial \zeta} \text{ and } U_\nu = \frac{\partial U}{\partial \nu}$$

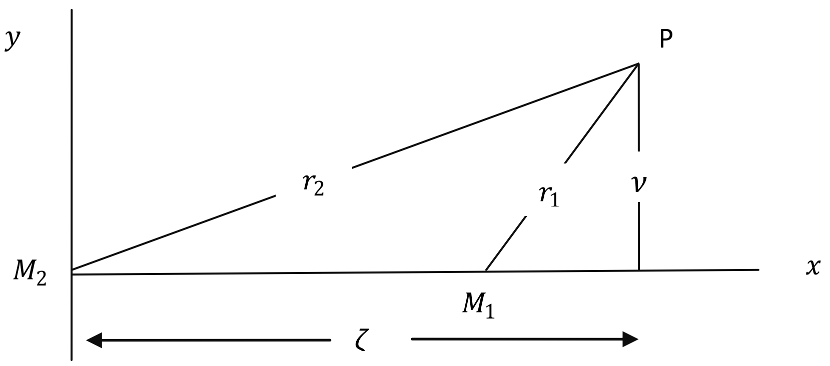
$$U = \frac{\omega^2}{2}((\zeta+\mu-1)^2 + \nu^2) + \frac{\omega q_1}{(1-\mu)}\{((\zeta+\mu-1)^2 + \nu^2) - \mu(\zeta+\mu-1)\}\left(\frac{(1-\mu)}{r_1^3} + \frac{I_1}{2r_1^5}\right) + \frac{\omega\lambda}{\mu}\{((\zeta+\mu-1)^2 +$$

$\nu 2 + (1-\mu)\zeta + \mu - 1\mu r23 + I22r25 + q11-\mu r1 + q1I12r13 + \mu r2 + I22r23$ (7)

$r_1^2 = (\zeta-1)^2 + \nu^2$, $r_2^2 = \zeta^2 + \nu^2$

And new Jacobi constant, C_n, is given by

$$\dot{\zeta}^2 + \dot{\nu}^2 = 2U - C_n \qquad (8)$$



Fig(2)

III. Equilibrium points

The location of equilibrium points in general is given by

$U_{\zeta} = 0$ (9)

$U_{\nu} = 0$ (10)

The solution of equations (9) and (10) results the equilibrium points on the x-axis ($\nu = 0$), called the collinear equilibrium points and other are on xy-plane ($\nu \neq 0$) called the non-collinear equilibrium points(ncep). Here we have solved these equations numerically by use the software mathematica. The tables (1----12) shows that for $\lambda > 0$ and for different values of q_1 there exist four collinear equilibrium points which we denoted by L_1, L_2, L_3 and L_4 but in some values of $q_1$there exist only two equilibrium points L_1 and L_4. In fig 3 and fig 4 we give the position of the points L_1 and L_4 respectively for various values of μ. In these figs the curve $q_1 = 1$ correspond to the case when radiation pressure is not taken into consideration and other curves corresponds to $q_1 = .3$, $q_1 = .5$ and $q_1 = .9$. Here we observed that due to the radiation pressure the both points $L_1$and L_4 shifted towards the origin. It may be also observed that the deviation of the location of the point L_4 from $q_1 = 1$ decreases as μ increases and this becomes zero when $\mu \approx .35$ and again this deviation increases.

$\mu = 3.37608 \times 10^{-4}$ $q_1 = 1$	L_1	L_2	L_3	L_4
$\lambda = 1$	2.39051	-	-	−1.74821
$\lambda = 3$	2.71276			−2.84499
$\lambda = 5$	3.10013			−3.58096
$\lambda = 7$	3.51476			−4.17329

Table (1)

$\mu = 3.37608 \times 10^{-4}$ $q_1 = .5$	L_1	L_2	L_3	L_4
$\lambda = 1$	2.13876	-	-	−1.78185
$\lambda = 3$	2.49977			−2.86665
$\lambda = 5$	2.94143			−3.59742
$\lambda = 7$	3.40119			−4.18691

Table(2)

$\mu = 3.37608 \times 10^{-4}$ $q_1 = .9$	L_1	L_2	L_3	L_4
$\lambda = 1$	2.34821	-	-	−1.75494
$\lambda = 3$	2.67596			−2.84931
$\lambda = 5$	3.07147			−3.580426
$\lambda = 7$	3.49348			−4.17611

Table(3)

$\mu = .132679$ $q_1 = 1$	L_1	L_2	L_3	L_4
$\lambda = 1$	2.45144	.662328	.870220	−1.78160
$\lambda = 3$	2.72307	.715733	.869666	−2.87033
$\lambda = 5$	3.06977	.742964	.868395	−3.60486
$\lambda = 7$	3.46219	.7617777	.866981	−4.14701

Table(4)

$\mu = .132679$ $q_1 = .5$	L_1	L_2	L_3	L_4
$\lambda = 1$	2.19628	.69686	.870099	−1.80221
$\lambda = 3$	2.49117	.753724	.867533	−2.88535
$\lambda = 5$	2.88831	.783278	.864287	−3.61684
$\lambda = 7$	3.33211	.805181	.851632	−4.20709

Table(5)

□ = .132679 $q_1 = .9$	L_1	L_2	L_3	L_4
$\lambda = 1$	2.40893	.667415	.790781	−1.78571
$\lambda = 3$	2.68390	.721353	.869445	−2.87334
$\lambda = 5$	3.03785	.748851	.867999	−3.60726
$\lambda = 7$	3.43826	.767909	.866363	−4.19903

Table(6)

□ = .230437 $q_1 = 1$	L_1	L_2	L_3	L_4
$\lambda = 1$	2.49180	.562939	.779097	−1.80690
$\lambda = 3$	2.73493	.649373	.772205	−2.88902
$\lambda = 5$	3.05355	.697523	.760817	−3.62218
$\lambda = 7$	3.42741	-	-	−4.213880

Table(7)

□ = .230437 $q_1 = .5$	L_1	L_2	L_3	L_4
$\lambda = 1$	2.23395	.618412	.775622	−1.81767
$\lambda = 3$	2.49202	.726268	.745650	−2.89914
$\lambda = 5$	2.85571	-	-	−3.63080
$\lambda = 7$	3.71960	-	-	−4.22138

Table(8)

□ = .230437 $q_1 = .9$	L_1	L_2	L_3	L_4
$\lambda = 1$	2.44904	.571296	.778739	−1.80904
$\lambda = 3$	2.69438	.658573	.770714	−2.89104
$\lambda = 5$	3.01936	.710819	.755144	−3.6239
$\lambda = 7$	3.40138	-	-	−4.21538

Table(9)

□ = .458505 $q_1 = 1$	L_1	L_2	L_3	L_4
$\lambda = 1$	2.57508	.438243	.585582	−1.86992
$\lambda = 3$	2.77151	-	-	−2.93591
$\lambda = 5$	3.03536	-	-	−3.66579
$\lambda = 7$	3.36433	-	-	−4.25648

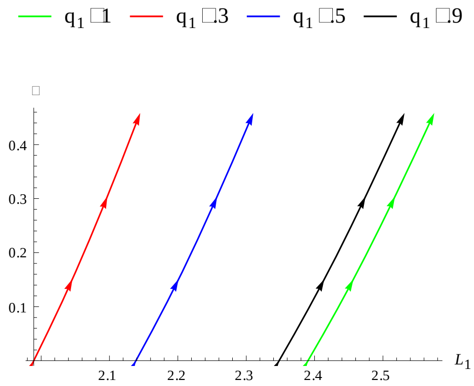
Table(10)

□ = .458505 $q_1 = .5$	L_1	L_2	L_3	L_4
$\lambda = 1$	2.31037	.533495	.552828	−1.85712
$\lambda = 3$	2.50977	-	-	−2.93435
$\lambda = 5$	2.80384	-	-	−3.66641
$\lambda = 7$	3.18660	-	-	−4.25784

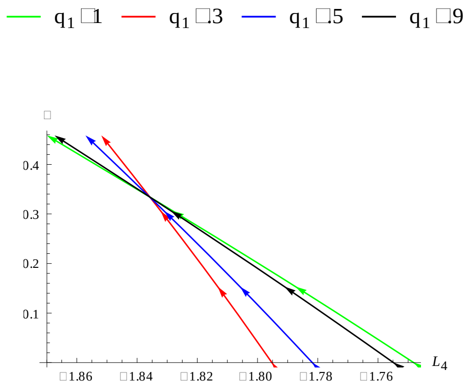
Table(11)

□ = .458505 $q_1 = .9$	L_1	L_2	L_3	L_4
$\lambda = 1$	2.53151	.449693	.583822	−1.86739
$\lambda = 3$	2.72855	-	-	−2.9356
$\lambda = 5$	2.99667	-	-	−3.66591
$\lambda = 7$	3.33327	-	-	−4.25675

Table(12)

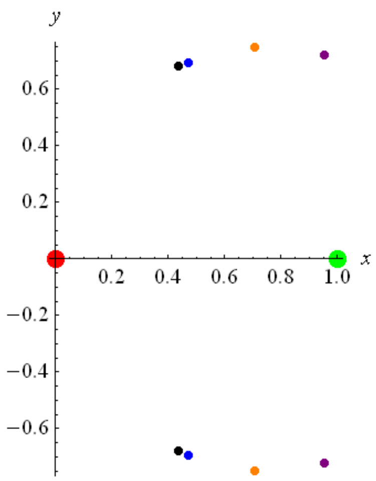


Fig(3)

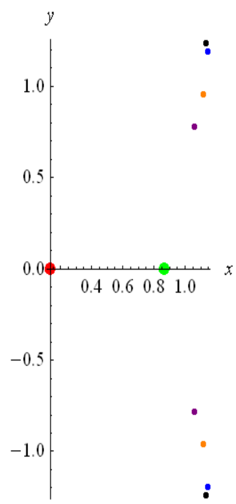


Fig(4)

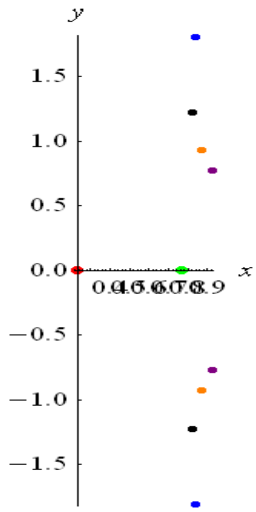
In Figs (5) , (6) and (7) we give the position of the non collinear equilibrium points L_5 and L_6 for □ = 3.37608×10^{-4}, □ = .132679 and □ = .230437 respectively. In these figs black dots denote the position of L_5 and L_6 when $q_1 = 1$ (no radiation) and blue, orange and purple dots denote the position when $q_1 = .9$, $q_1 = .5$ and $q_1 = .3$ respectively Here we observed that both L_5 and L_6 moves towards the small primary from left to right when □ = 3.37608×10^{-4} and come up to down when □ = .132679 but when □ = .230437 this shifting is deferent .



Fig(5)



Fig(6)



Fig(7)

IV. Stability of motion near equilibrium points

Let (x_0 , y_0) be the coordinate of any one of the equilibrium point and let ξ, η denote small displacement from the equilibrium point. Therefore we have

$$\xi = \zeta - x_0 ,$$
$$\eta = \nu - y_0,$$

Put this value of ζ and ν in equation (5) and (6), we have the variation equation as:

$$\ddot{\xi} - \dot{\eta} f_0 = \xi \left(U_{\zeta\zeta}\right)^0 + \eta \left(U_{\zeta\nu}\right)^0 \qquad (11)$$

$$\ddot{\eta} + \dot{\xi} f_0 = \xi \left(U_{\zeta\nu}\right)^0 + \eta (U_{\nu\nu})^0 \qquad (12)$$

Retaining only linear terms in ξ and η. Here superscript indicates that these partial derivative of $U$are to be evaluated at the equilibrium point (x_0 , y_0) . So the characteristic equation at the equilibrium points are

$$\lambda_1{}^4 + \lambda_1{}^2 \left\{ f^2 - \left(U_{\zeta\zeta}\right)^0 - \left(U_{\zeta\nu}\right)^0 \right\} + \left(U_{\zeta\zeta}\right)^0 (U_{\nu\nu})^0 - \left(U_{\zeta\nu}\right)^{0\,2} = 0 \qquad (13)$$

The equilibrium point (x_0 , y_0) is said to be stable if all the four roots of equation (13) are either negative real numbers or pure imaginary. In tables 13 to 22 we have given these roots for the collinear equilibrium points and in 23 to 25 for non-collinear equilibrium points.

⍰ $= 3.37608 \times 10^{-4}$ $q_1 = 1$	L_1	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
$\lambda = 1$	2.39051	± 0.240282	$\pm 1.66982i$
$\lambda = 3$	2.71276	$-0.75647 \pm 0.75567i$	$0.75647 \pm 0.75567i$
$\lambda = 5$	3.10013	$-1.22735 \pm 0.602771i$	$1.22735 \pm 0.60277i$
⍰ $= .132679$ $q_1 = 1$			
$\lambda = 1$	2.45144	± 0.0172224	$\pm 1.51261i$
$\lambda = 3$	2.72307	$-0.88978 \pm 0.692155i$	$0.88978 \pm 0.692155i$
$\lambda = 5$	3.06977	$-1.34709 \pm 0.43637i$	$1.34709 \pm 0.43637i$

Table(13)

⍰ $= 3.37608 \times 10^{-4}$ $q_1 = .5$	L_1	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
$\lambda = 1$	2.13876	± 0.454719	$\pm 1.75661i$
$\lambda = 3$	2.49977	$-0.374651 \pm 0.756143i$	$0.374651 \pm 0.756143i$
$\lambda = 5$	2.94143	$-0.890746 \pm 0.78258i$	$0.890746 \pm 0.78258i$
⍰ $= .132679$ $q_1 = .5$			
$\lambda = 1$	2.19628	± 0.436948	$\pm 1.82384i$
$\lambda = 3$	2.49117	$-0.350631 \pm 0.752211i$	$0.350631 \pm 0.752211i$
$\lambda = 5$	2.88831	$-0.914426 \pm 0.791328i$	$0.914426 \pm 0.791328i$

Table(14)

⍰ $= 3.37608 \times 10^{-4}$ $q_1 = .9$	L_1	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
$\lambda = 1$	2.34820	± 0.290048	$\pm 1.7208i$
$\lambda = 3$	2.67596	$-0.68180 \pm 0.771888i$	$0.68180 \pm 0.771888i$
$\lambda = 5$	3.07147	$-1.16170 \pm 0.656217i$	$1.16170 \pm 0.656217i$
⍰ $= .132679$ $q_1 = .9$			
$\lambda = 1$	2.40893	± 0.218523	$\pm 1.62633i$
$\lambda = 3$	2.68390	$-0.800298 \pm 0.727709i$	$0.800298 \pm 0.727709i$
$\lambda = 5$	3.03785	$-1.27069 \pm 0.535704i$	$1.27069 \pm 0.535704i$

Table(15)

⍰ $= 3.37608 \times 10^{-4}$ $q_1 = 1$	L_4	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
$\lambda = 1$	-1.74821	± 2.39908	± 0.791718
$\lambda = 3$	-2.84499	± 3.54145	± 0.94908
$\lambda = 5$	-3.58096	± 4.3150	± 0.967011
⍰ $= .132679$ $q_1 = 1$			
$\lambda = 1$	-1.78160	± 2.49069	± 0.84689
$\lambda = 3$	-2.87033	± 3.72008	± 0.95589
$\lambda = 5$	-3.60486	± 4.54733	± 0.96894

Table(16)

⍰ $= 3.37608 \times 10^{-4}$ $q_1 = .5$	L_4	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
$\lambda = 1$	-1.78185	$-1.24438 \pm 0.30131i$	$-1.24438 \pm 0.30131i$
$\lambda = 3$	-2.86665	± 2.20970	± 1.14979
$\lambda = 5$	-3.59792	± 2.82656	± 1.08871
⍰ $= .132679$ $q_1 = .5$			
$\lambda = 1$	-1.80221	$-1.28605 \pm 0.293247i$	$1.28605 \pm 0.293247i$
$\lambda = 3$	-2.88535	± 2.35634	± 1.13038
$\lambda = 5$	-1.78160	± 2.94064	± 1.14106

Table(17)

⍰ $= 3.37608 \times 10^{-4}$ $q_1 = .9$	L_4	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
$\lambda = 1$	-1.75494	± 2.23837	± 0.83362
$\lambda = 3$	-2.84931	± 3.32748	± 0.96769
$\lambda = 5$	-3.58426	± 4.06590	± 0.979341
⍰ $= .132679$ $q_1 = .9$			
$\lambda = 1$	-1.78571	± 2.32260	± 0.88322
$\lambda = 3$	-2.87733	± 3.49758	$\pm 0.97246i$
$\lambda = 5$	-3.60726	± 4.28720	$\pm 0.97991i$

Fig(18)

⍰ $= .132679$ $q_1 = 1$	L_2	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
$\lambda = 1$	0.662328	± 1.71992	$\pm 23.4554i$
$\lambda = 3$	0.715735	$\pm 0.55122i$	$\pm 44.8950i$
$\lambda = 5$	0.742964	$\pm 1.45132i$	$\pm 63.7128i$
⍰ $= .132679$ $q_1 = .5$			
$\lambda = 1$	0.69686	± 0.470315	$\pm 34.5159i$
$\lambda = 3$	0.753724	$\pm 0.917093i$	$\pm 69.0878i$
$\lambda = 5$	0.783278	$\pm 1.02355i$	$\pm 99.1660i$
⍰ $= .132679$ $q_1 = .9$			
$\lambda = 1$	.667415	± 1.46455	$\pm 24.8856i$
$\lambda = 3$	.721353	$\pm 0.73834i$	$\pm 47.7345i$
$\lambda = 5$	.748851	$\pm 1.45377i$	$\pm 68.0377i$

Fig(19)

⍰ $= .230427$ $q_1 = 1$	L_2	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
$\lambda = 1$	.562939	$\pm 2.51635i$	$\pm 12.4741i$
$\lambda = 3$	.649373	$\pm 2.96704i$	$\pm 27.999i$
$\lambda = 5$	.697523	$\pm 3.42845i$	$\pm 43.6595i$
⍰ $= .230427$ $q_1 = .5$			
$\lambda = 1$	.618412	$\pm 1.6177i$	$\pm 18.0079i$
$\lambda = 3$	.726268	$\pm 2.1378i$	$\pm 51.1495i$
⍰ $= .230427$ $q_1 = .9$			
$\lambda = 1$	.571296	$\pm 2.36742i$	$\pm 13.0801i$
$\lambda = 3$	.658573	$\pm 2.81635i$	$\pm 29.6179i$
$\lambda = 5$	.710819	$\pm 3.32662i$	$\pm 48.0887i$

Table(20)

$\mu = .132679$ $q_1 = 1$	L_3	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
$\lambda = 1$	.870822	$\pm 5.99808i$	$\pm 457.911i$
$\lambda = 3$	.869666	$\pm 5.90060i$	$\pm 448.533i$
$\lambda = 5$	.868395	$\pm 5.79689i$	$\pm 438.453i$
$\mu = .132679$ $q_1 = .5$			
$\lambda = 1$	.870099	$\pm 2.93265i$	$\pm 457.573i$
$\lambda = 3$	.867533	$\pm 2.84998i$	$\pm 433.943i$
$\lambda = 5$	.864287	$\pm 2.73754i$	$\pm 403.094i$
$\mu = .132679$ $q_1 = .9$			
$\lambda = 1$	.790781	$\pm 1.07964i$	$\pm 101.901i$
$\lambda = 3$	.869445	$\pm 5.28311i$	$\pm 447.697i$
$\lambda = 5$	.867999	$\pm 5.18223i$	$\pm 435.918i$

Table(21)

$\mu = .230427$ $q_1 = 1$	L_3	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
$\lambda = 1$	.779097	$\pm 5.62696i$	$\pm 82.8032i$
$\lambda = 3$	.772205	$\pm 5.25535i$	$\pm 79.8394i$
$\lambda = 5$	.760817	$\pm 4.77456i$	$\pm 74.2119i$
$\mu = .230427$ $q_1 = .5$			
$\lambda = 1$	.775622	$\pm 2.63302i$	$\pm 84.0107i$
$\lambda = 3$	.745650	$\pm 2.29635i$	$\pm 62.1799i$
$\mu = .230427$ $q_1 = .9$			
$\lambda = 1$	.778739	$\pm 4.9964i$	$\pm 83.4792i$
$\lambda = 3$	.770714	$\pm 4.6565i$	$\pm 79.3113i$
$\lambda = 5$	.755144	$\pm 4.15761i$	$\pm 70.649i$

Table(22)

$\mu = 3.37608 \times 10^{-4}$ $\lambda = 1$	$L_5, \quad L_6$ $\xi \qquad \eta$	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
$q_1 = 1$	$.435766$, $\pm$ .680940	± 3.5603	$\pm 3.53295i$
$q_1 = .3$	$.953254$, $\pm$ .722112	± 2.60299	$\pm 1.396864i$
$q_1 = .5$	$.704880$, $\pm$ .748658	± 3.26453	$\pm 2.295976i$
$q_1 = .9$	$.46976$, $\pm$ .695638	± 3.69993	$\pm 4.721654i$

Table(23)

$\mu = .132679$ $\lambda = 1$	$L_5, \quad L_6$ $\xi \qquad \eta$	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
$q_1 = 1$	1.13455 , $\pm$1.23929	± 2.33559	$\pm 0.78432i$
$q_1 = .3$	1.06213 , $\pm$ .780120	± 2.62362	$\pm 1.3502i$
$q_1 = .5$	1.11706, $\pm$ .958210	± 2.52099	$\pm 1.18674i$
$q_1 = .9$	1.14413, $\pm$ 1.19237	± 2.34015	$\pm 0.88217i$

Table(24)

$\mu = .230437$ $\lambda = 1$	$L_5, \quad L_6$ $\xi \qquad \eta$	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
$q_1 = 1$	$0.820584, \quad \pm 1.22540$	± 2.76584	$\pm 1.49003i$
$q_1 = .3$	$0.922591, \quad \pm .770110$	± 3.02968	$\pm 1.97359i$
$q_1 = .5$	$0.868804, \quad \pm .930507$	± 2.52099	$\pm 1.18674i$
$q_1 = .9$	$0.840901, \quad \pm 1.80790$	± 2.34015	$\pm 0.88217i$

Table(25)

V. Conclusion

In this article we have seen that for $\lambda > 0$ and for different values of q_1 and mass parameter $\mu = 3.37608 \times 10^{-4}$ (mars-Jupiter), $\mu = .132679$ (Saturn-Uranus), $\mu = .230437$ (Jupiter-Saturn), $\mu = .458505$ (Uranus-Neptune), there exists four or two collinear equilibrium points. Here we observed that due to the radiation pressure the points $L_1$and L_4 shifted towards the origin. It may be also observed that the deviation of the location of the point L_4 from $q_1 = 1$ decreases as μ increases and this becomes zero when $\mu \approx .35$ and again this deviation increases. We have also seen that the points L_1, L_2 and L_3 move away from the centre of mass and L_4 moves toward the centre of mass as λ increases. Here we have also observed that the non-collinear equilibrium points L_5 and L_6 moves towards the small primary from left to right when $\mu = 3.37608 \times 10^{-4}$ and come up to down when $\mu = .132679$ but when $\mu = .230437$ this shifting is deferent. We have observed that the points L_1, L_4, L_5 and L_6 are unstable while the point L_3 are stable for the given values of μ, q_1 and λ. We have also seen that the point L_2 is unstable for $\mu = .132679$ and for $q_1 = 1$, $q_1 = .5$, $q_1 = .9$ when $\lambda = 1$ and for other given values this point L_2 is stable.

References

[1] Arif. Mohd. (2010). Motion of a charged particle when the primaries are oblate spheroids international journal of applied math and Mechanics. 6(4), pp.94-106.

[2] Arif. Mohd. (2015). A study of the equilibrium points in the photogravitational magnetic binaries problem. International journal of modern sciencees and engineering technology, Volume 2, Issue 11, pp.65-70.

[3] Bhatnagar. K.B, J.M.Chawla.(1979) A study of the lagrangian points in the photogravitational restricted three body problem Indian J. pure appl. Math., 10(11): 1443-1451.

[4] Bhatnagar. K.B, Z. A.Taqvi, Arif. Mohd.(1993) Motion of a charged particle in the field of two dipoles with their carrier stars. Indian J. pure appl. Math., 24(7&8): 489-502.

[5] C. Ramesh Kumar, A. Narayan (2012) Existence and stability of collinear equilibrium points in elliptical restricted three body problem under the effects of photogravitational and oblateness primaries, International ournal of Pure and Applied Mathematics Volume 80 No. 4, 477-494.

[6] Chernikov,Yu. A. , (1970). The photogravitational restricted three body problem. Soviet Astronomy AJ. vol.14, No.1, pp.176-181.

[7] Mavraganis A (1978). Motion of a charged particle in the region of a magnetic-binary system. Astroph. and spa. Sci. 54, pp. 305-313.

[8] Mavraganis A (1978). The areas of the motion in the planar magnetic-binary problem. Astroph. and spa. Sci. 57, pp. 3-7.

[9] Mavraganis A (1979). Stationary solutions and their stability in the magnetic -binary problem when the primaries are oblate spheroids. Astron Astrophys. 80, pp. 130-133.

[10] Mavraganis A and Pangalos. C. A. (1983) Parametric variations of the magnetic -binary problem Indian J. pure appl, Math. 14(3), pp. 297- 306.

[11] Perezhogin, A.A.(1982). The stability of libration points in the restricted photogravitational circular three- body problem. Academy of Science, U.S.S.R. 20, Moscow.

[12] Radzievsky, V.V.(1950). The restricted problem of three bodies taking account of light pressure, Astron Zh, vol.27, pp.250.

[13] Ragos, O., Zagouras, C. (1988a). The zero velocity surfaces in the photogravitational restricted three body problem. Earth, Moon and Planets, Vol.41, pp. 257-278.

[14] Schuerman, D.W. (1980). The restricted three body problem including radiation pressure. Astrophys. J., Vol.238, pp.337.

[15] Shankaran et al. (2011),International Journal of Engineering, Science and Technology, Vol. 3, No. 2, pp. 63- 67.