Strong (Weak) Triple Connected Domination Number of a Fuzzy Graph

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ABSTRACT

A subset S of V of a nontrivial fuzzy graph G is said to be strong (weak) triple connected dominating set, if S is a strong (weak) dominating set and the induced sub graph <S> is a triple connected. The minimum cardinality taken over all strong (weak) triple connected dominating set is called the strong (weak) triple connected domination number and it denoted by γ_{stc} (γ_{wtc}).we introduce strong (weak) triple connected domination number of a fuzzy graphs and obtain some interesting results for this new parameter in fuzzy graphs.

Keywords: *Connected dominating set, Fuzzy graphs,triple connected dominating set, strong (weak) triple connected dominating set*

I. Introduction

In 1975, the notion of fuzzy graph and several fuzzy analogues of graph theoretical concepts such as paths cycles and connectedness are introduced by Rosenfeld[15] Bhattacharya[3] has established some connectivity regarding fuzzy cut node and fuzzy bridges. The concept of domination in fuzzy graphs are introduced by A.Somasudaram and S.Somasundaram[18] in 1998. In 2012, Bounds on connected domination in square of a graph is introduced by M.H.Muddabihal and G.Srinivasa.Triple connected domination number of a graph introduced by G.Mahadevan,Selvam. In this paper,We analyze bounds on strong (weak) triple connected dominating set of fuzzy graph and proves some results based on triple connected dominating fuzzy graph.

II.Preliminaries

Definition 2.1

A fuzzy subset of a nonempty set V is mapping $\sigma: V \rightarrow [0, 1]$ and A fuzzy relation on V is fuzzy subset of V x V. A fuzzy graph is a pair G: (σ,μ) where σ is a fuzzy subset of a set V and μ is a fuzzy relation on σ, where $\mu(u,v) \leq \sigma(u) \Lambda \sigma(v)\ \forall u,v \in V$

Definition 2.2

A fuzzy graph G=(σ, μ) is a strong fuzzy graph if $\mu(u,v) = \sigma(u) \Lambda \sigma(v)$ for all u, $v\in$V and is a complete fuzzy graph if $\mu(u,v) = \sigma(u) \Lambda \sigma(v)$ for all u, $v\in$V.The complement of a fuzzy graphG=(σ, μ) is a fuzzy graph $\bar{G} = (\bar{\sigma}, \bar{\mu})$where$\bar{\sigma}$=$\sigma$ and $\bar{\mu}(u,v) = \sigma(u) \Lambda \sigma(v) - \mu(u,v)$ for all u, $v\in$V. we denote a cycle on P vertices by C_p, a path by P_p a complete graph on P vertices by K_p.

Definition 2.3

Let G=(σ, μ) be a fuzzy graph. Then D $\subseteq$ V is said to be a fuzzy dominating set of G if for every $v\in$V-D, There exists u in D such that $\mu(u,v) = \sigma(u) \Lambda \sigma(v)$. The minimum scalar cardinality of D is called the fuzzy dominating number and is denoted by γ (G). Note that scalar cardinality of a fuzzy subsetD of V is|D| $=\sum_{v\in V} \sigma(v)$

Definition 2.4

A dominating set D of a fuzzy graph G = (σ, μ) is connected dominating set if the induced fuzzy sub graph <D> is connected. The minimum cardinality of a connected dominating set of G is called the connected domination number of G and is denoted by γ_c (G)

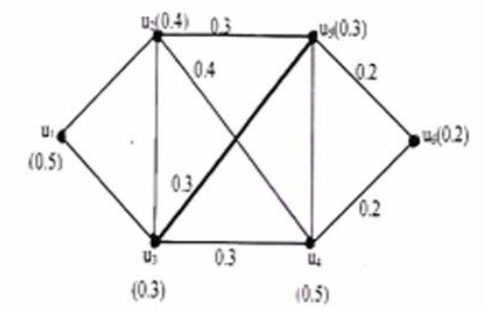

Fig.1 D= $\{u_3, u_5\}$, γ_c (G)=0.6

Definition 2.5

Let G be a fuzzy graph the neighborhood of a vertex v in V is defined by N(v) $=\{u \in V: \mu(u,v) = \sigma(u) \wedge \sigma(v)\}$.The scalar cardinality of N(V) is the neighborhood degree of V, Which is denoted by $d_{N(V)}$ and the effective degree of V is the sum of the weights of the edges incident on V denoted by $d_{E(v)}$

Definition2.6

Let u and v be any two vertices of a fuzzy graph G. Then u strongly dominates v(v weakly dominates u) if

i) $\mu(u,v) = \sigma(u) \wedge \sigma(v)$. and
ii) $d_{N(u)} > d_{N(v)}$

Definition2.7

Let G be a fuzzy graph, then D $\subseteq$V is said to be a strong(weak) fuzzy dominating set of G if every vertex $v \in V$-D is strongly (weakly) dominated by some vertex u in D. We denoted a strong (weak) fuzzy dominating set by sfd -set(wfd-set).

The minimum scalar cardinality of a sfd-set(wfd-set) is called the strong (weak) fuzzy domination number of G and it is denoted by γ_{sd}(G) (γ_{wd}(G))

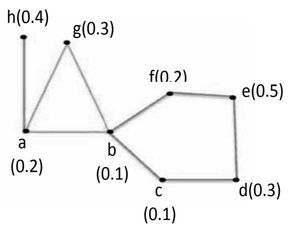

Fig:2

Example2.8

For the fuzzy graph G in Fig.2.γ_{sd}(G) =0.8 and γ_{wd}(G)=1.0.since {a,b,e} and {c,f,g,h} are the minimum sfd-set and wfd-set respectively.

Theorem 2.9

Let D be a minimal sfd- set of a fuzzy graph G. Then for each $u \in D$ of the following holds.

(i) No vertex in D strongly dominate v
(ii) There exists $v \in$ V-D such that v is the only vertex in D which strongly dominates u.

Proof

Suppose D is a minimal connected dominating set of G. Then for each node $u \in D$ the set D′=D-{u} is not a connected dominating set. Thus, there is a node $v \in V$-D′ which is not dominated by any node in D′. Now either u=v or $v \in V$-D, If v=u then no vertex in D strongly dominates v. If $v \in V$-D and v is not dominated by D-{u} but is dominated by D, Then u is the only strong neighbor of v and v is the only vertex in D which strongly dominates u.

Conversely suppose D is a dominating set and each node $u \in D$, one of the two stated conditions holds. Now we prove D is a minimal strong connected dominating set. Suppose D is not a minimal strong connected dominating set, then there exists a node $u \in D$ such that D-{u} is a dominating set. Therefore condition (i) does not hold. Also if D-{u} is a dominating set then every node in V-D is a strong neighbor to at least one node in

D-{u}. Therefore condition (ii) does not hold. Hence neither condition (i) nor (ii) holds which is a contradiction.

Definition2.10
A fuzzy graph G is said to be triple connected if any three vertices lie on a path in G. All paths, cycles, complete graphs are some standard examples of triple connected fuzzy graphs.

Definition2.11
A subset D of V of a nontrivial connected fuzzy graph G is said to be triple connected dominating set. If D is the dominating set and the induced fuzzy sub graph <D> is triple connected .The minimum cardinality taken over all triple connected dominating set of G is called the triple connected dominating number of G and is denoted by $\gamma_{tc}(G)$.

Example 2.12
For the fuzzy graph Fig.3, D = {v_1,v_2,v_5 } forms a γ_{tc} set of G

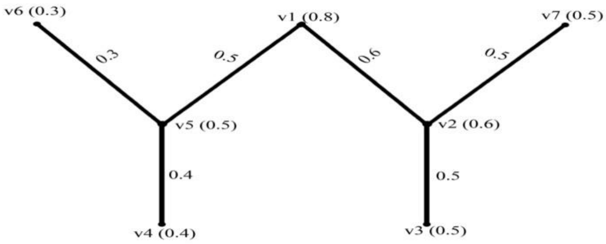

Fig.3, $\gamma_{tc}(G)$ =0.8+0.5+0.6 =1.9

III.Strong (weak) Triple connected domination number of a fuzzy graph

Definition3.1
A subset S of V of a nontrivial fuzzy graph G is said to be strong (weak) triple connected dominating set, if S is a strong (weak) dominating set and the induced sub graph <S> is a triple connected. The minimum cardinality taken over all strong (weak) triple connected dominating set is called the strong (weak) triple connected domination number and it denoted by γ_{stc} (γ_{wtc}). In this section we present few elementary bounds on strong(weak) triple connected domination number of a fuzzy graph and the correspondingsome results

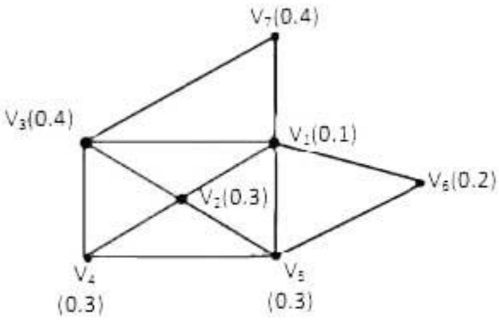

Fig:4

Example3.2 For the fuzzy graph in Fig : 4, s = **{**v_1,v_2,v_6 **}** forms γ_{stc} set and w = {v_3,v_4,v_5} formsγ_{wtc} set.
$\gamma_{stc}(G)$ = 0.6 and $\gamma_{wtc}(G)$ = 1

Theorem3.3
Strong (weak) triple connected dominating set does not exists for all fuzzy graphs.

Proof By definition,The connected graph is not strong (weak) triple connected.consider only connected fuzzy graphs for which strong (weak) triple connected dominating set exists.

Observation 3.4 Every strong (weak) triple connected dominating set is a dominating set but not conversely.

Observation 3.5 Every strong (weak) triple connected dominating set is a triple connected dominating set but not conversely.

Observation 3.6 The complement of strong (weak) triple connected dominating set need not be a strong (weak) triple connected dominating set.

Theorem3.7 For a fuzzy graph G of order p, (i) $\gamma(G) \leq \gamma_{sd}(G) \leq \gamma_{stc}(G)$ P-$\Delta_N(G) \leq$ p-$\Delta_E(G)$ and
(ii) $\gamma(G) \leq \gamma_{wd}(G) \leq \gamma_{wtc}(G) \leq P\text{-}\delta_N(G) \leq p\text{-}\delta_E(G)$ where $\Delta_N(G)$ $[\Delta_E(G)]$ and $\delta_N(G)$ $[\delta_E(G)]$ denote the maximum and minimum neighborhood degrees (effective degrees)of G

Proof Since every stcd set (wtcd) is a fuzzy dominating set of G, $\gamma(G) \leq \gamma_{sd}(G) \leq \gamma_{stc}(G)$ and $\gamma(G) \leq \gamma_{wd}(G) \leq \gamma_{wtc}(G)$.Let u ,v$\in$V, If $d_{N(u)} = \Delta_N(G)$ and $d_{N(v)} = \delta_N(G)$.Then clearly V-N(u) is a stcd- set and V-N(v) is a wtcd -set. Therefore $\gamma_{stc}(G) \leq |V\text{-}N(u)|$ and $\gamma_{wtc}(G) \leq |V\text{-}N(u)|$ that is $\gamma_{stc}(G) \leq P\text{-}\Delta_N(G)$ and $\gamma_{wtc}(G) \leq P\text{-}\delta_N(G)$ Further since $\Delta_E(G)] \leq \Delta_N(G)$and $\delta_E(G) \leq \delta_N(G)$

Hence $\gamma(G) \leq \gamma_{sd}(G) \leq \gamma_{stc}(G) \leq P\text{-}\Delta_N(G) \leq p\text{-}\Delta_E(G)$ and
$\gamma(G) \leq \gamma_{wd}(G) \leq \gamma_{wtc}(G) \leq P\text{-}\delta_N(G) \leq p\text{-}\delta_E(G)$

Theorem3.8 For any connected fuzzy graph G, $\gamma(G) \leq \gamma_c(G) \leq \gamma_{tc}(G) \leq \gamma_{stc}(G) \leq \gamma_{wtc}(G)$.

Proof Let G be a fuzzy graph and D be a minimum dominating set. D_{tc} is triple connected dominating set but need not be a minimum fuzzy dominating set, and alsoD_{stc} is a strong (weak) triple connected dominating set.

Therefore we get$|D| \leq |D_c| \leq |D_{tc}| \leq |D_{st}|$ and$|D_{stc}| \leq |D_{wtc}|$

That is $\gamma(G) \leq \gamma_c(G) \leq \gamma_{tc}(G) \leq \gamma_{stc}(G) \leq \gamma_{wtc}(G)$.

Theorem 3.9 If a spanning sub graph H of a graph G has a strong(weak) triple connected dominating set then G also has a strong (weak) triple connected dominating set.

Proof Let G be a connected fuzzy graph and H is the spanning sub graph of G. H has a strong (weak) triple connected dominating set and V(G) = V(H) therefore G is a strong (weak) triple connected dominating set.

Theorem 3.10 For any connected fuzzy graph G with P vertices and strong (weak) triple connected dominating vertices P if any only if $G \cong P_3$ or C_3.

Proof Suppose $G \cong P_3$, C_3 then the strong(weak) triple connected dominating vertices is 3, and P vertices. Conversely, Let G be a connected fuzzy graph with P vertices such that strong (weak) triple connected dominating vertices P then
$G \cong P_3$ or C_3

Theorem 3.11 For any connected fuzzy graph G with P≥3 vertices and exactly one vertex has$\Delta_N(G) \leq$P-2, the strong triple connected dominating vertices is 3.

Proof Let G be a connected graph with P≥3 vertices and exactly one vertex has maximum neighborhood degree $\Delta_N(G) \leq$P-2.Let v be the vertex of maximum neighborhood degree $\Delta_N(G) \leq$P-2. Let v_1,v_2…… and v_{p-2} be the vertices which are adjacent to v and Let v_{p-1} be the vertex which is not adjacent to v. Since G is connected, v_{p-1} is adjacent to a vertex v_i for some i, Then s = $\{v,v_i,v_{p-1}\}$ forms a minimum strong triple connected dominating set of G.

Theorem 3.12 For a connected fuzzy graph G with 5 vertices, the strong triple connected dominating vertices is P-2 if and only if G is isomorphic to P_5,C_5.

Proof Suppose G is isomorphic to P_5,C_5then clearly strong triple connected dominating vertices is P-2 conversely, let G be a connected fuzzy graph with 5 vertices and strong triple connected dominating vertices is 3. Let S=$\{ v_1,v_2 ,v_3\}$ be a γ_{stc} set then clearly <S> = P_3 or C_3. Let V-S= V(G)-V(S)= $\{ v_4,v_5 \}$ then $\{V\text{-}S\} = K_2$ or $\overline{k_2}$

Case (i)<S> = $P_3 = v_1v_2 v_3$

Sub case (i)<V-S> = $K_2 = v_4 v_5$

Since G is connected, there exists a vertex say v_1 (or v_3) in P_3 which is adjacent to v_4 (or v_5) in K_2, then s=$\{v_1, v_2, v_4\}$ forms γ_{stc} set of G so that strong triple connected dominating vertices is P-2. If v_4 is adjacent to v_1, if the vertices v_1 and v_2 are connected to two vertices, v_3 is connected to one vertex then G is isomorphic to P_5. Since G is connected there exists a vertex say v_2 in P_3 is adjacent to v_4 (or v_5) in K_2. Then S=$\{ v_2, v_4, v_5\}$ forms a γ_{stc} set of G so that strong triple connected dominating vertices is P-2. If the vertex v_1 and v_3 are connected to two vertices and v_2 is connected to one vertex. Then G is isomorphic to P_4. Now by increasing the degree of the vertices by the above arguments, we have $G \cong C_5$

subcase (ii)<V-S> = $\overline{k_2}$.

Since G is connected, then there exists a vertex say v_1(or v_3) in P_3 is adjacent to v_4 and v_5 in $\overline{k_2}$. Then S=$\{ v_1,v_2 ,v_3\}$ forms a γ_{stc} set of G so that strong triple connected dominating vertices is P-2. Since G is connected. There exists a vertex say v_2 in P_3 which is adjacent to $v_4$and v_5 in $\overline{k_2}$. Then S=$\{ v_1,v_2 ,v_3\}$ forms a γ_{stc} set of G so that strong triple connected dominating vertices is P-2.since G is connected, there exists a vertex say v_1 in P_3 which is adjacent v_2 in $\overline{k_2}$ and v_3 in P_3 is adjacent to v_5 in K_2. Then S=$\{ v_1,v_2 ,v_3\}$ form a γ_{stc} set of G so that strong triple connected dominating vertices is P-2. Since G is connected, then there exists a vertex say v_1 in P_3 is adjacent to v_4 in $\overline{k_2}$ and v_3 in P_3 is adjacent to v_5 in $\overline{k_2}$. Then S={ v_1,v_2 ,v_3} forms a γ_{stc} set of G so that strong triple connected dominating vertices is P-2

Case (ii)<S> = C_3= v_1v_2 v_3 v_1
Sub case (i)<V-S>= K_2 = v_4v_5
Since G is connected, there exists a vertex say v_1, (or v_2 ,v_3) in C_3 is adjacent to v_4 (or v_5). Then S={ v_1,v_2 ,v_4} forms a γ_{stc} set of G so that strong triple connected dominating vertices is P-2. If the vertex v_1, v_2, and v_3 are connected to two vertices than G ≅ C3
Sub case (ii)<V-S> = K_2

Since G is connected there exists a vertex say v_1(or v_2 ,v_3) in C_3 is adjacent to v_4 in $\overline{k_2}$ and v_2 (or v_3) in C_3 is adjacent to v_5 in $\overline{k_2}$. Then S={ v_1,v_2 ,v_3} forms aγ_{stc} set of G so that strong triple connected dominating vertices is P-2.

IV. Conclusion

The strong (weak) triple connected domination number of fuzzy a graph is defined. Theorems related to this concept are derived and the relation between triple connected domination number of fuzzy graphs and strong (weak) triple connected domination number of fuzzy graphs are established.

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