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On (1,2)*-πgθ-CLOSED SETS IN BITOPOLOGICAL SPACES

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Abstract
In this paper we introduce a new class of sets called (1,2)-πgθ-closed sets in bitopological spaces. Also we find some basic properties of (1,2)*-πgθ-closed sets. Further, we introduce a new space called (1,2)*-πgθ-$T_{½}$ space. Mathematics Subject Classification: 54E55, 54C55*

KEY WORDS: *(1,2)*-πgθ-closed set, (1,2)*-πgθ-open set, (1,2)*-πgθ-continuity and (1,2)*-πgθ-$T_{½}$ space.*

I. INTRODUCTION

Velicko[24] introduced the notions of θ-open subsets, θ-closed subsets and θ-closure, for the sake of studying the important class of H-closed spaces in terms of arbitrary filterbases. Dontchev and Maki [7] alone have explored the concept of θ-generalized closed sets. Regular open sets have been introduced and investigated by Stone [23]. Levine [4,14] introduced generalized closed sets and studied their properties. Bhattacharya and Lahiri [5], Arya and Nour [4], Maki et al.[15],[16] introduced semi-generalized closed sets, generalized semi-closed sets and α–generalized closed sets and generalized α–closed sets respectively. O.Ravi et al [21] have introduced the concepts of (1,2)*-semi-open sets, (1,2)*-α-open sets, (1,2)*-semi-generalized-closed sets and (1,2)*-α-generalized closed sets in bitopological spaces. This paper is an attempt to highlight a new type of generalized closed sets called (1,2)*-π generalized θ-closed (briefly (1,2)*-πgθ-closed) sets and a new class of generalized functions called (1,2)*-πgθ-continuous functions and (1,2)*-πgθ-irresolute functions. These findings result in procuring several characterizations of (1,2)*-πgθ-closed sets and as well as their application which leads to an introduction of a new space called (1,2)*-πgθ-$T_{½}$ space.

II. PRELIMINARIES

Throughout this paper (X,τ_1,τ_2) and (Y,σ_1,σ_2) represent bitopological spaces on which no separation axioms are assumed unless otherwise mentioned.

Definition 2.1 ([20]). A subset S of a bitopological space (X,τ_1,τ_2) is said to be $\tau_{1,2}$-open if S = $A\cup B$ where $A\in\tau_1$ and $B\in\tau_2$. A subset S of X is $\tau_{1,2}$-closed if the complement of S is $\tau_{1,2}$-open.

Definition 2.2 ([20]). Let S be a subset of X. Then

(i) The $\tau_1\tau_2$–interior of S , denoted by $\tau_1\tau_2$ –int (S) is defined by $\cup\{G / G\subset S$ and G is $\tau_{1,2}$-open $\}$.
(ii) The $\tau_1\tau_2$–closure of S denoted by $\tau_1\tau_2$-cl(S) is defined by $\cap\{F / S\subset F$ and F is $\tau_{1,2}$ -closed$\}$.

Definition 2.3 . A subset A of a bitopological space (X,τ_1,τ_2) is called
1. (1,2)* - semi-open[20] if $A\subset\tau_1\tau_2$-cl($\tau_1\tau_2$-int(A)).
2. (1,2)*- preopen [20] if $A\subset\tau_1\tau_2$-int ($\tau_1\tau_2$-cl(A)).
3. (1,2)*-α-open [20] if $A\subset\tau_1\tau_2$-int ($\tau_1\tau_2$-cl ($\tau_1\tau_2$-int(A))).
4. (1,2)*-generalised closed (briefly (1,2)*-g-closed) [20] if $\tau_1\tau_2$-cl (A)$\subset$ U whenever A $\subset$ U and U is $\tau_{1,2}$-open in X.
5. (1,2)*-regular open[20] if A= $\tau_1\tau_2$-int ($\tau_1\tau_2$-cl(A)).
6. (1,2)* - semi-generalised-closed (briefly (1,2)*-sg-closed) [20] if (1,2)*-scl(A) $\subset$ U whenever A $\subset$ U and U is (1,2)*-semi-open in X.
7. (1,2)*-generalized semi-closed (briefly (1,2)*-gs-closed)[20] if (1,2)*-scl(A)$\subset$ U, whenever $A\subset$ U and U is $\tau_{1,2}$-open in X .

8. (1,2)*-α-generalized-closed (briefly (1,2)*-αg-closed) [20] if (1,2)*-αcl(A)⊂ U, whenever A⊂ U and U is $\tau_{1,2}$ -open in X.
9. (1,2)*-generalized α-closed (briefly (1,2)*-gα-closed)[20] if (1,2)*- αcl(A)⊂ U, whenever A ⊂ U and U is (1,2)*-α -open in X.
10. a (1,2)*-θ-generalized closed (briefly, (1,2)*-θg-closed) set [11] if (1,2)*-cl_θ(A)⊂ U whenever A⊂ U and U is $\tau_{1,2}$ -open in (X,τ_1,τ_2).
11. (1,2)*-π generalized closed(briefly (1,2)*-πg-closed [8] if (1,2)*-cl(A)⊂ U, whenever A⊂ U and U is $\tau_{1,2}$ -π-open.
12. (1,2)*-π generalized α-closed (briefly (1,2)*-πgα-closed)[3] if (1,2)*-αcl(A)⊂ U, whenever A⊂ U and U is $\tau_{1,2}$ -π-open.
13. (1,2)*-π generalized semi-closed (briefly (1,2)*-πgs-closed)[4] if (1,2)*-scl(A)⊂ U, whenever A⊂ U and U is $\tau_{1,2}$ -π-open.
14. (1,2)*-π generalized b-closed (briefly (1,2)*-πgb-closed)[22] if (1,2)*-bcl(A)⊂ U, whenever A⊂ U and U is $\tau_{1,2}$ -π-open.
15. (1,2)*-π generalized pre-closed(briefly (1,2)*-πgp-closed)[19] if (1,2)*-pcl(A)⊂ U, whenever A⊂ U and U is $\tau_{1,2}$ -π-open.

The complement of a (1,2)*-semi-closed (resp. (1,2)*-α-closed, (1,2)*-g-closed, (1,2)*-sg-closed, (1,2)*-gs-closed, (1,2)*-αg-closed (1,2)*-gα-closed, (1,2)*-θg-closed , (1,2)* πg-closed, (1,2)*- πgα-closed, (1,2)*-πgs-closed, (1,2)*-πgb-closed, (1,2)*-πgp-closed) set is called (1,2)*-semi open(resp. (1,2)*-α-open,(1,2)*-g-open,(1,2)*-sg-open,(1,2)*-gs-open,(1,2)*-αg-open, (1,2)*-gα-open, (1,2)*-θg-closed, (1,2)*-πg-open,(1,2)*-πgα-open,(1,2)*-πgs-open,(1,2)*-πgb-open, (1,2)*-πgp-open).

Definition 2.4 The finite union of (1,2)*-regular open sets[5] is said to be $\tau_{1,2}$-π-open. The complement of $\tau_{1,2}$-π-open is said to be $\tau_{1,2}$-π-closed.

Definition2. 5: A function f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is called
(i) $\tau_{1,2}$-π-open map[3] if f(F) is $\tau_1\tau_2$-π-open map in Y for every $\tau_{1,2}$-openset F in X.
(ii) (1,2)*-θ-continuous[7] if f^{-1}(V) is (1,2)*-θ-closed in (X,τ_1,τ_2) for every (1,2)*-closed set V in (Y,σ_1,σ_2).
(iii) (1,2)*-θ-irresolute[7] if f^{-1}(V) is (1,2)*-θ-closed in (X,τ_1,τ_2) for every (1,2)*-θ-closed set V in (Y,σ_1,σ_2).

III. (1,2)*- πgθ-closed set

We introduce the following definition.

Definition 3.1. A subset A of (X,τ_1,τ_2) is called(1,2)*- π generalized θ-closed set (briefly (1,2)*-πgθ-closed) if $\tau_1\tau_2$-cl_θ(A)⊂ U whenever A⊂ U and U is $\tau_1\tau_2$ -π-open.
The complement of (1,2)*-πgθ-closed is (1,2)*-πgθ-open..

Theorem 3.2:
1. Every (1,2)*-θ- closed set is (1,2)*-πgθ-closed.
2. Every (1,2)*-θg-closed set is (1,2)*-πgθ-closed.
3. Every (1,2)*-πgθ-closed set is (1,2)*-πg-closed.
4. Every (1,2)*-πgθ-closed set is (1,2)*-πgα -closed.
5. Every (1,2)*-πgθ-closed set is (1,2)*-πgs -closed.
6. Every (1,2)*-πgθ-closed set is (1,2)*-πgb-closed.
7. Every (1,2)*-πgθ-closed set is (1,2)*-πgp-closed

Proof: Straight forward.

Converse of the above need not be true as seen in the following examples.

Example 3.3 Let X ={a,b,c}.τ_1={ ϕ, X ,{a},{c},{a,c} }; τ_2={ϕ, X,{ b,c }}:
Let A ={b}. Then A is πgθ-closed but not θ-closed.

Example3.4 Let X={a,b,c,d}.τ_1={ϕ,{b},{d},{b,d},{a,b,d},X}; τ_2={ϕ,{b,d},{b,c,d}, X}. Let A = {a,d}.
Then A is (1,2)*-πgθ-closed but not (1,2)*-θg-closed.

Example 3.5 Let $X = \{a,b,c,d\}$, $\tau_1 = \{\phi,\{a\},\{b\},\{a,b\},\{a,b,d\},X\}$, $\tau_{2=}\{\phi,\{a,c\},\{a,b,c\},X\}$. Let A = {c}. Then A is (1,2)*-πg-closed but not (1,2)*-πgθ-closed.

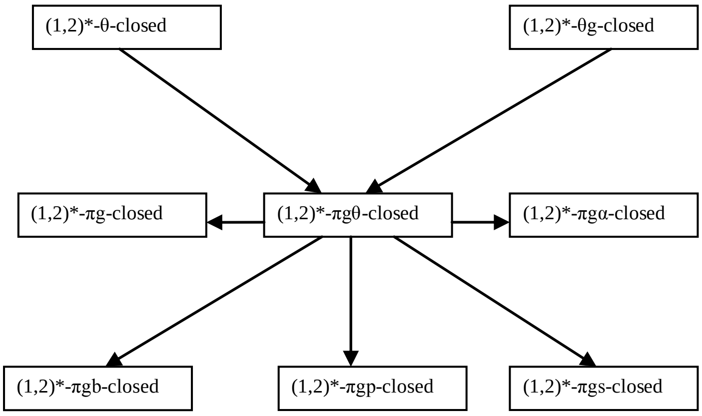
Example 3.6 Let $X = \{a,b,c,d\}$. $\tau_{1=}\{\phi,\{a\},\{d\},\{a,d\},X\}$; $\tau_2 = \{\phi,\{c,d\},\{a,c,d\},X\}$:
Let A ={c}. Then A is (1,2)*-πgα-closed set but not (1,2)*-πgθ-closed.

Example3.7 Let $X = \{a,b,c,d\}$. $\tau_1 = \{\phi,\{a\},\{b\},\{a,b\},X\}$; $\tau_2 = \{\phi,\{a,b\},\{a,b,d\},X,\}$;
Let A ={a}. Then A is (1,2)*-πgs-closed set but not (1,2)*-πgθ-closed.

Example 3.8 Let $X=\{a,b,c,d\}.\tau_1 = \{\phi,\{a\},\{d\},\{a,d\},X\}$; $\tau_2 = \{\phi,\{c,d\},\{a,c,d\},X\}$;
Let A ={a,c}. Then A is (1,2)*-πgb-closed set but not (1,2)*-πgθ-closed.

Example 3.9 Let $X = \{a,b,c,d\}$. $\tau_1 = \{\phi,\{a\},\{d\},\{a,d\},X\}$; $\tau_2 = \{\phi,\{c,d\},\{a,c,d\},X\}$:
Let A = {c}. Then A is (1,2)*-πgp-closed but not (1,2)*-πgθ-closed.

Remark 3.10 The above discussions are summarized in the following diagram.



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Remark 3.11 (1,2)*-πgθ-closed is independent of (1,2)*-closedness, (1,2)*-α-closedness, (1,2)*-semi-closedness, (1,2)*-sg-closedness, (1,2)*-gs-closedness, (1,2)*-g-closedness, (1,2)*-αg-closedness and(1,2)*-gα-closedness, as seen in the following examples.

Example 3.12 Let $X = \{a,b,c,d\}$, $\tau_1 = \{\phi,\{a\},\{b\},\{a,b\},X\}$; $\tau_2 = \{\phi,\{a,b,d\},X\}$:
Let A = {d}. Then A is (1,2)*-πgθ-closed but not (1,2)*-g-closed.

Example 3.13 Let $X = \{a,b,c,d,e\}$, $\tau_1 = \{\phi,\{a,b\},\{a,b,c,d\},X\}$; $\tau_{2=}\{\phi,\{c,d\},X\}$:
Let A = {e}. Then A is (1,2)*-g-closed but not (1,2)*-πgθ-closed.

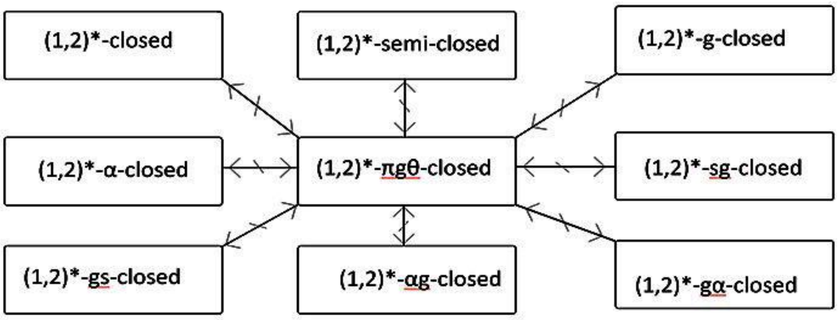
Example3.14. Let $X = \{a,b,c,\}$, $\tau_1 = \{\phi,\{a\},\{b\},\{a,b\},X\}$; $\tau_2 = \{\phi,\{b,c\},X\}$: Let A ={b}. Then A is (1,2)*-πgθ-closed but not (1,2)*-closed, (1,2)*-α-closed, (1,2)*-semi-closed.

Example 3.15 Let $X = \{a,b,c\}$, $\tau_1 = \{\phi,\{a\},\{b\},\{a,b\},X\}$; $\tau_2 = \{\phi,\{b,c\},X\}$: Let A ={a}. Then A is (1,2)*-closed, (1,2)*-α-closed, (1,2)*-semi-closed but not (1,2)*-πgθ-closed.

Example 3.16 Let $X = \{a,b,c,d\}$, $\tau_1 = \{\phi,\{a\},\{b\},\{a,b\},\{a,b,c\},X\}$; $\tau_2 = \{\phi,\{a,b,d\},X\}$:
(i) Let A={a,b,c}. Then A is (1,2)*-πgθ-closed but neither (1,2)*-sg-closed nor (1,2)*-gs-closed.
(ii) Let A = {a}. Then A is both (1,2)*-sg-closed and (1,2)*-gs-closed but not (1,2)*-πgθ-closed.

Example 3.17 Let $X = \{a,b,c,d\}$, $\tau_1=\{\phi,\{c,d\},\{a,c,d\},X)$, $\tau_{2=}\{\phi,\{a\},\{d\},\{a,d\},\{c,d\},X\}$.
(i) Let A={b,d}. Then A is (1,2)*-πgθ-closed but neither (1,2)*-αg-closed nor (1,2)*-gα- closed.
(ii)Let A ={c}.Then A is (1,2)*-αg-closed,(1,2)*-gα-closed but not (1,2)*-πgθ-closed.

Remark 3.18 The above discussions are summarized in the following diagram.



Remark 3.19 A finite union of (1,2)*-$\pi g\theta$ -closed sets is always a (1,2)*-$\pi g\theta$-closed.

Proof: Let A, B ϵ (1,2)*-$\pi G\theta C(X)$. Let U be any $\tau_1\tau_2$-π-open set such that $(A\cup B) \subseteq U$. Since (1,2)*-$cl_\theta(A\cup B)$ = (1,2)*-$cl_\theta(A) \cup$ (1,2)*-$cl_\theta(B) \subseteq U\cup U=U$. This implies (1,2)*-$cl_\theta(A\cup B) \subseteq U$. Hence $A\cup B$ is also a (1,2)*-$\pi g\theta$-closed set.

Remark 3.20 The intersection of two (1,2)*-$\pi g\theta$-closed sets need not be (1,2)*-$\pi g\theta$-closed as seen in the following example.

Example3.21 Let X = {a,b,c,d}, τ_1 = {ϕ,{a},{b},{a,b},X }; τ_2 = {ϕ,{a,b,d},X}:
Let A={a,b,c} and B ={ a,b,d}.Clearly A and B are (1,2}*-$\pi g\theta$-closed sets. But A∩B = { a,b} is not a (1,2)*-$\pi g\theta$-closed set.

Proposition 3.22 Let A be (1,2)*-$\pi g\theta$-closed in (X,τ). Then (1,2)*-$cl_\theta(A)$–A does not contain any non-empty $\tau_1\tau_2$-π-closed set.

Proof: Let F be a non-empty $\tau_1\tau_2$-π-closed set such that $F\subset$ (1,2)*-$cl_\theta(A)$–A. Since A is (1,2)*-$\pi g\theta$-closed, $A\subset$ X–F where X–F is $\tau_1\tau_2$-π-open implies (1,2)*-$cl_\theta(A)\subset$ (X–F). Hence $F\subset$ X−(1,2)*-$cl_\theta(A)$. Now, $F\subset$ (1,2)*-$cl_\theta(A)\cap$X–(1,2)*-$cl_\theta(A)$ implies F= ϕ which is a contradiction. Therefore $cl_\theta(A)$–A does not contain any non-empty $\tau_1\tau_2$-π-closed set.

Remark 3.23 The converse of Proposition 3.22 need not be true as shown in the following example.

Example 3.24 Let X = {a,b,c}. τ_1= {ϕ, X,{b} }; τ_2 = { ϕ, X,{c}}:
Let A = {b,c}. Then (1,2)*-$cl_\theta(A)$ −A contains no non-empty $\tau_1\tau_2$-π-closed set. However A is not (1,2)*-$\pi g\theta$-closed.

Proposition 3.25 If A is a $\tau_{1,2}$-regular open and (1,2)*-$\pi g\theta$-closed subset of (X,τ_1,τ_2), then A is a (1,2)*-θ-closed subset of (X,τ_1,τ_2).

Proof. Since A is $\tau_{1,2}$-regular open and (1,2)*-$\pi g\theta$-closed, (1,2)*-$cl_\theta(A)\subset$ A . Hence A is (1,2)*-θ-closed.

Proposition3.26 Let A be a (1,2)*-$\pi g\theta$-closed subset of (X,τ_1, τ_2). If $A \subset B \subset$ (1,2)*- $cl_\theta(A)$, then B is also a (1,2)*-$\pi g\theta$-closed subset of (X,τ_1,τ_2).

Proof. Let U be a $\tau_1\tau_2$-π-open set of (X,τ_1, τ_2) such that $B\subset U$. Then $A\subset U$. Since A is a (1,2)*-$\pi g\theta$-closed set, (1,2)*-$cl_\theta(A)\subset U$. Also since $B\subset$ (1,2)*-$cl_\theta(A)$, (1,2)*-$cl_\theta(B)\subset$ (1,2)* - cl_θ((1,2)*-$cl_\theta(A)$)=(1,2)*-$cl_\theta(A)$.Thus (1,2)*-$cl_\theta(B)\subset U$. Hence B is also a (1,2)*-$\pi g\theta$-closed subset of (X,τ_1,τ_2).

Theorem 3.27 Let A be a (1,2)*-$\pi g\theta$-closed sub set in X. Then A is (1,2)*-θ-closed if and only if (1,2)*-$cl_\theta(A)$–A is $\tau_{1,2}$ -π-closed.

Proof. Necessity: Let A be (1,2)*-θ-closed subset of X. Then (1,2)*-$cl_\theta(A)$=A and (1,2)*-$cl_\theta(A)$–A=ϕ which is $\tau_{1,2}$ -π-closed.
Sufficiency: Since A is (1,2)*-$\pi g\theta$-closed, by proposition 3.22 (1,2)*-cl_θ(A–A contains no non-empty $\tau_{1,2}$- π-closed set. But (1,2)*-$cl_\theta(A)$ – A is $\tau_{1,2}$-π-closed. This implies (1,2)*-$cl_\theta(A)$ – A=ϕ, which means (1,2)*-$cl_\theta(A)$=A and hence A is (1,2)*-θ-closed.

IV. (1,2)*-πgθ -open sets

Definition 4.1 A subset A of (X, τ_1, τ_2) is called (1,2)*-πgθ-open if and only if A^c is (1,2)*-πgθ-closed in (X,τ_1,τ_2).

Remark 4.2 For a subset A of (X, τ_1, τ_2), $(1,2)^*\text{-cl}_\theta(A^c) = [(1,2)^*\text{- int}_\theta(A)]^c$

Theorem 4.3 A subset A of (X,τ_1,τ_2) is (1,2)*-πgθ-open if and only if $F\subset (1,2)^*\text{-int}_\theta(A)$ whenever F is $\tau_1\tau_2$-π-closed and $F\subset A$.

Proof. Necessity: Let A be a (1,2)*-πgθ-open set in (X,τ_1,τ_2). Let F be $\tau_1\tau_2$-π-closed and $F\subset A$. Then $F^c\supseteq A^c$ and F^c is $\tau_1\tau_2$-π-open. Since A^c is (1,2)*-πgθ-closed, $(1,2)^*\text{-cl}_\theta(A^c)\subseteq F^c$. By remark 4.2, $[(1,2)^*\text{-int}_\theta(A)]^c\subseteq F^c$. That is $F\subset (1,2)^*\text{-int}_\theta(A)$.

Sufficiency: Let $A^c\subseteq U$ where U is $\tau_1\tau_2$-π-open. Then $U^c\subset A$ where U^c is (1,2)*-$\tau_1\tau_2$-π-closed. By hypothesis $U^c \subseteq (1,2)^*\text{-int}_\theta(A)$. That is $[(1,2)^*\text{-int}_\theta(A\}]^c\subseteq U$. By remark 4.2, $(1,2)^*\text{-cl}_\theta(A^c)\subseteq U$. This implies A^c is (1,2)*-πgθ-closed. Hence A is (1,2)*-πgθ-open.

Theorem 4.4 If $(1,2)^*\text{-int}_\theta(A)\subseteq B\subseteq A$ and A is (1,2)*-πgθ-open, then B is also (1,2)*-πgθ-open.

Proof. $(1,2)^*\text{-int}_\theta(A)\subseteq B\subseteq A$ implies $A^c\subseteq B^c\subseteq [(1,2)^*\text{-int}_\theta(A)]^c$. By remark 4.2 $A^c\subseteq B^c\subseteq (1,2)^*\text{-cl}_\theta(A^c)$. Also A^c is (1,2)*-πgθ-closed. By Proposition 3.26 B^c is (1,2)*-πgθ-closed. Hence B is (1,2)*-πgθ-open.

As an application of (1,2)*-πgθ-closed sets we introduce the following definition.

Definition 4.5 A space (X,τ_1, τ_2) is called a (1,2)*-πgθ-$T_{½}$ space if every (1,2)*-πgθ-closed set is (1,2)*-θ-closed.

Theorem 4.6 For a space (X,τ_1,τ_2) the following conditions are equivalent.
(i) (X,τ_1,τ_2) is a (1,2)*-πgθ-$T_{½}$ space.
(ii) Every singleton set of (X,τ_1,τ_2) is either $\tau_1\tau_2$-π-closed or (1,2)*-θ-open.

Proof. (i) ⇒ (ii). Let $x \in X$. Suppose {x} is not a $\tau_1\tau_2$-π- closed set of (X, τ_1, τ_2). Then X–{x} is not a $\tau_1\tau_2$-π-open set. So X is the only $\tau_1\tau_2$-π-open set containing X–{x}.So X-x is a (1,2)*-πgθ-closed set of (X,τ_1,τ_2). Since (X,τ_1,τ_2) is a (1,2)*-πgθ-$T_{½}$ space, X– x is a (1,2)*-θ-closed set of (X,τ_1,τ_2) or equivalently {x} is a (1,2)*-θ-open set of (X,τ_1, τ_2).
(ii)⇒ (i). Let A be a (1,2)*-πgθ-closed set of X. Trivially $A\subset (1,2)^*\text{-cl}_\theta(A)$. Let $x\epsilon 1,2)^*\text{-cl}_\theta(A)$. By (ii) {x} is either $\tau_1\tau_2$-π-closed or (1,2)*-θ-open.
(a) Suppose that {x} is $\tau_1\tau_2$-π-closed. If $x\notin A$, then $x\epsilon(1,2)^*\text{-cl}_\theta(A)–A$ contains a non-empty $\tau_1\tau_2$-π-closed set {x}. By proposition 3.6 we arrive at a contradiction. Thus $x\epsilon A$.
(b) Suppose that {x} is a (1,2)*-θ-open. Since $x\epsilon(1,2)^*\text{-cl}_\theta(A)=A$ or equivalently A is (1,2)*-θ-closed. Hence (X,τ_1,τ_2) is a (1,2)*-πgθ-$T_{½}$ space.

V. (1,2)*-πgθ-continuous and (1,2)*-πgθ-irresolute functions

Definition 5.1 A function $f:(X,\tau_1,\tau_2) \to(Y,\sigma_1,\sigma_2)$ is called (1,2)*-πgθ-continuous if every $f^{-1}(V)$ is (1,2)*-πgθ-closed in (X,τ_1, τ_2) for every(1,2)*-$\sigma_1\sigma_2$-closed set V of (Y,σ_1,σ_2).

Definition 5.2 A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called (1,2)*-πgθ-irresolute if $f^{-1}(V)$ is (1,2)*-πgθ-closed in (X,τ_1,τ_2) for every (1,2)*-πgθ-closed set V in (Y,σ_1,σ_2).

Remark 5.3 (1,2)*-πgθ-irresolute function is independent of (1,2)*-θ-irresoluteness, as seen in the following examples.

Example 5.4 Let X=Y={a,b,c,d}, τ_1={ ϕ,{a},{d},{a,d},{a,c,d},X},τ_2= {ϕ,{a,d},{a,b,d}, X}, σ_1 = {ϕ,{a},{d},{a,d},X},σ_2= {ϕ,{c,d},{a,c,d},X}. Let f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be an identity function. Then f is (1,2)*-θ-irresolute but not (1,2)*-πgθ-irresolute, since f^{-1} [{b,c,d}]={b,c,d} is not (1,2)*-πgθ-closed in (X,τ_1,τ_2).

Example 5.5 Let $X=Y=\{a,b,c,d\}$, $\tau_1=\{\phi,\{a\},\{d\},\{a,d\},\{a,c,d\},X\}$, $\tau_2=\{\phi,\{a,d\},\{a,b,d\},X\}$, $\sigma_1=\{\phi,\{a\},\{c\},\{a,c\},\{c,d\},\{a,c,d\},X\}$, $\sigma_2=\{\phi,\{d\},\{a,b,d\},X\}$, Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be an identity function. Then f is (1,2)*-πgθ-irresolute but not (1,2)*-θ-irresolute, since $f^{-1}[\{a,b,d\}]=\{a,b,d\}$ is not (1,2)*-θ-closed in (X,τ_1,τ_2).

Remark 5.6: Every (1,2)*-θ-continuous is (1,2)*-πgθ-continuous.
The converse of the above need not be true as seen in the following examples.

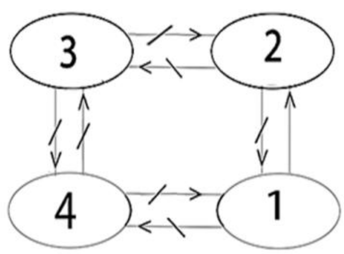
Example 5.7: Let $X=Y=\{a,b,c,d,e\}$, $\tau_1=\{\phi,\{a,b\},\{a,b,c\},X\}$, $\tau_2=\{\phi,\{c\},X\}$, $\sigma_1=\{\phi,\{a,b\},\{a,b,c,d\},X\}$, $\sigma_2=\{\phi,\{c,d\},\{a,b,c,d\},X\}$. Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be an identity function. Then f is (1,2)*-πgθ-continuous but not (1,2)*-θ-continuous, since $f^{-1}[\{c,d,e\}]=\{c,d,e\}$ is not (1,2)*-θ-closed in (X,τ_1,τ_2).

Remark 5.8 (1,2)*-πgθ-continuous is independent of (1,2)*- πgθ-irresolute as seen in the following examples.

Example 5.9 Let $X=Y=\{a,b,c,d,e\}$, $\tau_1=\{\phi,\{a,b\},\{a,b,c,d\},X\}$, $\tau_2=\{\phi,\{c,d\},X\}$, $\sigma_1=\{\phi,\{b\},\{b,c\},X\}$, $\sigma_2=\{\phi,\{c\},\{a,b\},\{a,b,c\},X\}$. Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be an identity function. Then f is (1,2)*-πgθ-continuous but not (1,2)*-πgθ-irresolute, since $f^{-1}[\{d\}]=\{d\}$ is not (1,2)*-πgθ-closed in (X,τ_1,τ_2) where {d}is πgθ-closed in (Y,σ_1,σ_2).

Example 5.10: Let $X=Y=\{a,b,c,d\}$, $\tau_1=\{\phi,\{a\},\{b\},\{a,b\},\{a,b,c\},X\}$, $\tau_2=\{\phi,\{b\},\{c\},\{b,c\},\{a,c\},\{a,b,c\},\{a,b,d\},X\}$, $\sigma_1=\{\phi,\{a\},\{b\},\{a,b\},\{a,d\},\{a,b,d\},X\}$, $\sigma_2=\{\phi,\{c\},\{a,c\},\{b,c\},\{a,b,c\},\{a,c,d\},X\}$. Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be an identity function. Then f is (1,2)*-πgθ-irresolute but not (1,2)*-πgθ-continuous, since $f^{-1}[\{b\}]=\{b\}$ is not (1,2)*-πgθ-closed in (X,τ_1,τ_2) where {b}is closed in (Y,σ_1,σ_2).

Remark 5.11 The above discussions are summarized in the following diagram.



Where **(1) ⇒ (1,2)*-θ-continuous; (2) ⇒ (1,2)*-πgθ-continuous;**
(3) ⇒ (1,2)*-πgθ-irresolute; (4) ⇒ (1,2)*-θ-irresolute.

Remark 5.12 Composition of two (1,2)*-πgθ-continuous function need not be (1,2)*-πgθ-continuous.

Example 5.13 Let $X=Y=Z=\{a,b,c,d,e\}$, $\tau_1=\{\phi,\{a,b\},\{a,b,c\},X\}$, $\tau_2=\{\phi,\{c\},X\}$, $\sigma_1=\{\phi,\{a,b\},X\}$, $\sigma_2=\{\phi,\{c,d\},\{a,b,c,d\},X\}$, $\eta_1=\{\phi,\{a,b,c,d\},X\}$, $\eta_2=\{\phi,\{e\},X\}$. Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ be the identity functions. Both f and g are (1,2)*-πgθ-continuous but gof is not (1,2)*-πgθ-continuous, since $(gof)^{-1}[\{a,b,c,d\}]=\{a,b,c,d\}$ is not (1,2)*-πgθ-closed in (X,τ_1,τ_2).

Theorem 5.14 Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a function .
(i) If f is (1,2)*-πgθ-irresolute and X is(1,2)*-πgθ-$T_{½}$ space, then f is(1,2)*-θ-irresolute.
(ii) If f is (1,2)*-πgθ-continuous and X is (1,2)*-πgθ-$T_{½}$ space then f is (1,2)*-θ-continuous.

Proof: (i) Let V be (1,2)*-θ-closed in Y. Since f is (1,2)*-πgθ-irresolute, $f^{-1}(V)$ is (1,2)*- πgθ-closed in X. Since X is (1,2)*-πgθ-$T_{½}$ space, $f^{-1}(V)$ is (1,2)*-θ-closed in X. Hence f is (1,2)*-θ-irresolute.

(ii)Let V be closed in Y. Since f is (1,2)*-πgθ-continuous, $f^{-1}(V)$ is (1,2)*-πgθ-closed in X. By assumption, it is (1,2)*-θ-closed. Therefore f is (1,2)*-θ-continuous.

Theorem 5.15 If the bijective $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is (1,2)*-θ-irresolute and $\tau_1\tau_2$-π-open map, then f is (1,2)*-πgθ-irresolute.

Proof: Let V be (1,2)*-πgθ-closed in Y. Let $f^{-1}(V) \subset U$ where U is $\tau_1\tau_2$-π-open in X. Then $V \subset f(U)$ and f(U) is $\tau_1\tau_2$-π-open implies (1,2)*-$cl_\theta(V) \subset f(U)$. This implies f^{-1}((1,2)*-$cl_\theta(V)) \subset U$. Since f is (1,2)*-θ-irresolute, f^{-1}((1,2)*-cl_θ(V)) is (1,2)*-θ-closed. Hence (1,2)*- $cl_\theta(f^{-1}(V)) \subset$ (1,2)*- $cl_\theta(f^{-1}$((1,2)*-$cl_\theta(V)))=f^{-1}$((1,2)*-$cl_\theta(V)) \subset U$. Therefore f is (1,2)*-πgθ-irresolute.

Theorem 5.16 :If f: $(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is (1,2)*-πgθ-irresolute map and g: $(Y,\sigma_1,\sigma_2)\rightarrow(Z,\eta_1,\eta_2)$ is (1,2)*-πgθ-continuous map, the composition gof: $(X,\tau_1,\tau_2)\rightarrow(Z,\eta_1,\eta_2)$ is a (1,2)*-πgθ-continuous map.

Proof: Let V be $\eta_1\eta_2$–closed set in Z. Since g:$(Y,\sigma_1,\sigma_2)\rightarrow(Z,\eta_1,\eta_2)$ is a (1,2)*-πgθ-continuous map,$g^{-1}(V)$ is (1,2)*-πgθ-closed in Y. By hypothesis ,$f^{-1}(g^{-1}(V))=(gof)^{-1}(V)$ is (1,2)*-πgθ-closed in X. Hence gof: $(X,\tau_1,\tau_2)\rightarrow(Z,\eta_1,\eta_2)$ is a (1,2)*-πgθ-continuous map.

V. CONCLUSION

Through the above findings, this paper has attempted to compare (1,2)*-πgθ-closed with the other closed sets in bitopological spaces. An attempt of this paper is to state that the several definitions and results that shown in this paper, will result in obtaining several characterizations and enable to study various properties as well. It brings to limelight that the weaker form of continuity in bitopological settings is the future scope of study.

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