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Test of scale Parameter of the two-parameters exponential distribution using Ranked set sampling

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Abstract

The testing of the scale parameter of the 2-parameter exponential distribution was considered when the two parameters are unknown. Three tests were derivedand the modified rank set sampling procedure was used one of the tests was truncated sequential test .
The power functions were derived and calculations of them were reported . The truncated sequential test led to earlier decision whereby the expected value of the number of cycles was less than the number of cycles used in the fixed sample size .

Key worlds *: Two –parameters exponential ,Modified Ranked set Sampling Truncated sequential Test .*

I. Introduction

Exponential distribution has been used widely in life testing applications . Estimation and testing of the parameters of the two-parameters exponential distribution were addressed by many anthers
Hirano (1984) gave a preliminary test procedure for the scale parameters of exponential distribution when selection par parameters is unknown .
Wright et al (1978) gave inferences for the two – parameters exponential distribution under Type 1 censored sampling
Abu-Salih (2015) derived different tests for the scale parameters of the 2–parameter exponentialdistribution and composed the power and the expected simple size of these tests .
Different sampling methods were used in the estimation and testing procedures for this distribution .
Median ranked set sampling was used by Hossain and Muttlak(2006) . Abu-Dayyeh and Muttlak (1996) used Ranked set Sampling (RSS)for their tests on the scale parameter of the exponential distribution .
Shahi (2012) used extremeRSS for hypothesis testing .
Ranked Set sampling (RSS) was originally used by Mcintyre(1952) to improve the estimation of population mean of agricultural Gropes . it is mainly used In situations whereby ranking the items is easy while measuring them is expensive .In this research the modified rank set sampling (MRSS) in testing the scale parameter ofthe2–parametersexponential distribution Since RSS relies on ranking samples the size of each set must be small for each of ranking and to minimize ranking errors Usually the size is around 3 or 4 .

The steps followed to draw a (MRSS) are :

1. Take a simple random sample of size r^2 from the population , and allocate them randomly in r sets of size r each .
2. Rank the units in each row without measuring them . The result of this step is given :-

This is called the first cycle , and the RSS of this cycle will be
$(\chi_{(1)1} , \chi_{(2)2} , \dots \chi_{(r)r})$

3. To get a modified RSS, the first column of the first cycle is taken , namely $(\chi_{(1)1} , \chi_{(1)2} , \dots \chi_{(1)r})$

The MRSS of cycle (C) will be denoted by $(\chi_{(1)1c} , \chi_{(1)2c} , \dots \chi_{(1)rc})$.

4. Repeat steps (1) to (3) Ltimes, which gives a sample of size n= rλ
   The MRSS obtained from the L cycles are independent and identically distributed .

Since $\chi_{(1)i}$, í=1,…,rL , is distributed as$\chi_{(1)k}$the first order statistic
in a random sample of size m from the original distribution . The MRSS ,using L cycles , each of size r^2 will be :
$(\chi_{(1)11} , \chi_{(1)21} , \dots \chi_{(1)r1},\dots \chi_{(1)1L}, \chi_{(1)2L} , \cdots \chi_{(1)rL})$

II. Statement of the problem

As is well known, these are many tests on the scale parameter of the 2- parameter exponential distribution because of the importance of this distribution In this research it is proposed to derive other tests and use MRSS instead of simple random sampling (SRS) .

The proposed tests are called truncated sequential tests in the sense that , not the whole sample of size n is used at once but the test is applied sequentially and if no decision is reached then the whole sample is used

These tests are expected to have smaller expected sample size than the fixed sample size .

Also,instead of using SRS of size n,MRSS is used for these tests .

Let X be a random variable having exponential distribution with pdf :

(1.1) $F(x)=\lambda e^{-\lambda(x-\theta)}$,$\theta\leq x<oo$

Where λ >o and$\theta\geq$oare the Scale and location parameters respectively . Based on the MRSS procedures discussed in the introduction, the tests will be based on a random sample of size n=mk , which is denoted by

$(\chi_{(1)11}, \chi_{(1)21}, \chi_{(1)31,\ldots} \chi_{(1)r1}, \chi_{(1)12}, \chi_{(1)22,\ldots,} \chi_{(1)r2}, \cdots, \chi_{(1)1k}, \chi_{(1)2k\ldots,} \chi_{(1)rL})$

Where $\chi_{(1)ij}$denotesthe first order statistic of the i^{th} row of the j^{th} cycle consisting of r^2independent observation's and ranked as in step (2)

It is seen that this sample is of size n =rLusually r is small to enable good ranking . The tests good ranking .

The tests considered are to test

$H_o: \lambda \geq \lambda_o$vs $H_1 : \lambda <\lambda_o$

For pdf given in (1.1), let $Y_{ij} = X(1)ij$, i=1,. . . r , j=1,2 . . . , L

Then Y_{ij}'sareindependent identically distributed with pdf

(1.2) $g(y) = r\lambda e^{-r\lambda(y-\theta)}$,$\theta\leq y<\infty$

Since Y_{ij} is the first order statistic of a random sample of size (r)from a 2-parameter exponential distribution of pdf given in (1. 1).

<u>2. Testing the scale parameter</u>

Let $Y_{ij}=X_{(1)ij}$, i=1,2,3,…r,and

j=1,2,3,…, L ,be a MRSS chosen

By the process defined in section (1) .

Y_{ij} are independentidentically distributed

as 2-paramenter exponential distribution with scale and location parametersrL and θrespectively as in (1.2)

Our interest is to test $H_o: \lambda \geq \lambda_o$ vs $H_1 : \lambda <\lambda_o$ when both parameters λ and θ are unknown .

The following tests will be considered .

(i) The UMP (uniformly most powerful) invariant size$\propto$testϕ_1

Let $U_n = \min_{\substack{1\leq i\leq r\\ 1\leq j\leq L}} Y_{ij}$, n=rL

Then the UMPinvariant size αtest ϕ_1 is defined by:

Reject H_o: if $S_1>C_1$where

$S_1 = \sum_{j=1}^{\lambda}\sum_{i=1}^{r}\left(Y_{ij}-U_n\right)$ and C_1 isdetermined from $P(S_1>C_1 | \lambda o)=\alpha$

Since $2r\lambda_o$ S_1 has α χ^2Distribution with 2(n-1) d.f. , C_1 is easily evaluated , where n = rL

The power function of $\phi_1$is easily found to be

(2.1) $P\phi_1(\lambda)=P(S_1<C_1 | \lambda)= P(2r\lambda S_1<2r\lambda C_1)$

Where $2r\lambda S_1$has aχ^2 distribution with (2n-1) d.f.

(ii) The size testϕ_2 based on the range .

ϕ_2 is defined as follows :

Let $M_n = \max_{\substack{1\leq i\leq r\\ 1\leq j\leq L}} Y_{ij}$, n=rL

And U_n as defined before .

Reject H_o if $R_n>c_2$, where

$R_n =m_n-U_n$ and C_2 is determined from $P(R_n> C_2 | \lambda_o)=\alpha$

To evaluate this probability and solve for C_2 and evaluate the power function of ϕ_2 , the pdf of R_n is needed .

David (1970) gave the joint pdf of M_n and U_n by

$V(u_n , m_n ; \lambda , \theta) = n(n-1)\left[G(m_n) - G(u_n)\right]^{n-2} \; g_{m_n} * g_{u_n}$

$m_n \geq u_n \geq \theta$

Where G is the cumulative distribution function of Y_{Ω} , i.e

(2.2) $G_{(y)} = \begin{cases} 0 & ; y < \theta \\ 1 - e^{-r\lambda(y-\theta)} & ; y \geq \theta \end{cases}$

It is easy to show that the pdf of R_n is given by

(2.3) $R_n(t) = r\lambda(n-1)e^{-r\lambda t}(1 - e^{-r\lambda t})^{n-2} \quad ; t \geq o$

The cumulative distribution of R_n is :

(2.4) $Q_{(t)} = \int_o^t r(n-1) \, e^{-r\lambda y}(1 - e^{-r\lambda y})^{n-2} dy$

$= (1 - e^{-r t y})^{n-1} \quad , t \geq o \quad n = r\ell$

From (2.3) , it can be seen that R_n tends to be small when λ is large and hence it is reasonable to use the test ϕ_2 for which C_2 is determined from

$P(R_n > C_2 \mid \lambda_o) = (1 - e^{-r\lambda_o C_2})^{n-1} = \alpha$

And hence :

(2.4) $C_2 = -1/ (r\lambda_o) \ln [1 - (1 - \alpha)^{1/(n-1)}]$

By determining C_2 , the power function of the test ϕ_2 is easilydetermined and is given by

(2.5) $P_{\phi 2}(\lambda) = 1 - (1 - e^{-r\lambda C_2})^{n-1}$

whereC_2 is defined in (2.4) and n=

III. Truncated sequential test

Instead of using data from all cycles in the MRSS process, it is suggested to use cycles , one by one , and when the test is significant , the null hypothesis is rejected, the test stops as shown in

Fig1. No decision of rejection is reached by **observing**the (ℓ-1) cycle , the L**th** cycle is observed and decision is reached .

Cycles	**1**	**2…**	**K…**	**(k+1)…**	**L**
Observation	**r**	**r**	**r**	**………**	**1**

Fig. 1

The test is defined as follows :

The k^{th} cycle is observed Fig.1 . Define

$$T_k = \sum_{i=1}^{k} \sum_{i=1}^{r} (Y_{ij} - U_k)$$

Where $U_k = \min_{\substack{1 \leq i \leq r \\ 1 \leq j \leq k}} Yij \qquad k < \ell$

T_k is monotone non-decreasing in k . To prove this fact, consider the two cases :

(i) $Y_{i(k+1)} \leq U_k \; 1 \leq i \leq r$

then $U_{k+1} = Y_{i_o (k+1)}$ for some $i_o , 1 \leq i_o \leq r$

and

$$T_{k+1} = \sum_{j=1}^{k+1}\sum_{i=1}^{r}\left(Y_{ij} - U_{k+1}\right) =$$

$$= \sum_{j=1}^{k+1}\sum_{i=1}^{r}\left(Y_{ij} - Y_{i_o(k+1)}\right)$$

$$= \sum_{j=1}^{k}\sum_{i=1}^{r}\left(Y_{ij} - Y_{i_o(k+1)}\right)$$

$$= \sum_{j=1}^{k}\sum_{i=1}^{r}\left(Y_{ij} - U_k\right) = T_k$$

(ii) $\min_{1\le i\le r} Y_{i(k+1)} > U_k$

then $U_{K+1} = U_K$ and

$$T_{K+1} = \sum_{j=1}^{k+1}\sum_{i=1}^{r}\left(Y_{ij} - U_k\right) =$$

$U_k) > T_k$

Since T_k is monotone non-decreasing in k , it is useful to use the
Truncated sequential test basedon T_k, t test H_o: $\lambda \geq \lambda_o$vs $H_1 : \lambda < \lambda_o$
With the possibility that this test will lead to early rejection of H_o.
Let sampling procedure be the MRSS with numbers of items in each
Cycle r^2 and the number of measured observations r as explained in the set up explained earlier . Let the number of cycles be L, where rL=n ,the total number of observations .
Consider the test ϕ_3 which rejects H_o: $\lambda \geq \lambda_o$atthe kth stage of sampling (the kthcycle) , where $2 \leq k \leq L-1$, if $T_k > C$ and takes one more observation if $T_k<C$ where C is given by $P(T_L>C \mid \lambda_o)=$
Notice that $T_L=S_1$and thereforeC=C_1 which was determined before .
If therule dose not reject H_o by using the(l-1)th cycle , then
The Lth cycle is used and a decision is reached
(3.1) Power of test ϕ_3
Let $E_k = (T_k>C)$ fork=2,3,…, L)
Then $E_2 \subset E_3 \subset \ldots \subset E_L$ since
T_k is monotone non-decreasing .
Hence,P(rejecting H_o)=$P(U_{k=2}E_k)=P(E_L)$
And hence the power of the test ϕ_3 is equal to the power of the fixed sample size test ϕ_1 defined by :
Reject H_o if $S_1>C_1$.

(3.2) <u>Expected sample size of ϕ_3</u>
let W be the number of cycles required for termination of the test in the early rejection procedure .
Since $E_2 \subset E_3 \subset \ldots \subset E_L$, then

$E_L = \cup_{(R=2)}^{(L-1)} (\bar{E}_k \cap E_{(k+1)}) \bigcup E_2$

And hence :
(3.3)$E(W \mid \lambda)=2P(E_2 \mid \lambda)+$
$\sum_{(R=2)}^{(L-1)} (k+1)P(\bar{E}_k \cap E_{(k+1)} \mid \lambda)+$
$nP(\bar{E}_L \mid \lambda)$
Following the same procedure given by Abu-Salih (2015) , $E(W \mid \lambda)$is evaluated to be given by:
$E(W \mid \lambda)=2[1-P(\chi^2_2 < 2r\lambda C]+$

$$\sum_{k=2}^{\lambda} \frac{(k+1)_{e^{-r\lambda c_{r\lambda}}c^{k-1}}}{(\square)} + nP(\chi^2_{2(n-1)} < 2r\lambda C)$$

Where n=rɻ

IV. Comparisonof power function of the tests

Power function of tests ϕ_1, ϕ_2 (and $\phi_{3)}$ are computed for λ_o=0.4 ,
,…,0.04 ,r=5 , and ɻ=2,4, and 6
Note that power of ϕ_3 isnot listed because it is equal to power ofϕ_1 .

Table 4.1 : powers of tests according to λand ɻ when r=5

λ	r λ	ϕ_1 ,ɻ=2	ϕ_2 ,ɻ=2	ϕ_1 ,ɻ=4	ϕ_2 ,ɻ=4	ϕ_1 ,ɻ=6	ϕ2 , ɻ=6
0.40	2	0.05	0.049974	0.05	0.05	0.05	0.05
0.32	1.6	0.1861499	0.134939	0.27595	0.154627	0.35444	0.1669
0.24	1.2	0.50111	0.338872	0.741097	0.425349	0.87104	0.4800
0.16	0.8	0.869687	0.703638	0.986541	0.846154	0.99878	0.9087
0.08	0.4	0.99703	0.98082	0.999996	0.99904	1	0.9999
0.04	0.2	0.999979	0.999715	1	1	1	1

It is clear that the values of the power of ϕ_1 and the corresponding truncated sequential test ϕ_3 are greater than the power of ϕ_2.
Also ,the powers increase with ɻ and the distance of λ from λ_o .
Expected value of number of cycles fo2 ϕ_3 , when r=3 and for values of λ_o =0.6 and values ofλ=0.60 to 0.07 and ɻ =5 and 10 are given in Table2

Table2 : E[k│λ] for test ϕ_3

λ	r λ	λ=5	λ=10
0.6	1.8	2.9699	2.96522
0.53	1.6	2.9231	2.89203
0.40	1.2	2.6597	2.462412
0.33	1	2.38734	2.11410
0.20	0.6	1.638552	1.35166
0.07	0.2	0.81337	0.58389

It is clear the E[K│λ]<L , meaning that the truncated sequential test of λ using MRSS is expected toreach decision earlier than the fixed sample size test using MRSS, with n= rL

V. Conclusion

It has been suggested to use these tests and modified rank set sampling to test
H_o: $\lambda \geq \lambda_o$vs H_1 : $\lambda < \lambda_o$
where the distribution of the population is the negative exponential with location and scale parameters θ and λ , respectively .
The tests are free of the location parameter θ and the proposed truncated sequential test leads to earlier decision where by it uses a number of cycle smaller than the assumed numberɻ .

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