ISSN (e): 2250 – 3005 || Vol, 04 || Issue, 9 || September – 2014 ||
International Journal of Computational Engineering Research (IJCER)

Global Domination Set in Intuitionistic Fuzzy Graph

R. JahirHussain [1] ,S. Yahya Mohamed [2]
[1]P.G and Research Department of Mathematics, Jamal Mohamed College (Autonomous), Tiruchirappalli-620 020, India
[2]P.G and Research Department of Mathematics, Govt. Arts College, Trichy-22, India

ABSTRACT

In this paper, We define global Intuitionistic fuzzy domination set and its number of IFGs. Also connected Intuitionistic fuzzy domination number of IFGs are discussed. Some results and bounds of global Intutionistic fuzzy domination number of IFGs are established.

KEYWORDS: *Intuitionistic fuzzy graph, connected Intuitionistic fuzzy dominating set, global Intuitionistic fuzzy dominating set, effective degree.*
2010Mathematics Subject Classification: *05C69, 03F55, 05C72, 03E72.*

I. INTRODUCTION

Atanassov [1] introduced the concept of intuitionistic fuzzy (IF) relations and intuitionistic fuzzy graphs (IFGs). Research on the theory of intuitionistic fuzzy sets (IFSs) has been witnessing an exponential growth in Mathematics and its applications. R. Parvathy and M.G.Karunambigai's paper [7] introduced the concept of IFG and analyzed its components. Nagoor Gani, A and Sajitha Begum, S [5] defined degree, Order and Size in intuitionistic fuzzy graphs and extend the properties. The concept of Domination in fuzzy graphs is introduced by A. Somasundaram and S. Somasundaram [8] in the year 1998. Parvathi and Thamizhendhi[6] introduced the concepts of domination number in Intuitionistic fuzzy graphs. Study on domination concepts in Intuitionistic fuzzy graphs are more convenient than fuzzy graphs, which is useful in the traffic density and telecommunication systems. The Global domination number of a Graph was discussed by E. Sampathkumar [10] in 1989.In this paper, We define global Intuitionistic fuzzy domination set of IFG and discuss the situation of this concept used in network. Also some thereoms and bounds of global Intuitionistic fuzzy domination number of IFGs are established.

II. PRELIMINARIES

Definition 2.1: An Intuitionistic fuzzy graph is of the form G = (V, E) where

(i) V={v1,v2,….,vn} such that μ_1: V→[0,1]and γ_1: V →[0,1] denote the degree of membership and non-membership of the element vi ∈V, respectively, and

$0 \le \mu_1(vi) + \gamma_1(vi) \le 1$ for every vi ∈V, (i = 1,2, ……. n),

(ii) E ⊆ V x V where μ_2: VxV →[0,1] and γ_2: VxV →[0,1] are such that

$\mu_2(vi, vj) \le \min[\mu_1(vi), \mu_1(vj)]$ and $\gamma_2(vi, vj) \le \max[\gamma_1(vi), \gamma_1(vj)]$

and $0 \le \mu_2(vi, vj) + \gamma_2(vi, vj) \le 1$ for every (vi ,vj) ∈ E, (i, j = 1,2, ……. n)

Definition 2.2 An IFG H = < V', E' > is said to be an Intuitionistic fuzzy subgraph (IFSG) of the IFG, G = < V, E > if V' ⊆V and E' ⊆ E. In other words, if $\mu_{1i}' \le \mu_{1i}$; $\gamma_{1i}' \ge \gamma_{1i}$ and $\mu_{2ij}' \le \mu_{2ij}$; $\gamma_{2ij}' \ge \gamma_{2ij}$ for every i, j = 1,2………n.

Definition 2.3: Let G = (V,E) be a IFG. Then the cardinality of G is defined as

$$|G| = \left| \sum_{vi \in V} \frac{1+\mu1(vi)-\gamma1(vi)}{2} + \sum_{vi,vj \in E} \frac{1+\mu2(vi,vj)-\gamma2(vi,vj)}{2} \right|$$

Definition 2.4: The vertex cardinality of IFG G is defined by

$$|V| = \left| \sum_{vi \in V} \frac{1+\mu1(vi)-\gamma1(vi)}{2} \right| = p \text{ and}$$

The edge cardinality of IFG G is defined by $|E| = \left| \sum_{vi,vj \in E} \frac{1+\mu2(vi,vj)-\gamma2(vi,vj)}{2} \right| = q.$

The vertex cardinality of IFG is called the order of G and denoted by O(G). The cardinality of G is called the size of G, denoted by S(G).

Definition 2.5: An edge e = (x, y) of an IFG G = (V, E) is called an effective edge if $\mu_2(x, y) = \mu_1(x) \wedge \mu_1(y)$ and $\gamma_2(x, y) = \gamma 1(x) \vee \gamma 1(y)$.

Definition 2.6: An Intuitionistic fuzzy graph is complete if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} = \max(\gamma_{2i}, \gamma_{2j})$ for all (vi , vj) $\in$ V.

Definition 2.7: An Intuitionistic fuzzy graph G is said to be strong IFG if $\mu_2(x, y) = \mu_1(x) \wedge \mu_1(y)$ and $\gamma_2(x, y) = \gamma 1(x) \vee \gamma 1(y)$ for all (vi , vj) $\in$ E. That is every edge is effective edge.

Definition 2.8 : The complement of an IFG G = < V, E > is denoted by $\bar{G} = (\bar{V}, \bar{E})$ and is defined as i) $\bar{\mu}_1(v) = \mu_1(v)$ and $\bar{\gamma}_1(v) = \gamma_1(v)$
ii) $\bar{\mu}_2(u,v) = \mu_1(u) \wedge \mu_1(v) - \mu_2(u,v)$ and $\bar{\gamma}_2(u,v) = \gamma_1(u) \vee \gamma_1(v) - \gamma_2(u,v)$ for u,v in V

Definition 2.9: Let G = (V,E) be an IFG. The neighbourhood of any vertex v is defined as N(v) = ($N_\mu(v)$, $N_\gamma(v)$), Where $N_\mu(v) = \{w \in V;\ \mu_2(v,w) = \mu_1(v) \wedge \mu_1(w)\}$ and $N_\gamma(v) = \{w \in V;\ \gamma_2(v,w) = \gamma_1(v) \vee \gamma_1(w)\}$. N[v] = N (v) $\cup$ {v} is called the closed neighbourhood of v.

Definition 2.10: The neighbourhood degree of a vertex is defined as $d_N(v) = (d_{N\mu}(v), d_{N\gamma}(v))$ where $d_{N\mu}(v) = \sum_{w \in N(v)} \mu_1(w)$ and $d_{N\gamma}(v) = \sum_{w \in N(v)} \gamma_1(w)$.

The minimum neighbourhood degree is defined as $\delta_N(G) = (\delta_{N\mu}(v), \delta_{N\gamma}(v))$, where $\delta_{N\mu}(v) = \wedge\{d_{N\mu}(v): v \in V\}$ and $\delta_{N\gamma}(v) = \wedge\{d_{N\gamma}(v): v \in V\}$.

Definition 2.11: The effective degree of a vertex v in a IFG. G = (V, E) is defined to be sum of the effective edges incident at v, and denoted by $d_E(v)$. The minimum effective degree of G is $\delta_E(G) = \wedge\{d_E(v)/v \in V\}$

Definition 2.12: Let G= (V, E) be an IFG. Let u, v $\in$ V, we say that u dominated v in G if there exist a strong arc between them. A Intuitionistic fuzzy subset D $\subseteq$V is said to be dominating set in G if for every v$\in$ V-D, there exist u in D such that u dominated v. The minimum scalar cardinality taken over all Intuitionisitc fuzzy dominating sets is called Intuitionistic fuzzy domination number and is denoted by γ. The maximum scalar cardinality of a minimal domination set is called upper Intuitionistic fuzzy domination number and is denoted by the symbol Γ .

Definition 2.13: A Intuitionistic fuzzy dominating set D$\subseteq$V of IFG G is said to be a Intutitionisitc fuzzy connected dominating set of G if the subgraph <D> induced by D is connected. The minimum cardinality taken over all minimal Intuitionistic fuzzy connected dominating sets is called Intuitionistic fuzzy domination number of G and it is denoted by $\gamma_c(G)$.

Definition 2.14: An independent set of an Intuitionistic fuzzy graph G = (V, E) is a subset S of V such that no two vertices of S are adjacent in G.

Definition 2.15: A Bipartite IFG, G= (V,E) is said to be complete Bipartite IFG, if $\mu_2(v_i, v_j) = \mu_1(v_i) \wedge \mu_1(v_j)$ and $\gamma_2(v_i, v_j) = \gamma_1(v_i) \vee \gamma_1(v_j)$ for all $v_i \in V_1$ and $v_j \in V_2$. It is denoted by $K_{V1i,V2j}$.

III. GLOBAL INTUITIONISTIC FUZZY DOMINATION SET IN IFG

Definition 3.1: Let G = (V, E) be an IFG. A Intuitionistic fuzzy dominating set S $\subseteq$ V is said to be global Intuitionistic fuzzy dominating set of G if S is also a Intuitionistic fuzzy dominating set of $\bar{G}$.

The minimum cardinality of global Intuitionistic fuzzy dominating sets is global Intuitionistic fuzzy domination number and is denoted by $\gamma_g(G)$.

Example:

In case of transportation and road networks, the travel time is mostly used as weight. The travel time is a function of the traffic density on the road and/or the length of the road. The length of a road is a crisp quantity but the traffic density is fuzzy. In a road network, we represent crossings as nodes and roads as edges. The traffic density is mostly calculated on the road between adjacent crossings. These numbers can be represented as intuitionistic fuzzy numbers. Road network represented as an intuitionistic fuzzy graph $R^* = (C, L)$, where C is an intuitionistic fuzzy set of crossings at which the traffic density is calculated and L is an intuitionistic fuzzy set of roads between two crossings. The degrees of membership, $\mu_L(xy)$, and non membership, $v_L(xy)$, are calculated as $\mu_L(xy) = \min(\mu_C(x), \mu_C(y))$, $\gamma_L(xy) = \max(\gamma_C(x), \gamma_C(y))$.

Some essential goods are being supplied to some crossings from supplying stations located some other crossings. It may happen that the roads(edges of G) may be closed for some reason or the other. So, we have to think of maintaining the supply of goods to various crossing uninterrupted through secret links (i.e. edges of the complement of network). We have to find minimum number of supplying stations(crossings) needed which is called global Intuitionistic fuzzy domination number.

Example 3.2: Let G = (V,E) be IFG be defined as follows

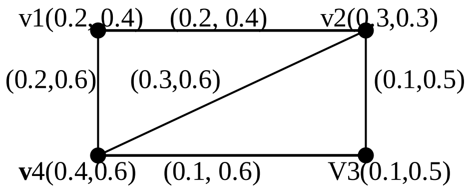

Fig- 1: Intuitionistic fuzzy graph(G)

Here |v1| = 0.4 , |v2| = 0.5, |v3| = 0.3, |v4| = 0.4 and minimum γ_g-set is { v2, v3, v4} and therefore $\gamma_g(G) = 1.2$.

Observations 3.3:

(i) $\gamma_g(Kn) = \gamma_g(\bar{K}_n) = p$

(ii) $\gamma_g(K_{v1i,v2j}) = \text{Min}\{|vi|\} + \text{Min}\{|vj|\}$, where $v_i \in V_1$ and $v_j \in V_2$.

Proposition 3.4: The global Intuitionistic fuzzy dominating set is not singleton.

Proof: Since gifd-set contain dominating set for both G and G^{sc} then at least two vertices are in the set.
i.e) The gifd-set containing at least two vertices.

Theorem 3.5: For any IFG G= (V, E) with effective edges, $\text{Min}\{|V_i|+|V_j|\} \le \gamma_g(G) \le p$, $i \neq j$

Proof: We know that global Intuitonistic fuzzy dominating set has at least two vertices. Let {vi, vj} are the vertices, then $\text{Min}\{|Vi|+|Vj|\} = \gamma_g(G)$
If the set contains other than {vi, vj} then $\text{Min}\{|Vi|+|Vj|\} < \gamma sg(G)$, $i \neq j$
If the given G is complete IFG then gifd-set contains all the vertices of the G, that is $\gamma_g(G) \le O(G) = p$
i.e.) We get, $\text{Min}\{|Vi|+|Vj|\} \le \gamma_g(G) \le p$.

Theorem 3.6: Let G = (V, E) be the IFG and the Intuitionistic fuzzy dominating set S of G is global Intuitionistic fuzzy dominating set if and only if, for each v ∈ V-S, there exists a u ∈S such that u is not adjacent to v.

Proof: Let S is global dominating set and also dominating set.
Suppose u is adjacent to v then we get S is not a dominating set.
Which is contradiction. That is u is not adjacent to v.
Conversely, for each v ∈ V-S and u is not adjacent to v then the set S is dominating both G and $\bar{G}$. That is S is global Intuitionistic fuzzy dominating set.

Theorem 3.7: Let G = (V, E) be an IFG then (i) $\gamma_g(G) = \gamma_g(\bar{G})$ (ii) $\gamma(G) \le \gamma_g(G)$

Proof: G is connected IFG and γ_g-set dominating vertices of G and $\bar{G}$. Clearly $\gamma_g(G) = \gamma_g(\bar{G})$.
Suppose D is the γ-set of G then the number of vertices in the dominating set is less than or some time equal to γ_g-set. That is $\gamma(G) \le \gamma_g(G)$.

Theorem 3.8: Let S be the minimum Intuitionisitc fuzzy dominating set of IFG G containing t vertices. If there exist a vertex v ∈ V-S adjacent to only vertices in S then γ_g-set contain atmost t+1 vertices.

Proof: Since S is the γ-set and v ∈ V-S adjacent to only vertices in S then we get S∪{v} is a global Intuitionistics fuzzy dominating set.
That is γ_g-set contain atmost t+1 vertices.

Theorem 3.9: Let G = (V, E) be strongly connected IFG then, at least one of the following holds.

(i) $\gamma_c(G) \le \gamma_g(G)$. (ii) $\bar{\gamma}_c(G) \le \gamma_g(G)$.

Proof: Since γ_c-set also dominating set and induced Intuitionistic fuzzy subgraph is connected then the Gc may be disconnected and it is less than or equal to γ_g-set.
That is $\gamma_c(G) \le \gamma_g(G)$. Similarly, we have $\bar{\gamma}_c(G) \le \gamma_g(G)$.

Definition 3.10: G = (V, E) be a connected IFG with effective edges which is said to be semi complete IFG, if every pair of vertices have a common neighbor in G .

The IFG G is said to be purely semi complete IFG if G is semi complete IFG but not complete IFG.

Example 3.11:

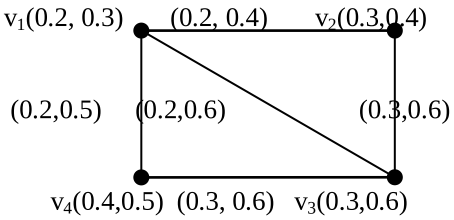

Fig-2: Purely Semi complete IFG

Theorem 3.12: Let G =(V, E) be the purely semi complete IFG. Then γg-set contains at least three vertices.

Proof: Since G is purely semi complete IFG then it contains triangles with common vertex.
Let v be the common vertex then, In G^c , the vertex v is isolated vertex. Also global Intuitionistic dominating set contains Intuitionistic fuzzy dominating vertices of G and G^c.

Suppose gifd-set contains less than three vertices
We know that gifd-set not a singleton. i.e) gifd-set contains at least two vertices
Let D = { v_1, v_2} be a gifd-set in G.

Case 1: <D> is connected in G
Then v_1v_2 is an effective edge in G. By the definition of semi complete IFG, there is a v_3 in G such that $<v_1v_2v_3>$ is triangle in G, i.e) D is not a Intuitonistic fuzzy domination set in G^c.
Which is contradiction to D is a gifd-set in G.

Case 2: <D> is disconnected in G. i.e.) There is no effective edge between v_1 and v_2 .
Since G is semi complete IFG, there is v_3 in G such that $v_1v_3$and v_3v_2 are the effective edges in G
Therefore, In G^c, v_3 is not dominated by a vertex in D.
Which implies, D is not a gifd-set in G
Which is contradiction to our assumption.
That is γ_g-set contains at least three vertices

Theorem 3.13: Let G = (V, E) be the IFG with effective edges . $\gamma_g(G) = \text{Min}\{|Vi|+|Vj|\}$ $i \neq j$ if and only if there is an effective edge uv in G such that each vertex in V – {u, v} is adjacent to u or v but not both.

Proof: Suppose $\gamma_g(G) = \text{Min}\{|Vi|+|Vj|\}$ $i\neq j$, We assume D = {u, v} be the gifd-set in G
Let <D> is connected in G, then uv is an effective edge in G.
If any vertex w in V-{u, v} is adjacent to both u and v.
Which implies D is not a dominating set for G^c. which is contradiction to our assumption. i.e) effective edge uv in G such that each vertex in V – {u, v} is adjacent to u or v but not both.
Conversely, each vertex in V – {u, v} is adjacent to u or v but not both, then we get $\gamma g(G) = \text{Min}\{|Vi|+|Vj|\}$ $i\neq j$.

IV. CONCLUSION

Here ,We defined global Intuitionistic fuzzy domination set of IFG and discussed the situation of this concept used in network. Also some theorems and bounds of global Intuitionistic fuzzy domination number of IFGs are established. Further we going to establish more results and bounds on this gifd number with other domination parameters.

REFERENCES

[1]. Atanassov. KT. Intuitionistic fuzzy sets: theory and applications. Physica, New York, 1999.
[2]. Bhattacharya, P. Some Remarks on fuzzy graphs, Pattern Recognition Letter 5: 297-302,1987.
[3]. Harary,F., Graph Theory, Addition Wesley, Third Printing, October 1972.
[4]. Kulli,V.R., Theory of domination in graph, Vishwa International Publications, 2010.math. Phys. Sci., 13 (1979), 607-613.
[5]. Nagoor Gani. A and Shajitha Begum.S, Degree, Order and Size in Intuitionistic Fuzzy Graphs, International Journal of Algorithms, Computing and Mathematics, (3) 3 (2010).
[6]. Parvathi,R., and Thamizhendhi, G. Domination in Intuitionistic fuzzy graphs, Fourteenth Int. conf. on IFGs, Sofia, NIFS Vol.16, 2, 39-49, 15-16 May 2010.
[7]. Parvathi, R. and Karunambigai, M.G., Intuitionistic Fuzzy Graphs, Computational Intelligence,Theory and applications,International Conference in Germany, Sept 18 -20, 2006.
[8]. Somasundaram, A and Somasundaram, S., Domination in Fuzzy graph-I, Patter Recognition Letter 19(9), 1998, 787-791.
[9]. Sampathkumar.E, and Walikar.H.B., The connected domination number of a graph, J. math. Phys. Sci., 13 (1979), 607-613.
[10]. Sampathkumar. E., The Global Domination number of a Graph, Jour. Math. Phy. Sc., vol. 23, No. 5, pp. 377-385.
[11]. Tamizh Chelvam. T and Robinson Chelladuari. S, Complementary nil domination number of a graph, Tamkang Journal of mathematics, Vol.40, No.2 (2009),165-172.