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Special Double Sampling Plan for truncated life tests based on the Marshall-Olkin extended exponential distribution

D.Malathi[1] ,Dr.S.Muthulakshmi[2]

[1] *Assistant Professor (Sr.Gr), Department of Mathematics, Velalar College of Engineering & Technology, Erode.*

[2] *Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore.*

ABSTRACT

In this study , we propose the designing of special double sampling plan for truncated life tests using percentiles under the assumption that the life time of the product follows Marshall-Olkin extended exponential distribution . The minimum sample sizes for the first and second samples are determined for the specified consumer's confidence level with minimum average sample number. The operating characteristics are analysed and the minimum percentile ratios of life time are obtained so as to meet the specified producer's risk. The efficiency of special double sampling plan is analysed with zero-one double sampling.

***Keywords**: Special double sampling, Consumer's risk, Operating Characteristic function, Producer's risk, Truncated life tests, Average sample number.*

I. INTRODUCTION

The reliability of the product has become a vital factor in a global business market. Acceptance sampling ensures the quality of the product.The acceptance sampling for life tests is concerned with accepting or rejecting a submitted lot of products on the basis of life time of the sample units taken from the lot. The producer and the consumer are subject to risks due to decision based on sample results. By increasing the sample size, Consumer's risk of accepting bad lots and producer's risk of rejecting good lots may be minimized to a certain level but this will increase the cost of inspection. Truncation of life test time may be introduced to reduce the cost of inspection.

Many studies have been carried out for designing single sampling plan and double sampling plans for the truncated life tests under various statistical distributions for life time.

Epstein (1954) first introduced single acceptance sampling plans for the truncated life test based on the exponential distribution. Goode and Kao (1961) developed an acceptance sampling plan using the Weibull distribution as a lifetime distribution. Gupta and Groll (1961) derived the acceptance sampling plan for the gamma distribution and Gupta (1962) designed the plan for the lifetime of the product having Log-normal distribution. All these authors considered the design of acceptance sampling plans based on the population mean.

Duncan (1986) pointed out that double sampling plan reduce the sample size to attain the same result as compared to single sampling plan. Aslam and Jun (2010) introduced double acceptance sampling for the truncated life test based on percentiles of the generalized log-logistic distribution.

The purpose of this paper is to propose the special double acceptance sampling plan for the truncated life test assuming that the lifetime of a product follows Marshall-Olkin extended exponential distribution. This distribution plays a vital role , in studying the life time of electrical component such as memory disc, mechanical component such as bearings and systems such as aircraft, automobiles. The minimum sample sizes of special double sampling plan are determined to meet the specified consumer's confidence level by incorporating minimum average sample number. The operating characteristics are analysed and the minimum percentile ratio's of life time are obtained for the specified producers risk. The accomplishment of this plan is studied by comparing with zero-one double sampling plan.

Marshall- Olkin extended exponential distribution

Assume that the lifetime of a product follows Marshall- Olkin extended exponential distribution. The probability density function and cumulative distribution function of Marshall- Olkin extended exponential distribution are given by

$$f(t) = \frac{(a/\sigma)\exp(-t/\sigma)}{[1-(1-a)\exp(-t/\sigma)]^2}, \quad t > 0, a, \sigma > 0 \tag{1}$$

and

$$F(t) = \frac{1-\exp(-t/\sigma)}{1-(1-a)\exp(-t/\sigma)}, \quad t > 0, a, \sigma > 0 \tag{2}$$

where σ is the scale parameter and **a** is the shape parameter. The $100q^{th}$ percentile is given by

$$t_q = \sigma \ln[(1-(1-a)q)/(1-q)] \tag{3}$$

where 0<q<1. When $q=0.5$, t_q reduces to σln(1+a) which is the median of Marshall- Olkin extended exponential distribution. It is seen that, t_q depends only on **a** and **σ** .Also it is seen that t_q is increasing with respect to **a** for $q> 0.5$ and decreasing with respect to **a** for $q< 0.5$.

Taking

$$\eta = \ln[(1-(1-a)q)/(1-q)]$$

and $\delta = t/t_q$,

equation (2) becomes

$$F(t) = \frac{1-\exp(-\delta\eta)}{1-(1-a)\exp(-\delta\eta)}, t > 0 \tag{4}$$

The designing of special double acceptance sampling plan using percentiles under a truncated life test is to set up the minimum sample sizes for the given shape parameter **a** such that the consumer's risk, the probability of accepting a bad lot, does not exceed 1- P* . A bad lot means that the true $100q^{th}$ percentile t_q , is below the specified percentile t_q^0 .

Design of the proposed sampling plan

Assume that the acceptable quality of a product is represented by its percentile lifetime t_q^0 . The lot will be accepted if the data supports the null hypothesis, H_0: $t_q \geq t_q^0$ against the alternative hypothesis, H_1: $t_q < t_q^0$. The significance level for the test, is 1−P*, where P*is the consumer's confidence level.

The operating procedure of special double sampling plan for the truncated life test has the following steps:

- Draw a sample of size n_1 from the lot and put on the test for pre-assigned experimental time t_0 and observe the number of defectives d_1. If $d_1 \geq 1$ reject the lot.
- If $d_1 = 0$, draw a second random sample of size n_2 and put them on the test for time t_0 and observe the number of defectives d_2.If $d_2 \leq 1$ accept the lot, Otherwise reject the lot.

In a special double sampling plan the decision of acceptance is made only after inspecting the second sample. This aspect differs from usual double sampling plan in which decision of acceptance can be made even before the inspection of the second sample.

It is more convenient to make a termination time interms of acceptable percentiles of lifetime t_q^0 which depend only on $\delta = t/t_q^0$. For a given P*, the proposed acceptance sampling plan can be characterized by $(n_1, n_2, a, \eta, \delta)$.

The minimum sample sizes n_1 and n_2 are determined for

$$P_a = (1-p)^{n_1+n_2}\left[1+\frac{n_2 p}{1-p}\right] \le 1-P^* \quad (5)$$

where, p is the probability that an item fails before t_0, which is given by

$$p = \frac{1-\exp(-\delta\eta)}{1-(1-a)\exp(-\delta\eta)} \quad (6)$$

Equation (5) provides multiple solutions for sample sizes n_1 and n_2 satisfying the specified consumer's confidence level. In order to find the optimal sample sizes the minimum of ASN is incorporated along with the probability of the acceptance of the lot less than or equal to 1-P* and $n_2 \le n_1$.

Determination of the minimum sample sizes for special double sampling plan reduces to

Minimize $ASN = n_1 + n_2(1-p)^{n_1}$

subject to $(1-p)^{n_1+n_2}\left[1+\frac{n_2 p}{1-p}\right] \le 1-P^*$ (7)

where n_1 and n_2 are integers.The minimum sample sizes satisfying the condition (7) can be obtained by search procedure. Table 1 is constructed for the minimum sample sizes of zero-one double sampling plan for q=0.05 and 0.1 with various values of a (=2,3,4,5), P*(=0.75,0.90,0.95,0.99) and δ (=0.5,0.7,1.0, 1.5,2.0,2.5,3.0,3.5) . Numerical values in Tables 1 reveals that

(i) increase in consumers confidence level increases the first and the second sample sizes quite rapidly, when the test time is short
(ii) increase in shape parameter **a** increases sample sizes for any P*.
(iii) increase in δ decreases the sample size for any P* .

Operating characteristics values of the sampling plan

OC values depict the performance of the sampling plan according to the quality of the submitted product. The probability of acceptance will increase more rapidly if the true percentile increases beyond the specified life .we need to know the operating characteristic values for the proposed plan according to the ratio of the true percentile to the specified life t_q/t_q^0. These plans are desirable since operating characteristic values increase more sharply to nearly one. Tables 2 and 3 are constructed to give the operating characteristic values corresponding to q=0.05 and 0.1 for fixed **a**.

Minimum Percentile ratio

Producer wants to know the minimum product quality level in order to maintain the producers risk at the specified level. At the specified producers risk α=0.05 the minimum ratio t_q/t_q^0 are obtained by solving $P_a \ge 1-\alpha$ and presented in table 4 by using the sample sizes in Table 1 for specified consumers confidence level. From Table 4 it is seen that with increase in consumers confidence level, decreases the ratio.

Comparitive study

The following table shows the gradual increase of OC values for the increase in the percentiles for special double sampling plan with confidence level P*=0.90, t/t_q^0=3.5 and t_q/t_q^0=15 with a=3 .

q	0.05	0.01	0.1	0.15	0.2	0.25
OC values	0.9738 (13,12)	0.9845 (47,46)	0.9901 (6,1)	0.9895 (4,1)	0.9888 (3,10)	0.9912 (2,1)

Also, the ASN values of zero-one double sampling plan and special double sampling plan for a=2,q=0.1 P*=0.75 are obtained as follows:

plan \ δ	2	2.5	3	3.5
DSP(0,1)	10.366	8.2671	6.8	6.5171
SDSP	6.7981	5.2441	4.5018	4.1927

On comparing the values of life test plans,the zero-one double sampling plan using percentiles provides minimum sample size and hence economical.

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Table-1 Minimum sample sizes for Special double sampling plan

q	a	δ P*	0.5	0.7	1	1.5	2	2.5	3
0.05	2	0.75	43,40	31,28	22,19	15,11	11,9	9,7	8,4
		0.9	66,63	47,45	33,31	22,20	17,14	13,12	11,10
		0.95	82,81	59,57	41,40	28,25	21,18	17,14	14,12
		0.99	119,118	85,84	60,57	40,37	30,27	24,22	20,18
	3	0.75	43,41	31,28	22,19	14,13	11,8	9,6	7,6
		0.9	66,65	47,46	33,31	22,20	16,15	13,11	11,8
		0.95	83,82	59,58	41,40	27,26	20,19	16,14	13,12
		0.99	121,119	86,84	60,57	39,38	29,27	23,21	19,17
	4	0.75	44,41	31,29	22,19	14,13	11,8	8,7	7,5
		0.9	67,66	48,45	33,31	22,19	16,14	13,10	10,9
		0.95	84,83	60,57	41,40	27,25	20,18	16,13	13,11
		0.99	122,121	86,85	60,57	39,37	28,27	22,21	18,17
	5	0.75	44,42	31,29	22,19	14,12	10,9	8,6	7,4
		0.9	68,66	48,46	33,31	22,19	16,13	12,11	10,8
		0.95	85,84	60,58	41,40	27,25	19,18	15,13	12,11
		0.99	124,121	87,85	60,57	39,37	28,26	21,20	17,16
0.1	2	0.75	21,20	15,14	11,9	8,4	6,3	5,1	4,2
		0.9	33,31	24,21	17,14	11,10	8,7	7,5	6,3
		0.95	41,39	29,28	21,18	14,12	10,9	8,7	7,5
		0.99	59,58	42,41	30,27	20,18	15,13	12,10	10,8
	3	0.75	22,20	16,13	11,9	7,6	6,2	4,3	4,1
		0.9	34,31	24,22	17,14	11,9	8,7	7,4	5,4
		0.95	42,40	30,28	21,18	14,11	10,8	8,6	7,4
		0.99	61,59	43,41	30,27	19,18	14,13	11,10	9,8
	4	0.75	22,21	16,14	11,9	7,6	5,4	4,3	4,1
		0.9	34,33	24,23	17,14	11,8	8,6	6,5	5,3
		0.95	43,41	30,29	21,18	13,12	10,7	8,5	6,5
		0.99	62,60	43,42	30,27	19,17	14,11	11,8	8,7
	5	0.75	23,20	16,14	11,9	7,5	5,4	4,2	3,2
		0.9	35,33	24,23	17,14	11,8	8,5	6,4	5,2
		0.95	44,42	30,29	21,18	13,11	9,8	7,6	6,4
		0.99	63,62	44,42	30,27	19,16	13,12	10,8	8,6

Table-2 Operating Characteristic values of special double sampling plan with a=2 & q=0.05

P*	t/t_q^0	n_1	n_2	t_q/t_q^0					
				3	6	9	12	15	18
0.75	0.5	43	40	0.6794	0.8342	0.8888	0.9164	0.9331	0.9442
	0.7	31	28	0.6833	0.8368	0.8907	0.9179	0.9343	0.9453
	1	22	19	0.689	0.8406	0.8935	0.9202	0.9361	0.9468
	1.5	15	11	0.7167	0.8578	0.9058	0.9297	0.9439	0.9533
	2	11	9	0.7001	0.8476	0.8985	0.9241	0.9393	0.9494
	2.5	9	7	0.7041	0.8506	0.9008	0.9258	0.9407	0.9507
	3	8	4	0.7679	0.8893	0.9281	0.9469	0.9579	0.9652
0.9	0.5	66	63	0.5279	0.7445	0.8268	0.8693	0.8951	0.9125
	0.7	47	45	0.5279	0.7446	0.8268	0.8693	0.8951	0.9125
	1	33	31	0.5316	0.7475	0.8291	0.8711	0.8966	0.9137
	1.5	22	20	0.5398	0.7536	0.8336	0.8747	0.8996	0.9163
	2	17	14	0.5544	0.7649	0.8422	0.8816	0.9053	0.9212
	2.5	13	12	0.5401	0.7541	0.8339	0.8749	0.8997	0.9164
	3	11	10	0.5377	0.7532	0.8334	0.8746	0.8996	0.9163
0.95	0.5	82	81	0.4345	0.6798	0.7804	0.8335	0.6611	0.8881
	0.7	59	57	0.4345	0.6827	0.7828	0.8355	0.8677	0.8897
	1	41	40	0.4341	0.6824	0.7825	0.8352	0.8675	0.8893
	1.5	28	25	0.4487	0.6952	0.7926	0.8435	0.8744	0.8952
	2	21	18	0.4594	0.7038	0.7992	0.8488	0.8788	0.899
	2.5	17	14	0.4648	0.7088	0.8032	0.8521	0.8816	0.9014
	3	14	12	0.4577	0.7034	0.7991	0.8487	0.8788	0.8991
0.99	0.5	119	118	0.2749	0.5565	0.6885	0.7614	0.8071	0.8382
	0.7	85	84	0.2754	0.5573	0.6892	0.7621	0.8076	0.8387
	1	60	57	0.2821	0.5648	0.6955	0.7674	0.8122	0.8427
	1.5	40	37	0.2881	0.5716	0.7012	0.7721	0.8162	0.8462
	2	30	27	0.2944	0.5785	0.7071	0.7769	0.8203	0.8498
	2.5	24	22	0.2884	0.5733	0.7029	0.7736	0.8175	0.8474
	3	20	18	0.2923	0.5778	0.7068	0.7768	0.8203	0.8497

Table-3 Operating Characteristic values of special double sampling plan with a=2 & q=0.1

P*	t/t_q^0	n_1	n_2	t_q/t_q^0					
				3	6	9	12	15	18
0.75	0.5	21	20	0.6798	0.8342	0.8888	0.9164	0.9331	0.9442
	0.7	15	14	0.6841	0.8371	0.8907	0.9181	0.9343	0.9452
	1	11	9	0.6993	0.8472	0.8982	0.9238	0.9391	0.9493
	1.5	8	4	0.7673	0.8891	0.9279	0.9468	0.9578	0.9651
	2	6	3	0.7673	0.8892	0.9281	0.9469	0.9579	0.9651
	2.5	5	1	0.8628	0.9428	0.9651	0.9751	0.9606	0.9842
	3	4	2	0.7676	0.8896	0.9283	0.9471	0.9581	0.9652
0.9	0.5	33	31	0.5306	0.7469	0.8286	0.8708	0.8963	0.9135
	0.7	24	21	0.5426	0.7561	0.8386	0.8764	0.9011	0.9175
	1	17	14	0.5534	0.7643	0.8418	0.8813	0.9051	0.9211
	1.5	11	10	0.5368	0.7526	0.8331	0.8744	0.8993	0.9161
	2	8	7	0.5576	0.7666	0.8431	0.8822	0.9057	0.9214
	2.5	7	5	0.5756	0.7821	0.8553	0.8921	0.9141	0.9286
	3	6	3	0.6457	0.8288	0.8892	0.9185	0.9356	0.9468
0.95	0.5	41	39	0.4404	0.6874	0.7864	0.8383	0.8701	0.8915
	0.7	29	28	0.4394	0.6867	0.7858	0.8378	0.8696	0.8911
	1	21	18	0.4583	0.7031	0.7987	0.8484	0.8785	0.8987
	1.5	14	12	0.4566	0.7027	0.7986	0.8483	0.8785	0.8987
	2	10	9	0.4614	0.7054	0.8001	0.8493	0.8792	0.8992
	2.5	8	7	0.4679	0.7111	0.8046	0.8529	0.8821	0.9017
	3	7	5	0.5021	0.7388	0.8259	0.8701	0.8964	0.9141
0.99	0.5	59	58	0.2786	0.5607	0.6919	0.7642	0.8095	0.8403
	0.7	42	41	0.2809	0.5635	0.6943	0.7662	0.8112	0.8416
	1	30	27	0.2933	0.5776	0.7063	0.7764	0.8199	0.8494
	1.5	20	18	0.2912	0.5769	0.7061	0.7763	0.8198	0.8494
	2	15	13	0.2993	0.5862	0.7138	0.7827	0.8253	0.8541
	2.5	12	10	0.3079	0.5956	0.7217	0.7892	0.8308	0.8589
	3	10	8	0.3169	0.6054	0.7297	0.7959	0.8364	0.8637

Table -4 Minimum percentile ratio of Special double sampling plan with a=2

q	a	P*	t/t_q^0						
			0.5	7	1	1.5	2	2.5	3
0.05	2	0.75	0.1999	0.1894	0.1871	0.2031	0.2105	0.2095	0.2089
		0.9	0.1198	0.1277	0.1295	0.1364	0.1368	0.1382	0.1357
		0.95	0.0987	0.0985	0.0971	0.0981	0.0982	0.0987	0.0995
		0.99	0.0693	0.0673	0.0663	0.0693	0.0683	0.0682	0.0681
	3	0.75	0.1921	0.1931	0.1899	0.1959	0.2009	0.2013	0.2093
		0.9	0.1278	0.1248	0.1258	0.1268	0.1277	0.1298	0.1339
		0.95	0.0992	0.0989	0.0984	0.0999	0.1028	0.1094	0.1108
		0.99	0.0732	0.0702	0.0714	0.0712	0.0782	0.0772	0.0724
	4	0.75	0.1985	0.1965	0.1952	0.1982	0.2079	0.2192	0.2299
		0.9	0.1238	0.1299	0.1278	0.1299	0.1329	0.1393	0.1391
		0.95	0.1053	0.0991	0.0999	0.1129	0.1168	0.1198	0.1212
		0.99	0.0691	0.0699	0.0736	0.0721	0.0792	0.0791	0.0993
	5	0.75	0.1949	0.1981	0.1982	0.2192	0.2312	0.2432	0.2391
		0.9	0.1291	0.1269	0.1299	0.1309	0.1392	0.1409	0.1463
		0.95	0.1094	0.1053	0.1099	0.1109	0.1172	0.1163	0.1193
		0.99	0.0717	0.0754	0.0783	0.0778	0.0784	0.0799	0.0873
0.1	2	0.75	0.1909	0.1899	0.1901	0.2099	0.2095	0.2395	0.2229
		0.9	0.1276	0.1277	0.1275	0.1274	0.1278	0.1272	0.1379
		0.95	0.1037	0.1085	0.1087	0.1091	0.1082	0.1077	0.1075
		0.99	0.0699	0.0672	0.0673	0.0693	0.0683	0.0691	0.0709
	3	0.75	0.1909	0.1991	0.1999	0.2099	0.2491	0.2283	0.2459
		0.9	0.1279	0.1262	0.1298	0.1398	0.1397	0.1398	0.1469
		0.95	0.1099	0.1089	0.1074	0.1062	0.1088	0.1124	0.1178
		0.99	0.0732	0.0762	0.0709	0.0729	0.0759	0.0772	0.0794
	4	0.75	0.1985	0.1965	0.2069	0.2099	0.2292	0.2312	0.2499
		0.9	0.1258	0.1279	0.1378	0.1429	0.1432	0.1493	0.1631
		0.95	0.1093	0.1045	0.1091	0.1139	0.1168	0.1298	0.1312
		0.99	0.0731	0.0791	0.0796	0.0799	0.0852	0.0891	0.0973
	5	0.75	0.1999	0.2099	0.2092	0.2292	0.2312	0.2592	0.2661
		0.9	0.1339	0.1329	0.1459	0.1499	0.1592	0.1663	0.1891
		0.95	0.1094	0.1053	0.1099	0.1189	0.1299	0.1293	0.1399
		0.99	0.0717	0.0754	0.0793	0.0899	0.0924	0.0973	0.0993