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Frequency Identification Approach For Wiener Systems

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Abstract:
Wiener systems identification is studied in presence of possibly infinite-order linear dynamics and static nonlinearity. The problem of identifying Wiener models is addressed in the presence of hard nonlinearity. This latter is not required to be invertible of arbitrary-shape. Moreover, the prior knowledge of the nonlinearity type, being e.g. saturation effect, dead zone or preload, is not required. Using sine excitations, and getting benefit from model plurality, the identification problem is presently dealt with by developing a two-stage frequency identification method.

Keywords: *Wiener model, Hard nonlinearity, frequency identification.*

1. Introduction

The Wiener model is a series connection of a linear dynamic bloc and a memoryless nonlinearity (Fig. 1). When both parts are parametric, the identification problem has been dealt with using stochastic methods (e.g. Wigren, 1993, 1994; Wills and Ljung, 2010; Vanbeylen et al., 2009; Vanbeylen and Pintelon, 2010) as well as deterministic methods (e.g. Vörös, 1997, 2010; Bruls et al., 1999). The stochastic methods enjoy local or global convergence properties under various assumptions e.g. the system inputs should be persistently exciting (PE) or Gaussian and the system nonlinearity is invertible. The last limitation has recently been overcome by Wills et al. (2011). Multi-stage methods, involving two or several stages, have been proposed in (e.g. Westwick and Verhaegen, 1996; Lovera et al., 2000) and their consistency was ensured if the inputs are Gaussian and the nonlinearity is odd. Deterministic parameter identification methods consist in reformulating the problem as an optimization task, generally coped with using various relaxation techniques. Then, local convergence properties ensured in presence of PE inputs. Nonparametric Wiener systems (where none of the linear subsystem or the nonlinear element assumes a priori known structure) have been dealt with using both stochastic and frequency methods. In the stochastic methods (e.g. Greblicki and Pawlak, 2008; Mzyk, 2010), the nonlinearity is generally determined using variants of the kernel regression estimation technique while the (unknown) coefficients of a FIR/IIR approximation of the linear part are estimated using cross-correlation analysis. Several assumptions are needed e.g. Gaussian inputs, FIR linear dynamics, Lipschitzian nonlinearity. In frequency methods, the linear subsystem frequency response and the nonlinearity map are determined in two or several stages (e.g. Giri et al., 2013, 2014; Brouri et al., 2014).

In this paper, the problem of identifying Winer systems is addressed, for simplicity, in the continuous-time. Unlike many previous works, the model structure of the linear subsystem is entirely unknown. Furthermore, the system nonlinearity is of hard (Figs. 2a-b) type and is not required to be invertible.

The present strategy is allowed to interest a wide range of the system nonlinearity (Figs. 2a-b). The identification problem amounts to determining an accurate estimate of the (nonparametric) frequency response $G(j\omega)$, for a set of frequencies $(\omega_1 \dots \omega_m)$, and the nonlinearity. The present identification method is a two-stage: the system nonlinearity is identified first, using simple constant inputs, and based upon in the second stage to identify the linear subsystem.

The paper is organized as follows: the identification problem is formulated in Section 2; the nonlinear operator identification is coped with in Section 3; the linear subsystem frequency response determination is investigated in Section 4; simulation results are presented in Section 6.

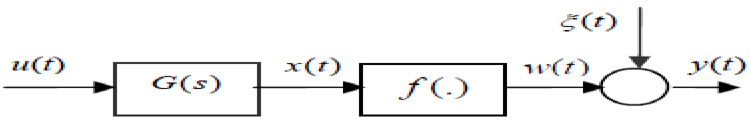

Fig. 1. Wiener model with hard nonlinearity $f(.)$

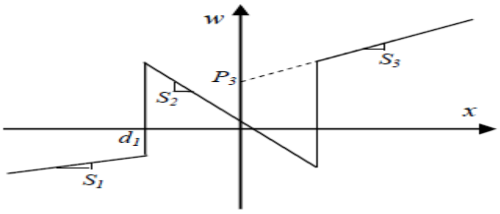

Fig. 2a. Nonlinearity hard

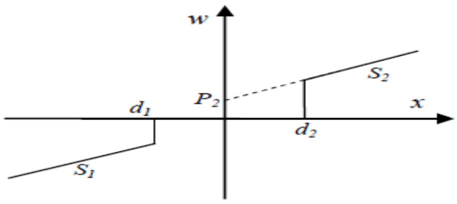

Fig. 2b. Nonlinearity hard with preload and dead zone

2. Identification of system nonlinearity

Standard Wiener systems consist of a linear dynamic subsystem $G(s)$ followed in series by a memoryless nonlinear operator $f(.)$ (Fig. 1). More specifically, the Wiener system under study is analytically described by the following equations:

$$x(t) = g(t) * u(t) \quad \text{with} \quad g(t) = L^{-1}(G(s)) \ (1)$$

$$y(t) = w(t) + \xi(t) \quad \text{with} \quad w(t) = f(x(t)) \tag{2}$$

where $u(t)$ and $y(t)$ denote the control input and the measured output; $x(t)$ and $w(t)$ are inner signals not accessible to measurement. The extra input $\xi(t)$ accounts for measurement noise and other modeling effects. It is supposed to be zero-mean ergodic and uncorrelated with the control input $u(t)$. The symbol * in (1) refers to the convolution operator and L^{-1} to the Laplace transform-inverse.

Accordingly, $g(t)$ denotes the impulse response of the linear subsystem and $G(s)$ its transfer function. It is just supposed that $g \in L_1$, so that the whole system becomes BIBO stable making possible open-loop system identification, with nonzero static gain (i.e. $G(0) \neq 0$). Interestingly, $G(s)$ is allowed to be infinite order. The system nonlinearity $f(.)$ is of hard (Figs. 2a-b) type and is not required to be invertible. Except for the above assumption $f(.)$ and $G(s)$ are arbitrary. In particular, the transfer function $G(s)$ is allowed to be non-parametric and of unknown structures. The static nonlinear element $f(.)$ may be noninvertible.

The problem complexity also lies in the fact that the non-accessible internal signals ($x(t)$, $w(t)$ and ξ) are not uniquely defined from an input-output viewpoint. In effect, if the couple $(G(s), f(x))$ is solution of the identification problem then, any model of the form $\left(\frac{G(s)}{K}, f(Kx)\right)$ is also solution of this problem whatever the real number $K \neq 0$. Therefore, without reducing the problem generality, one can assume $G(0) = 1$.

Accordingly, the system to be identified is described by the transfer functions:

$$\overline{G}(s) = \frac{G(s)}{G(0)} \tag{3}$$

and the nonlinearity:

$$\bar{f}(x(t)) = f(G(0)x(t)) \tag{4}$$

The modified system to be identified (i.e. $\bar{f}(.)$ and $\overline{G}(s)$) is the unique system satisfied the property: $\overline{G}(0) = 1$.

Then, the considered system is excited by simple constant inputs:

$$u(t) = U_j \quad \text{for} \quad j = 1 \ldots n \tag{5}$$

where the number n is arbitrarily chosen by the user, preferably sufficiently large. On the other hand, using the assumption of asymptotic stability of the linear subsystem and (3)-(5), the internal signal $x(t)$ is constant (i.e. $x(t) \xrightarrow[t\to\infty]{} X_j$). Then, one has, in the steady-state:

$$x(t) = U_j \quad \text{for} \quad j = 1\dots n \tag{6}$$

Accordingly, it is readily seen, using (2), (4) and (6), that the undisturbed output is also constant, in the steady-state, i.e. $w(t) \underset{t\to\infty}{\to} W_j$. This latter can be expressed as:

$$W_j = \bar{f}(U_j) \quad \text{for} \quad j = 1\dots n \tag{7}$$

Then, the system output is constant up to noise (in the steady-state). Finally, we can conclude using (7) that W_j (for $j = 1\dots n$) only depends on the system nonlinearity $\bar{f}(.)$ and the input signal. Therefore, using the fact $\xi(t)$ is zero-mean, it follows from (2) and (7) that, the estimate of the steady-state undisturbed output W_j $(j = 1\dots n)$ can be easily recovered using the following estimator:

$$\hat{W}_j(N) = \frac{1}{N}\sum_{k=1}^{N} y(k) \quad \text{for} \quad j = 1\dots n \tag{8}$$

where N is any sufficiently large integer. Specifically, W_j can be recovered by averaging $y(t)$ on a sufficiently large interval. Then, a set of points $(U_j, \bar{f}(U_j)) = (U_j, W_j)$ (with $j = 1\dots n$) belonging to nonlinearity $\bar{f}(.)$ can be determined. Finally, an accurate estimate $\hat{\bar{f}}_N(.)$ of $\bar{f}(.)$ can be easily obtained. These results lead to the following proposition:

Proposition 1. Consider the problem statement described by equations (1)-(2) and excited by the constant inputs (5). Then, one has:

1) $\hat{W}_j(N)$ converges in probability to W_j (as $N \to \infty$).
2) The nonlinearity $\hat{\bar{f}}_N(.)$ converges in probability to $\bar{f}(.)$ (as $N \to \infty$).

Proof. Part1. From the expressions (2) and (8) one easily gets:

$$\hat{W}_j(N) = \frac{1}{N}\sum_{k=1}^{N} W_j + \frac{1}{N}\sum_{k=1}^{N}\xi(k) = W_j + \frac{1}{N}\sum_{k=1}^{N}\xi(k) \quad \text{for} \quad j = 1\dots n \tag{9}$$

Using the fact that, the noise signal $\{\xi(.)\}$ is zero-mean ergodic sequence, the last term in (9) converges to zero.
This establishes Part 1.

Part2. It readily follows from Part 1 and (7) that, the estimate points $(U_j, \hat{\bar{f}}_N(U_j))$ converge to $(U_j, \bar{f}(U_j))$ ($j = 1\dots n$). This completes the proof of Proposition 1.

3. Frequency gain identification

The aim of this subsection is to establish an estimator of the linear subsystem. The complex gain $G(j\omega)$ is characterized by the modulus gain $|G(j\omega)|$ and the phase $\varphi(\omega) = \angle G(j\omega) = \arg(G(j\omega))$. The identification problem under study is dealt using a method based on the frequency approach. Firstly, by successively connecting the estimated points $\{(U_j, \hat{\bar{f}}_N(U_j)); j = 1\cdots n\}$, a set of segments of $\bar{f}(.)$ is obtained (Figs. 2a-b). For reasons of identifiability, at least one segment must have a non-zero slope. Let q designates any segment have a non-zero slope. Then, the nonlinear system described in subsection 1 is excited with a given sine input:

$$u(t) = u_0 + U\sin(\omega t) \tag{10}$$

The choice of u_0 in (10) can be performed using the experimental data of nonlinearity estimator. It can take any value in the segment q. Accordingly, all resulting signals depend on the amplitude/frequency couple (U, ω). In steady state, these signals write:

$$x_{U,\omega}(t) = u_0 + U|G(j\omega)|\sin(\omega t + \varphi(\omega)) \tag{11a}$$

$$w_{U,\omega}(t) = f(x_{U,\omega}(t)) \tag{11b}$$

$$y_{U,\omega}(t) = w_{U,\omega}(t) + \xi(t) \tag{11c}$$

Note that, the signal $w_{U,\omega}(t)$ is periodic with period $2\pi/\omega$ and, accordingly, the working point $(x_{U,\omega}(t), w_{U,\omega}(t))$ describes a closed cycle. Consequently, if $x(t)$ spans only the segment q, one gets:

$$w_{U,\omega}(t) = S_q\, x_{U,\omega}(t) + P_q \tag{12}$$

where (S_q, P_q) is the couple parameters of segment q (Figs. 2a-b). S_q and P_q can be determined using the experimental data of nonlinearity estimator. If necessary, excite the system to be identified with other constant inputs U_j. Then, it readily follows from (11a) and (12):

$$w_{U,\omega}(t) = S_q\, U|G(j\omega)|\sin(\omega t + \varphi(\omega)) + P_q + S_q\, u_0 \tag{13}$$

Then, under these conditions, the undisturbed output $w_{U,\omega}(t)$ is a sine signal (in the steady state) with an offset $P_q + S_q\, u_0$. Furthermore, the curve $(U|G(j\omega)|\sin(\omega t + \varphi(\omega)), w_{U,\omega}(t))$ is a straight line segment with slope $S_q\, U|G(j\omega)|$. If $S_q > 0$ (resp. $S_q < 0$), let $\bar{t}$ the first time, in the increasing (resp. in the decreasing) stages, satisfying:

$$w_{U,\omega}(\bar{t}) = P_q + S_q\, u_0 \tag{14a}$$

The expression (14a) implies:

$$S_q\, U|G(j\omega)|\sin(\omega\bar{t} + \varphi(\omega)) = 0 \tag{14b}$$

Then, one has the following relation between $\varphi(\omega)$ and $\bar{t}$:

$$\bar{t} = -\varphi(\omega)/\omega \quad (\text{modulo } 2\pi) \tag{15}$$

These results imply that, the complex frequency gains of the linear subsystem can be recovered if the undisturbed output $w_{U,\omega}(t)$ is accessible to measurement. At this stage, $w_{U,\omega}(t)$ is not measurable.

Fortunately, an accurate estimator of $w_{U,\omega}(t)$ exists, thanks to the (steady-state) periodicity of $w_{U,\omega}(t)$ and the ergodicity of the noise $\xi(t)$. The estimator, denoted $\hat{w}_{U,\omega,N}(t)$, is obtained by performing a T-periodic averaging with $T = 2\pi/\omega$ (Ljung, 1999, p. 232):

$$\hat{w}_{U,\omega,N}(t) = \frac{1}{N}\sum_{k=1}^{N} y_{U,\omega}(t + k\frac{2\pi}{\omega}) \quad \text{for} \quad t \in \left[0 \;\; \frac{2\pi}{\omega}\right) \tag{16a}$$

$$\hat{w}_{U,\omega,N}(t + k\frac{2\pi}{\omega}) = \hat{w}_{U,\omega,N}(t) \quad \text{for} \quad k = 1, 2, 3\ldots \tag{16b}$$

where N is any sufficiently large integer. The estimator (16a-b) is uniformly consistent i.e. $\hat{w}_{U,\omega,N}(t)$ converges (w.p.1 as $N \to \infty$) to $w_{U,\omega}(t)$, whatever t. Finally, it follows (14b) and (15) the gain modulus and the phase of the linear subsystem can be easily obtained using (16a-b).

4. Conclusions

The problem of system identification is addressed for Wiener systems where the linear subsystem, described by (1-2), may be parametric or not, finite order or not. The nonlinear element is of hard type. The latter are allowed to be noninvertible. The identification problem is dealt with using a two-stage approach combining frequency. Data acquisition in presence of constant inputs is performed in the first stage following the procedure of Section 2. Then, an accurate estimate of the system nonlinearity can be obtained. Data acquisition in presence of sine input excitations is performed in the first stage following the procedure of Table 1. Finally, the transfer function response is identified in the second stage using the algorithm described Section 3 and the estimator (16a-b). All involved estimators are consistent. To the author's knowledge no previous study has solved the identification problem for a so large class of Wiener systems

References

[1] T. Wigren. Recursive prediction error identification using the nonlinear Wiener model. Automatica, vol. 29, pp. 1011- 1025, 1993.

[2] T. Wigren. Convergence analysis of recursive identification algorithm based on the nonlinear Wiener model. IEEE Trans. Automatic Control, vol. 39, pp. 2191-2205, 1994.

[3] A. Wills and L. Ljung. Wiener system identification using the maximum likelihood method. In Giri F. & E.W. Bai (2010). Block-oriented nonlinear system identification. Springer, U.K, 2010.

[4] L. Vanbeylen, R. Pintelon, and J. Schoukens. Blind maximum-likelihood identification of wiener systems. IEEE Transactions on Signal Processing, vol. 57(8), pp. 3017–3029, 2009.

[5] L. Vanbeylen and R. Pintelon. Blind maximum-likelihood Identification of Wiener and Hammerstein nonlinear block structures. In Giri & Bai (eds), Block-Oriented Nonlinear System Identification, Springer, pp. 279-298, 2010.

[6] J. Vörös. Parameter identification of Wiener systems with discontinuous nonlinearities, Systems and Control Letters, vol. 44, pp. 363-372, 1997.

[7] J. Vörös. Compound operator decomposition and its application to Hammerstein and Wiener systems. In Giri & Bai, Block-Oriented Nonlinear System Identification, Springer, U.K, 2010.

[8] J. Bruls, C.T. Chou, B.R.J. Heverkamp and M. Verhaegen. Linear and nonlinear system identification using separable least squares, European Journal of Control, vol 5, pp.116-128, 1999.

[9] A. Wills, T.B. Schön, L. Ljung, B. Ninness. Blind Identification of Wiener Models. IFAC World Congress, Milan, Italy, 2011.

[10] D. Westwick and M. Verhaegen. Identifying MIMO Wiener systems using subspace model identification methods. Signal Processing, 52, pp. 235–258, 1996.

[11] M. Lovera, T. Gustafsson T., and M. Verhaegen. Recursive subspace identification of linear and non-linear Wiener statespace models. Automatica, vol. 36, pp.1639-1650, 2000.

[12] W. Greblicki, and M. Pawlak. Nonparametric System Identification. Cambridge University Press, 2008.

[13] G. Mzyk. Stochastic Identification of Wiener Systems. In Giri F. and E.W. Bai (2010). Block-oriented nonlinear system identification. Springer, U.K, 2010.

[14] F. Giri, A. Radouane, A. Brouri, F.Z. Chaoui. Combined frequency-prediction error identification approach for Wiener systems with backlash and backlash-inverse operators. Automatica, vol. 50, pp. 768-783, 2014.

[15] F. Giri, Y. Rochdi, A. Radouane, A. Brouri, F.Z. Chaoui. Frequency Identification of nonparametric Wiener systems containing backlash nonlinearities. Automatica, vol. 49, pp. 124-137, 2013.

[16] A. Brouri, O. Amdouri, F.Z. Chaoui, F. Giri. Frequency Identification of Hammerstein-Wiener Systems with Piecewise Affine Input Nonlinearity. 19th IFAC World Congress, Cape Town, South Africa, August 24-29, pp. 10030-10035, 2014.

[17] L. Ljung. System identification. Theory for the user. Prentice Hall, NJ, USA, 1999.