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The effect of Hall current on an unsteady MHD free convective Couette flow between two permeable plates in the presence of thermal radiation

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ABSTRACT:
An investigation on the non – linear problem of the effect of Hall current on the unsteady magneto hydrodynamic free convective Couette flow of incompressible, electrically conducting fluid between two permeable plates is carried out, when a uniform magnetic field is applied transverse to the plate, while the thermal radiation, viscous and Joule's dissipations are taken into account. The fluid is considered to be a gray, absorbing – emitting but non – scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The dimensionless governing coupled, non – linear boundary layer partial differential equations are solved by an efficient, accurate, and extensively validated and unconditionally stable finite difference scheme of the Crank – Nicolson method. The effects of thermal radiation and Hall current on primary and secondary velocity, skin friction and rate of heat transfer are analyzed in detail for heating and cooling of the plate by convection currents. Physical interpretations and justifications are rendered for various results obtained.

KEYWORDS: *Hall current, free convection, MHD, Couette flow, Thermal radiation, Finite difference method.*

I. INTRODUCTION:

a) *Magnetohydrodynamics (MHD):*

The influence of magnetic field on electrically conducting viscous incompressible fluid is ofimportance in many applications such as extrusion of plastics in the manufacture of rayon andnylon, the purification of crude oil, and the textile industry, etc. In many process industriesthe cooling of threads or sheets of some polymer materials is important in the productionline. The rate of the cooling can be controlled effectively to achieve final products of desiredcharacteristics by drawing threads, etc., in the presence of an electrically conducting fluidsubjected to magnetic field.The study of magnetohydrodynamic (MHD) plays an important role in agriculture, engineeringand petroleum industries. The MHD has also its own practical applications. Forinstance, it may be used to deal with problems such as the cooling of nuclear reactors by liquidsodium and induction flow meter, which depends on the potential difference in the fluid in thedirection perpendicular to the motion and to the magnetic field.

b) *Free Convection:*

The problem of free convection under the influence of themagnetic field has attracted the interest of many researchers in view of its applications in geophysicsand astrophysics. The problem under consideration has important applications in thestudy of geophysical formulations, in the explorations and thermal recovery of oil, and in theunderground nuclear waste storage sites. The unsteady natural convection flow past a semi – infinitevertical plate was first solved by Hellums and Churchill[1], using an explicit finite differencemethod. Because the explicit finite difference scheme has its own deficiencies, a more efficientimplicit finite difference scheme has been used by Soundalgekar and Ganesan[2]. A numericalsolution of transient free convection flow with mass transfer on a vertical plate by employingan implicit method was obtained by Soundalgekar and Ganesan[3]. Takhar*et al.*[4] studied thetransient free convection past a semi – infinite vertical plate with variable surface temperatureusing an implicit finite difference scheme of Crank Nicolson type.Soundalgekar*et al.*[5] analyzed the problem of free convection effects on Stokes problem fora vertical plate under the action of transversely applied magnetic field. Sacheti*et al.*[6] obtainedan exact solution for the unsteady MHD free convection flow on an impulsively started verticalplate with constant heat flux. Shanker and Kishan[7] discussed the effect of mass transfer onthe MHD flow past an impulsively started vertical plate with variable temperature or constantheat flux. Elbashbeshy[8] studied the heat and mass transfer along a vertical plate under thecombined buoyancy effects of thermal and species diffusion, in the presence of the magneticfield. Ganesan and Palani[9] obtained a numerical solution of the unsteady MHD flow past asemi – infinite isothermal vertical plate using the finite difference method.

c) *Thermal radiation:*

Radiation effects on free convection flow have becomevery important due to its applications in space technology,processes having high temperature, and design of pertinent equipment. Moreover, heat transfer withthermal radiation on convective flows is very importantdue its significant role in the surface heat transfer. Recentdevelopments in gas cooled nuclear reactors, nuclear powerplants, gas turbines, space vehicles, and hypersonic flightshave attracted research in this field.The unsteady convectiveflow in a moving plate with thermal radiation were examinedby Cogley*et al.* [10] andMansour [11].The combined effectsof radiation and buoyancy force past a vertical plate wereanalyzed by Hossain and Takhar [12]. Hossain*et al.* [13]analyzed the influence of thermal radiation on convectiveflows over a porous vertical plate. Seddeek [14] explainedthe importance of thermal radiation and variable viscosityon unsteady forced convection with an align magnetic field.Muthucumaraswamy and Senthil [15] studied the effects ofthermal radiation on heat and mass transfer over a movingvertical plate. Pal [16] investigated convective heat andmasstransfer in stagnation – point flow towards a stretching sheetwith thermal radiation. Aydin and Kaya [17] justified theeffects of thermal radiation on mixed convection flow overa permeable vertical plate with magnetic field. Mohamed[18] studied unsteady MHD flow over a vertical movingporous plate with heat generation and Soret effect. Chauhanand Rastogi [19] analyzed the effects of thermal radiation,porosity, and suction on unsteady convective hydromagneticvertical rotating channel. Ibrahim and Makinde [20] investigatedradiation effect on chemically reactionMHDboundarylayer flow of heat and mass transfer past a porous verticalflat plate. Pal and Mondal [21] studied the effects of thermalradiation on MHD DarcyForchheimer convective flow pasta stretching sheet in a porous medium. Palani and Kim[22] analyzed the effect of thermal radiation on convectionflow past a vertical cone with surface heat flux. Recently,Mahmoud and Waheed [23] examined thermal radiation onflow over an infinite flat plate with slip velocity.

d) *Viscous Dissipation:*

The viscous dissipation effects are important in geophysical flows andalso in certain industrial operations and are usually characterized by the Eckert number. Inthe literature, extensive research work is available to examine the effect of natural convectionon flow past a plate.Gebhart [24] has shown the importance of viscous dissipativeheat in free convection flow in the case of isothermal and constant heat flux at the plate. Gebhartand Mollendorf [25] have considered the effects of viscous dissipation for external natural convection flow over a surface. Soundalgekar [26] has analyzed viscous dissipative heat on thetwo – dimensional unsteady free convective flow past an infinite vertical porous plate when thetemperature oscillates in time and there is constant suction at the plate. Maharajan and Gebhart[27] have reported the influence of viscous dissipation effects in natural convective flows,showing that the heat transfer rates are reduced by an increase in the dissipation parameter.Israel Cookey*et al.* [28] have investigated the influence of viscous dissipation and radiation on an unsteady MHD free convection flow past an infinite heated vertical plate in a porous mediumwith time dependent suction. Suneetha*et al.* [29] have analyzed the effects of viscous dissipation and thermal radiationon hydromagnetic free convective flow past an impulsively started vertical plate. Suneetha*et al.* [30] have studied the effects of thermal radiation on the natural convective heat and masstransfer of aviscous incompressible gray absorbingemitting fluid flowing past an impulsivelystarted moving vertical plate with viscous dissipation. Ahmed and Batin [31] have obtainedan analytical model of MHD mixed convective radiating fluid with viscous dissipative heat.Babu*et al.* [32] have studied the radiation and chemical reaction effects on an unsteady MHDconvective flow past a vertical moving porous plate embedded in a porous medium with viscousdissipation. Kishore *et al.* [33] have analyzed the effects of thermal radiation and viscousdissipation on MHD heat and mass diffusion flow past an oscillating vertical plate embeddedin a porous medium with variable surface conditions.

e) *Joule's Dissipation:*

Apart from the viscous dissipation, in the MHD flows, Joule's dissipation also acts a volumetric heat source [34], [35] and this represents the electromagnetic energy dissipated on account of heating of the medium by the electric current. It depends on the strength of the applied magnetic field. Indeed, in MHD flows there is not only energy transfer between the electromagnetic field and the fluid flow, but also a portion of the kinetic energy is converted to thermal energy by means of Joule heating. When a stronger magnetic field is applied the flow is retarded severely and also there is a considerable heating of the fluid due to Joule effect. Joule heating causes, in general, an increase of temperature and its gradient, mainly in the temperature boundary layer [36]. Hence, one must consider this effect while modeling problems related to atmospheric flights.

f) *Hall Effect:*

When the strength of the appliedmagnetic field is sufficiently large, Ohm's law needs to be modified to includeHall current. The Hall Effect is due merely to the sideways magnetic force on thedrafting free charges .The electric field has to have a component transverse to thedirection of the current density to balance this force. In many works of plasmaphysics, it is not paid much attention to the effect caused due to Hall current.However, the Hall Effect cannot be completely ignored if the strength of themagnetic field is high and number of density of electrons is small as it isresponsible for the change of the flow pattern of an ionized gas. Hall Effect resultsin a development of an additional potential difference between opposite surfacesof a conductor for which a current is induced perpendicular to both the electricand magnetic field. This current is termed as Hall current. It was discovered in1979 by Edwin Herbert Hall while working on his doctoral degree at the JohnsHopkins University in Baltimore, Maryland, USA.

Katagiri [37] has studied the effect of Hall currents on the magnetohydrodynamic boundary layer flow past a semi – infinite flat plate. Hall effects on hydromagnetic free convection flow along a porous flat plate with mass transfer have been analyzed by Hossain and Rashid [38]. Hossain and Mohammad [39] have discussed the effect of Hall currents on hydromagnetic free convection flow near an accelerated porous plate. Pop and Watanabe [40] have studied the Hall effects on the magnetohydrodynamic free convection about a semi – infinite vertical flat plate. Halleffects on magnetohydrodynamic boundary layer flow over a continuous moving flat plate have been investigatedby Pop and Watanabe [41]. Sharma *et al.* [42] have analyzed the Hall effects on an MHD mixed convective flow of a viscous incompressible fluid past a vertical porous plate immersed in a porous medium with heat source/sink. Effects of Hall current and heat transfer on the flow in a porous medium with slip condition have been described by Hayat and Abbas [43]. Guria*et al.* [44] have investigated the combined effects of Hall current and slip condition on unsteady flow of a viscous fluid due to non – coaxial rotation of a porous diskand a fluid at infinity. Shit [45] has studied the Hall effects on MHD free convective flow and mass transfer over a stretching sheet. Ghara*et al.* [46] have discussed Hall effects on oscillating flow due to eccentrically rotating porous disk and a fluid at infinity. Hall effects on an MHD Couette flow between two infinite horizontal parallel porous plates in a rotating system under the boundary layer approximations have been studied by Das *et al.* [47].

g) *Couette flow:*

The problem of MHD Couette flow and heat transfer between parallel plates is a classical one that has several applications in MHD accelerators, MHD pumps and power generators, and in many other industrial engineering designs. Thus such problems have been much investigated by researchers such as, Seth *et al.* [48], Singh *et al.* [49], Chauhan and Vyas [50], Attia and Ewis [51], Seth *et al.* [52], and Attia*et al.* [53].Hall currents in MHDCouette flow and heat transfereffects have been investigated in parallel plate channelswith or without ion – slip effects by Soundalgekar*et al.* [54], Soundalgekar and Uplekar [55], and Attia [56].Hall effects on MHD couette flow between arbitrarilyconducting parallel plates have been investigated in arotating system by Mandal and Mandal [57]. The sameproblem of MHD couette flow rotating flow in a rotatingsystem with Hall current was examined by Ghosh [58] inthe presence of an arbitrary magnetic field.The study of hydromagnetic couette flow in a porouschannel has become important in the applications offluid engineering and geophysics. Krishna *et al.* [59] investigatedconvection flow in a rotating porous mediumchannel. Beg *et al.* [60] investigated unsteady magnetohydrodynamiccouette flow in a porous medium channelwith Hall current and heat transfer. When the viscousfluid flows adjacent to porous medium, Ochoa – Tapia[61, 62] suggested stress jump conditions at the fluidporous interface when porous medium is modelled byBrinkman equation. Using these jump conditions,Kuznetsov [63] analytically investigated the couette flowin a composite channel partially filled with a porousmedium and partially with a clear fluid. Chauhan andRastogi [64], heat transfer effects on MHD conductingflow with Hall current in a rotating channel partiallyfilled with a porous material using jump conditions at thefluid porous interface. Chauhan and Agrawal [65] investigatedHall current effects in a rotating channel partiallyfilled with a porous medium using continuity of velocitycomponents and stresses at the porous interface. Chauhanand Agrawal [66] further studied effects of Hall currenton couette flow in similar geometry and matching conditionsat the fluid porous interface.

Motivated the above research work, we have proposed in the present paper to investigate the effect of Hall current on the unsteady magnetohydrodynamic free convective Couette flow of incompressible, electrically conducting fluid between two permeable plates is carried out, when a uniform magnetic field is applied transverse to the plate, while the thermal radiation, viscous and Joule's dissipations are taken into account. The fluid is considered to be a gray, absorbing – emitting but non – scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The dimensionless governing coupled, non – linear boundary layer partial differential equations are solved by an efficient, accurate, and extensively validated and unconditionally stable finite difference scheme of the Crank – Nicolson method which is more economical from computational view point. The behaviors of the velocity, temperature, skin friction coefficient and Nussult number have been discussed in detail for variations in the important physical parameters.In section 2, the mathematical formulation of the problem and dimension less forms of the governing equations are established. Solution method tothese equations for the flow variables are briefly examined in section 3. Theresults of the previous sections are discussed in section 4. In section 5, generalconcluding remarks of the results of the previous sections are given.

II. MATHEMATICAL FORMULATION:

An unsteady free convection flow of an electrically conducting, viscous, incompressible fluid past an impulsively started infinite vertical porous plate, in the presence of a transverse magnetic field with the effect of Hall current and thermal radiation are considered.

We made the following assumptions:

1. The $x'-$ axis is taken along the infinite vertical porous wall in the upward direction and $y'-$ axis normal to the wall.
2. A constant magnetic field of magnitude B_o is applied in $y'-$ direction. Since the effect of Hall current gives rise to a force in z' direction, which induces a cross flow in that direction, the flow becomes three dimensional.
3. Let u', v' and w' denote the velocity components in x', y' and z' directions respectively. Let v_o be the constant suction velocity.
4. A uniform magnetic field B_o is applied in the positive $y'-$ direction and is assumed undisturbed as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number.
5. It is assumed that the external electric field is zero and the electric field due to the polarization of charges is negligible.
6. The initial temperature of the fluid is the same as that of the fluid, but at time $t' > 0$, the porous plate starts moving impulsively in its own plane with a constant velocity U_o and its temperature instantaneously rises or falls to T'_w which thereafter is maintained as such.
7. The fluid is assumed to have constant properties except that the influence of the density variations with the temperature, following the well – known Boussinesq approximation [68] is considered only in the body force terms.

The governing equations of the problem are as follows:

Continuity Equation:

$$\nabla'.\bar{q}' = 0 \tag{1}$$

Momentum Equation:

$$\frac{\partial \bar{q}'}{\partial t'} + (\bar{q}.\nabla')\bar{q}' = -\frac{1}{\rho} + \bar{g}\beta(T' - T'_\infty) + \frac{1}{\rho}(\bar{J}' \times \bar{B}') \tag{2}$$

Energy Equation:

$$\rho C_p\left[\frac{\partial T'}{\partial t'} + (\bar{q}'.\nabla')T'\right] = \kappa\nabla'^2 T' + \mu\phi' + \frac{\bar{J}'^2}{\sigma} - \frac{\partial q_r}{\partial y'} \tag{3}$$

Generalized Ohm's Law:

$$\bar{J}' = \sigma\left[\bar{E}' + \bar{q}' \times \bar{B}'\right] - \frac{\sigma}{en_e}\left[\bar{J}' \times \bar{B}' - \nabla' p'_e\right] \tag{4}$$

Maxwell's Equations:

$$\nabla'.\bar{E}' = -\frac{\partial \bar{B}'}{\partial t'} \tag{5}$$

$$\nabla'.\bar{B}' = 0 \tag{6}$$

Here, $\bar{q}'$ is the velocity field, T' the temperature field, T'_∞ the temperature of the fluid at infinity, $\bar{B}'$ the magnetic induction vector, $\bar{E}'$ the electric field vector, $\bar{J}'$ the current density vector, p' the pressure of the fluid, p'_e the electron pressure, ρ the density of the fluid, μ the coefficient of viscosity, ν the kinematic coefficient of viscosity, κ the thermal conductivity, e the electron charge, n_e number density of electron, C_p the specific heat capacity at constant pressure, ϕ' the viscous dissipation function and t' the time. The viscous dissipation function, for an incompressible fluid, is given by

$$\phi' = 2\left[\left(\frac{\partial u'}{\partial x'}\right)^2 + \left(\frac{\partial v'}{\partial y'}\right)^2 + \left(\frac{\partial w'}{\partial z'}\right)^2\right] + \left(\frac{\partial v'}{\partial x'} + \frac{\partial u'}{\partial y'}\right)^2 + \left(\frac{\partial w'}{\partial y'} + \frac{\partial v'}{\partial z'}\right)^2 + \left(\frac{\partial u'}{\partial z'} + \frac{\partial w'}{\partial x'}\right)^2 \quad (7)$$

The term $\frac{\overline{J}'^2}{\sigma}$ in the energy equation is the Joule's dissipation.

The magnetic Reynolds number is considered to be small and hence the induced magnetic field is neglected in comparison to the transversely applied magnetic field $\overline{B}' = B_o' \hat{j}$, which is assumed to be uniform [69]. Further, since no external electric field is applied, and the effect of polarization of ionized fluid is negligible, it can be assumed that the electric field is zero. As the plate is infinite, all variables in the problem are functions of y' and t' only. Hence, by the usual considerations of the impulsively started vertical flat plate problem, the basic equations become

Momentum Equation:

$$\frac{\partial u'}{\partial t'} - v_o \frac{\partial u'}{\partial y'} = \nu \frac{\partial}{\partial y'}\left(\frac{\partial u'}{\partial y'}\right) + g\beta\left(T' - T'_\infty\right) - \frac{\sigma B_o^2 u'}{\rho\left(1 + m^2\right)}\left(u' + m w'\right) \quad (8)$$

$$\frac{\partial w'}{\partial t'} - v_o \frac{\partial w'}{\partial y'} = \nu \frac{\partial}{\partial y'}\left(\frac{\partial w'}{\partial y'}\right) - \frac{\sigma B_o^2 u'}{\rho\left(1 + m^2\right)}\left(m u' - w'\right) \quad (9)$$

Energy Equation:

$$\rho C_p\left[\frac{\partial T'}{\partial t'} - v_o \frac{\partial T'}{\partial y'}\right] = \kappa \frac{\partial}{\partial y'}\left(\frac{\partial T'}{\partial y'}\right) + \mu\left[\left(\frac{\partial u'}{\partial y'}\right)^2 + \left(\frac{\partial w'}{\partial y'}\right)^2\right] + \frac{\sigma B_o^2}{\rho\left(1 + m^2\right)}\left(u'^2 + w'^2\right) - \frac{\partial q_r}{\partial y'} \quad (10)$$

The second and third terms on the right – hand side of (10) represent the viscous and Joule dissipations respectively. We notice that each of these terms has two components.

Where $m = \frac{\sigma B_o'}{e n_e}$ is the Hall current parameter.Obviously $v' = v_o'$.

The radiative heat flux term is simplified by making use of the Rosseland approximation [70]as

$$q_r = -\frac{4\overline{\sigma}}{3k^*}\frac{\partial T'^4}{\partial y'} \quad (11)$$

Here $\overline{\sigma}$ is Stefan – Boltzmann constant and k^* is the mean absorption coefficient. It is assumed that the temperature differences within the flow are sufficiently small so that T'^4 can be expressed as a linear function of T' after using Taylor's series to expand T'^4 about the free stream temperature T'_∞ and neglecting higher – order terms. This results in the following approximation:

$$T'^4 \cong 4T_\infty'^3 T' - 3T_\infty'^4 \quad (12)$$

Using equations (11) and (12) in the last term of equation (10), we obtain:

$$\frac{\partial q_r}{\partial y'} = -\frac{16\,\overline{\sigma} T_\infty'^3}{3k^*}\frac{\partial^2 T'}{\partial y'^2} \quad (13)$$

Introducing (13) in the equation (10), the energy equation becomes:

$$\rho C_p\left[\frac{\partial T'}{\partial t'} - v_o \frac{\partial T'}{\partial y'}\right] = \kappa \frac{\partial}{\partial y'}\left(\frac{\partial T'}{\partial y'}\right) + \mu\left[\left(\frac{\partial u'}{\partial y'}\right)^2 + \left(\frac{\partial w'}{\partial y'}\right)^2\right] + \frac{\sigma B_o^2}{\rho\left(1 + m^2\right)}\left(u'^2 + w'^2\right) + \frac{16\,\overline{\sigma} T_\infty'^3}{3k^*}\frac{\partial^2 T'}{\partial y'^2} \quad (14)$$

The initial and boundary conditions of the problem are given by

$$\left.\begin{cases} t' \le 0 : u' = w' = 0,\ T' = T'_\infty \quad for \quad all \quad y' \\ t' > 0 : \begin{cases} u' = U_o,\ w' = 0,\ T' = T'_w \ at \quad y' = 0 \\ u' \to 0,\ w' \to 0,\ T' \to T'_\infty \ at \quad y' \to \infty \end{cases} \end{cases}\right\} \tag{15}$$

The physical quantities are cast in the non – dimensional form by using the following dimensionless scheme:

$u = \dfrac{u'}{U_o},\ w = \dfrac{w'}{U_o},\ y = \dfrac{y' v_o}{\nu},\ t = \dfrac{t' v_o^2}{\nu},\ \theta = \dfrac{T' - T'_\infty}{T'_w - T'_\infty},\ Gr = \dfrac{\nu g \beta (T'_w - T'_\infty)}{v_o^2 U_o}$ is the Grashof number for heat transfer, $M = \dfrac{\sigma B_o'^2 \nu}{v_o^2 \rho}$ is the Magnetic field (Hartmann number), $\Pr = \dfrac{\mu C_p}{\kappa}$ is the Prandtl number, $Ec = \dfrac{U_o^2}{C_p (T'_w - T'_\infty)}$ is the Eckert number, $R = \dfrac{\kappa k^*}{4 \sigma T_\infty'^3}$ is the thermal radiation parameter.

In terms of the above non – dimensional variables and parameters equations (8), (9) and (14) are, respectively, written as

Momentum Equation:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) + Gr\,\theta - \frac{M}{1+m^2}(u + mw) \tag{16}$$

$$\frac{\partial w}{\partial t} - \frac{\partial w}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}\right) + \frac{M}{1+m^2}(mu - w) \tag{17}$$

Energy Equation:

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\left(\frac{3R+4}{3R}\right)\frac{\partial}{\partial y}\left(\frac{\partial \theta}{\partial y}\right) + (Ec)\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{M\,(Ec)}{1+m^2}(u^2 + w^2) \tag{18}$$

And the corresponding boundary conditions are

$$\left.\begin{cases} t \le 0 : u = w = \theta = 0 \quad for \quad all \quad y \\ t > 0 : \begin{cases} u = 1,\ w = 0,\ \theta = 1 \ at \quad y = 0 \\ u \to 0,\ w \to 0,\ \theta \to 0 \ as \quad y \to \infty \end{cases} \end{cases}\right\} \tag{19}$$

It is now important to calculate the physical quantities of primary interest, which are the local wall shear stress (Skin friction), the local surface heat flux (Rate of heat transfer or Nussult number). Given the velocity field in the boundary layer, we can now calculate the local wall shear stress (i.e., skinfriction) is given by and in dimensionless form, we obtain Knowing the temperature field, it is interesting to study the effect of the free convection and radiation on the rate of heat transfer. This is given by which is written in dimensionless form as

$$\tau = \frac{\tau_w}{\rho u_w^2},\ \tau_w = -\left[\mu \frac{\partial u}{\partial y}\right]_{y'=0} = -\rho U_o^2 u'(0) = -\left[\frac{\partial u}{\partial y}\right]_{y=0} \tag{20}$$

The dimensionless local surface heat flux (i.e., Nusselt number) is obtained as

$$N_u(x') = -\left[\frac{x'}{(T'_w - T'_\infty)}\frac{\partial T'}{\partial y'}\right]_{y'=0} \quad \text{then } Nu = \frac{N_u(x')}{R_{e_x}} = -\left[\frac{\partial \theta}{\partial y}\right]_{y=0} \tag{21}$$

These two are calculated by numerical differentiation using Newton's forward Interpolation formula. During computation of the above quantities, the non – dimensional time is fixed at $t = 1.0$.

III. NUMERICAL SOLUTION BY CRANK – NICHOLSON METHOD:

Equations (16), (17) &(18) represent coupled system of non – linear partial differential equations which are solved numerically under the initial and boundary conditions (19) using the finite difference approximations. A linearization technique is first applied to replace the non – linear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. Then the Crank – Nicolson implicit method is used at two successive time levels [71]. An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step and the iterations are continued till convergence, within a prescribed accuracy. Finally, the resulting block tri-diagonal system is solved using the generalized Thomas – algorithm [71]. The energy equation (18) is a linearnon – homogeneous second order partial differential equation whose right hand side is known from the solutions of the flow equations (16) and (17) subject to the conditions (19). The values of the velocity components are substituted in the right hand side of equation (16) which is solved numerically with the initial and boundary conditions (19) using central differences and Thomas algorithm to obtain the temperature distribution. Finite difference equations relating the variables are obtained by writing the equations at the midpoint of the computational cell and then replacing the different terms by their second order central difference approximations in the y – direction. The diffusion terms are replaced by the average of the central differences at two successive time – levels. The computational domain is divided into meshes of dimension Δt and Δy in time and space respectively as shown in figure 2. We define the variables $B = u_y$, $D = w_y$ and $L = \theta_y$ to reduce the second order differential equations (16), (17) and (18) to first order differential equations. The finite difference representations for the resulting first order differential equations (16) and (17) take the following forms:

$$\left(\frac{u_{i+1,\,j+1} - u_{i,\,j+1} + u_{i+1,\,j} - u_{i,\,j}}{2(\Delta t)}\right) - \left(\frac{B_{i+1,\,j+1} + B_{i,\,j+1} + B_{i+1,\,j} + B_{i,\,j}}{4}\right) = \left(\frac{\left(B_{i+1,\,j+1} + B_{i,\,j+1}\right) - \left(B_{i+1,\,j} + B_{i,\,j}\right)}{2(\Delta y)}\right)$$
$$- \left(\frac{M}{1+m^2}\right)\left\{\left(\frac{u_{i+1,\,j+1} + u_{i,\,j+1} + u_{i+1,\,j} + u_{i,\,j}}{4}\right) + m\left(\frac{w_{i+1,\,j+1} + w_{i,\,j+1} + w_{i+1,\,j} + w_{i,\,j}}{4}\right)\right\}$$
$$+ Gr\left(\frac{\theta_{i+1,\,j+1} + \theta_{i,\,j+1} + \theta_{i+1,\,j} + \theta_{i,\,j}}{4}\right) \tag{22}$$

$$\left(\frac{w_{i+1,\,j+1} - w_{i,\,j+1} + w_{i+1,\,j} - w_{i,\,j}}{2(\Delta t)}\right) - \left(\frac{D_{i+1,\,j+1} + D_{i,\,j+1} + D_{i+1,\,j} + D_{i,\,j}}{4}\right) = \left(\frac{\left(D_{i+1,\,j+1} + D_{i,\,j+1}\right) - \left(D_{i+1,\,j} + D_{i,\,j}\right)}{2(\Delta y)}\right)$$
$$- \left(\frac{M}{1+m^2}\right)\left\{m\left(\frac{u_{i+1,\,j+1} + u_{i,\,j+1} + u_{i+1,\,j} + u_{i,\,j}}{4}\right) - \left(\frac{w_{i+1,\,j+1} + w_{i,\,j+1} + w_{i+1,\,j} + w_{i,\,j}}{4}\right)\right\} \tag{23}$$

The variables with bars are given initial guesses from the previous time step and an iterative scheme is used at every time to solve the linearized system of difference equations. Then the finite difference form for the energy equation (18) can be written as

$$\left(\frac{\theta_{i+1,\,j+1} - \theta_{i,\,j+1} + \theta_{i+1,\,j} - \theta_{i,\,j}}{2(\Delta t)}\right) - \left(\frac{L_{i+1,\,j+1} + L_{i,\,j+1} + L_{i+1,\,j} + L_{i,\,j}}{4}\right) =$$
$$\frac{1}{\Pr}\left(\frac{3R+4}{3R}\right)\left(\frac{\left(L_{i+1,\,j+1} + L_{i,\,j+1}\right) - \left(L_{i+1,\,j} + L_{i,\,j}\right)}{4}\right) + QZFO \tag{24}$$

Where $QZFO$ represents the Joule and viscous dissipation terms which are known from the solution of the momentum equations and can be evaluated at the midpoint $\left(i + \frac{1}{2},\ j + \frac{1}{2}\right)$ of the computational cell.Computations have been made for $Gr = 2.0$, $\Pr = 0.71$, $m = 2.0$, $M = 2.0$, $Ec = 0.03$ and $R = 2.0$. Grid – independence studies show that the computationaldomain $0 < t < \infty$ and $-1 < y < 1$ can be divided intointervals with step sizes $\Delta t = 0.0001$ and $\Delta y = 0.005$ fortime and space respectively. The truncation error of thecentral difference schemes of the governing equations is $O\left(\Delta t^2,\ \Delta y^2\right)$. Stability and rate of convergence are functions of the flow and heat parameters. Smaller step sizesdo not show any significant change in the results. Convergenceof the scheme is assumed when all of the unknowns u, B, w, D, θ and L for the last two approximations differ from unity by less than 10^{-6} for all values of y in $-1 < y < 1$ at every time step. Less than 7 approximationsare required to satisfy this convergence criteria forall ranges of the parameters studied here.

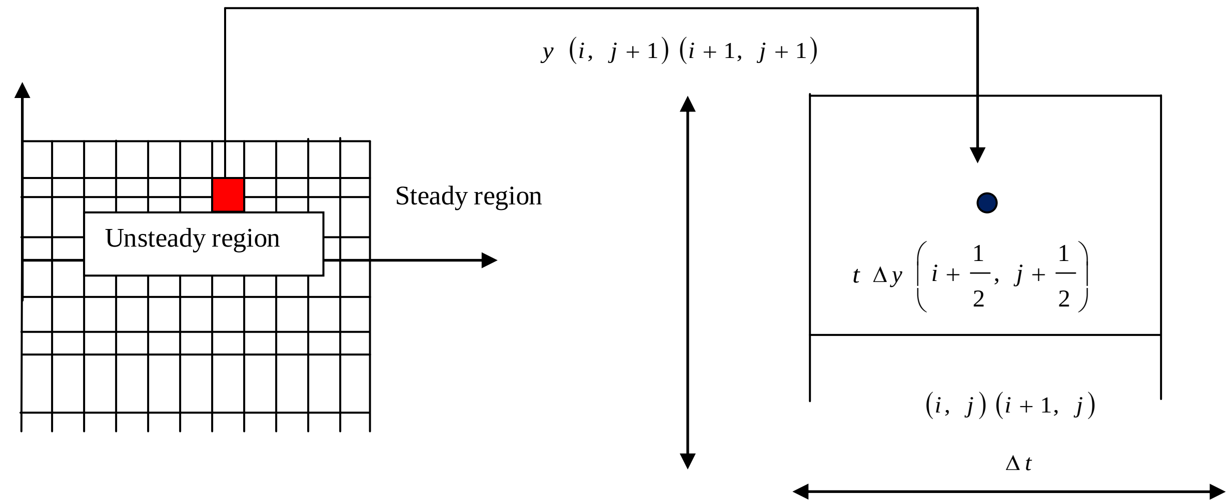

Figure 1. Mesh Layout

IV. RESULTS AND DISCUSSIONS:

The results of our present investigation reduce to those of [73] in the absence of Hall current, thermal radiation when viscous and Joule's dissipation are neglected. The results of [74] can also be recovered from our results by appropriate choice of values for the physical parameters. When our numerical results were compared with the earlier analytical work like [74] and [68], there was an excellent agreement with a maximum error of less than 5% , as stated above. Qualitatively, the results obtained are in good agreement with the earlier works, like those of [75] and [76].

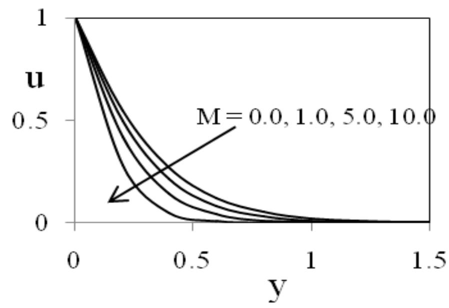

Figure 2. Effect of Magnetic field (Hartmann number) on Primary velocity profiles

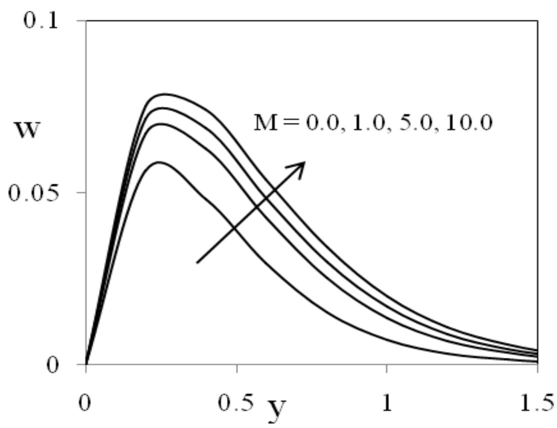

Figure 3. Effect of Magnetic field (Hartmann number) on Secondary velocity profiles

The quantitative differences arise because of different initial and boundary conditions used owing to different physical situations considered. Graphical illustrations of such comparisons are not presented due to paucity of space. For further details on the accuracy of the present numerical method, one may refer to [76].It must be noted that negative values of the parameters Gr and Ec correspond to the case of the plate being heated by the convection currents and similarly their positive values correspond to the case of the plate being cooled by the convection currents.We refer to the values of $Gr = \pm 1$ as moderate cooling and heating $Gr = \pm 2$ and as greater cooling and greater heating respectively. In the following discussion, the value of the non – dimensional time is fixed at $t = 1.0$. The value of Prandtl number is taken as 0.71 which corresponds to air. Air is assumed to be incompressible since all the velocities considered are less than the velocity of sound in the medium (air) so that the Mach number is less than unity [72]. The values for the various parameters are chosen approximately to correspond to a sufficiently ionized air, the flow of which can be modified by an applied magnetic field.

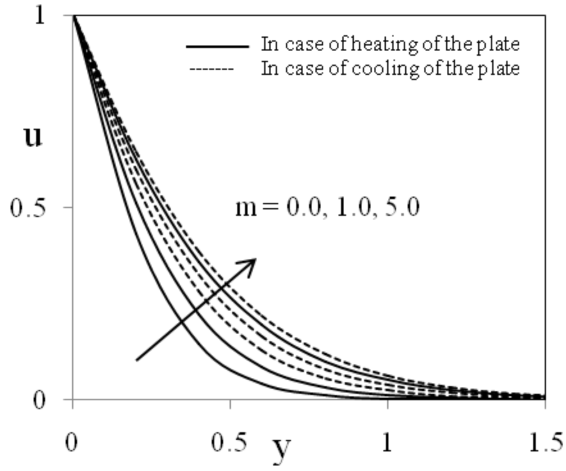

Figure 4. Effect of Hall Current on the primary velocity profiles for cooling of the plate ($Gr = 1.0$ and $Ec = 0.001$) and heating of the plate ($Gr = -1.0$ and $Ec = -0.001$)

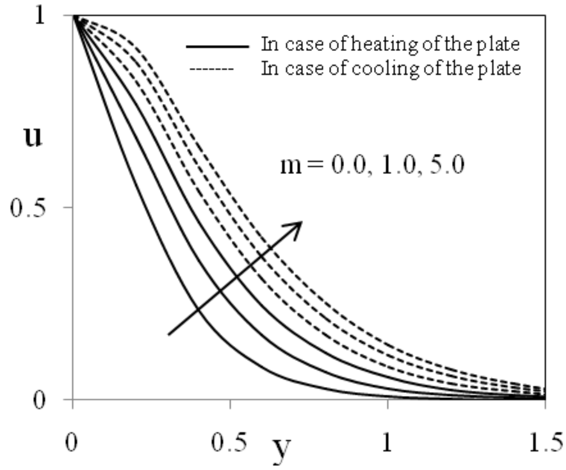

Figure 5. Effect of Hall Current on the primary velocity profiles for cooling of the plate ($Gr = 5.0$ and $Ec = 0.003$) and heating of the plate ($Gr = -5.0$ and $Ec = -0.003$)

Figures (2) and (3) show that the primary velocity u is diminished and the secondary velocity u is increased due to the applied magnetic field. This is in good agreement with the results of [77]. Indeed, when a transverse magnetic field is applied it is well known that the Lorentz force acts in a direction opposite to the flow and offers resistance to the flow and such a phenomenon is described by the term "magnetic – viscosity". From figures (4) and (5), it is inferred that the Hall current promotes the flow along the plate, both when the fluid is heated or cooled. This is because, in general, the Hall current reduces the resistance offered by the Lorentz force. It is also observed that the primary velocity u is greater in the case of cooling of the plate than in the case of heating of the plate. Flow reversal is also noticed in the case of greater heating of the plate. The rise and fall in velocity due to cooling and heating of the plate can be explained as follows. In the process of external cooling of the plate, the free convection currents travel away from the plate. As the fluid is also moving with the plate in the upward direction, the convection currents tend to help the velocity to increase. But, in the case of heating of the plate, as the free convection currents are traveling towards the plate, the motion is opposed by these currents and hence there is a decrease in velocity. In the case of greater heating, this opposition is large enough to counteract the upward push offered by the movement of the plate on the fluid particles just outside the thermal boundary layer and the net force acts downwards and hence the flow becomes downwards.

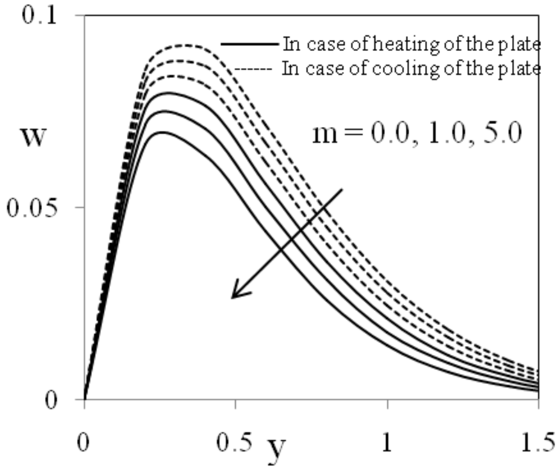

Figure 6. Effect of Hall Current on the secondary velocity profiles for cooling of the plate ($Gr = 1.0$ and $Ec = 0.001$) and heating of the plate ($Gr = -1.0$ and $Ec = -0.001$)

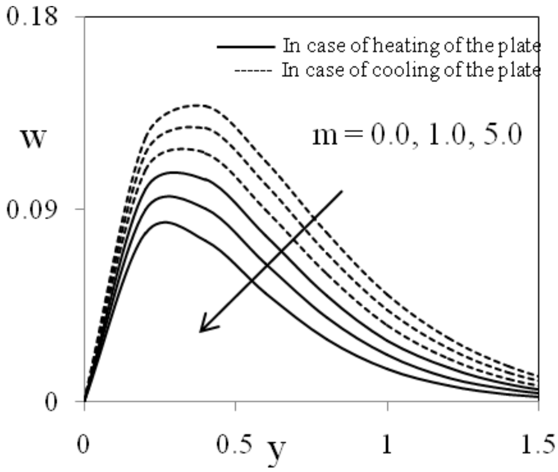

Figure 7. Effect of Hall Current on the secondary velocity profiles for cooling of the plate ($Gr = 5.0$ and $Ec = 0.003$) and heating of the plate ($Gr = -5.0$ and $Ec = -0.003$)

The effect of Hall current on the secondary velocity w isdepicted through figures (6) and (7). The secondary velocity isinduced by the component of the Lorentz force in the $z-$ direction which arises solely due to the Hall current. Fromequation (17), it is clear that is the term $\frac{Mm}{1+m^2}u$ whichdecides the flow in the $z-$ direction. If the Hall parameter $m = 0$, then the term mentioned above is zero and hence thereis no force to induce the flow in the $z-$ direction. Thatis $w = 0$. Further, $\frac{m}{1+m^2}$ increases as m increases in therange $0 \le m \le 1$ and it decreases as m increases in the range $m > 1$. This means that the magnitude of the component of theLorentz force in the $z-$ direction increases as m increases inthe range $0 \le m \le 1$ and hence the secondary velocity w isincreased, while it decreases when m increases in the range $m > 1$ and hence the secondary velocity w is decreased. Theseresults are observed graphically in figures (6) and (7).

The secondary velocity w is observed to be greater in thecase of cooling of the plate than in the case of heating of theplate. This result also can be inferred from the term $\frac{Mm}{1+m^2}u$ of equation (17). The primary velocity u isgreater in the case of cooling than in the case of heating of theplate and so is the term mentioned above, which is thedeciding factor so far as the secondary velocity is concerned.This ultimately results in the secondary velocity being greaterin the case of cooling of the plate than in the case of heating ofthe plate. By similar arguments, the flow reversal observed inthe secondary velocity in the case of greater heating can alsobe attributed to the flow reversal in the primary velocity.

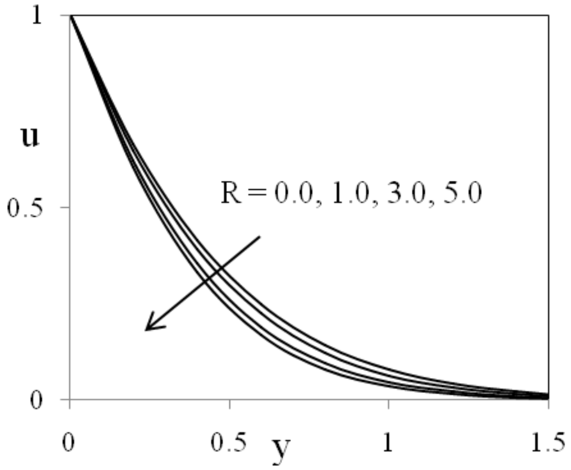

Figure 8. Effect of thermal radiation on the primary velocity profiles

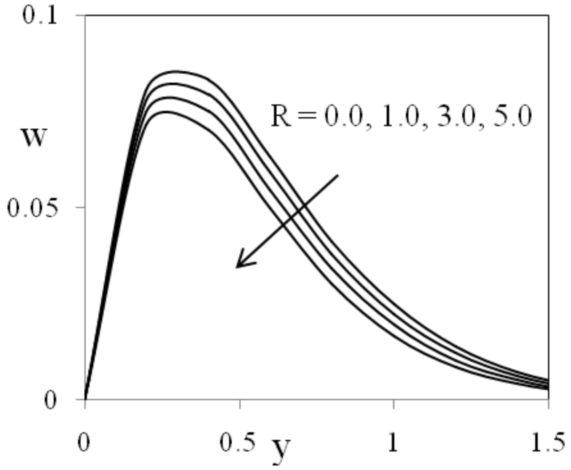

Figure 9. Effect of thermal radiation on the secondary velocity profiles

Figures (8) – (10) display the effects of the radiation parameter R on the time development of the primary and secondary velocities andtemperature of the fluid at the center of the channel. It is observed that increasing R decreases primary and secondary velocities andtemperature of the fluid. An increase in the radiation emission, which is represented by R , reduces the rate of heat transferthrough the fluid. This accounts for the decrease in temperature with increasing R . The velocity decreases through thereduction in buoyancy forces associated with the decreased temperature.The temperature θ is not significantly affected by themagnetic field and Hall current, except in the very closevicinity of the plate. This is because the effect of theHartmann number and Hall parameter can be felt only in theHartmann layer and the thermal boundary layer respectively.

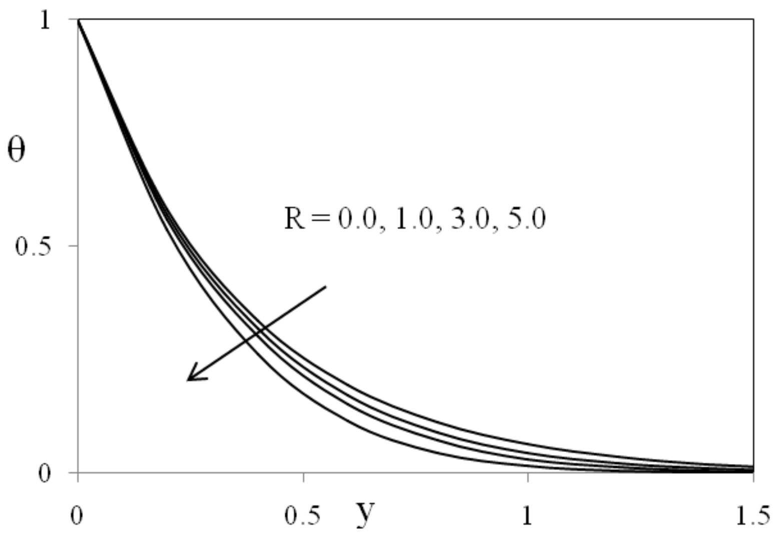

Figure 10. Effect of thermal radiation on the temperature profiles

Hence, graphical illustrations of the effect of these parameters on the temperature are not presented. However, appreciable changes in the slope of the temperature profiles very close to the plate, $y = 0$, were observed and hence the effects of these parameters on rate of heat transfer are presented below.

Table – 1: Numerical values of skin friction and rate of heat transfer for air $\Pr = 0.71$ and $t = 1.0$

Gr	Ec	M	m	τ	Nu
– 2.0	0.00000	1.0	0	2.0826551	1.9276342
– 2.0	0.00000	1.0	1	2.0826551	1.9276342
– 2.0	–0.0030	1.0	0	2.0824187	2.1958092
– 2.0	–0.0030	1.0	1	2.0824187	2.1958094
– 2.0	–0.0030	10	0	5.8822109	2.2069059
– 2.0	–0.0030	10	1	4.0288761	2.2035722
– 2.0	–0.0030	10	10	2.2001876	2.1960976
2.0	0.00000	1.0	0	1.9338129	2.1901224
2.0	0.00000	1.0	1	1.9338129	2.1901224
2.0	0.00300	1.0	0	1.8326618	2.1858731
2.0	0.00300	1.0	1	1.8326618	2.1858731
2.0	0.00300	10	0	4.5714483	2.1731947
2.0	0.00300	10	1	3.8263385	2.1769209
2.0	0.00300	10	10	2.2347811	2.1855300
– 2.0	–0.0030	10	0	5.7265224	2.2076842
– 2.0	–0.0030	10	1	4.8826478	2.2053844
– 2.0	–0.0030	10	10	3.6900763	2.2009206
2.0	0.00300	10	0	2.8969776	2.1716345
2.0	0.00300	10	1	2.2717939	2.1761598
2.0	0.00300	10	10	– 0.8053912	2.1867814

Table – 2: Numerical values of skin friction and rate of heat transfer for different values of R

R	τ	Nu
1.0	2.1096325	1.9023312
2.0	1.9266354	1.7844694
3.0	1.8122746	1.6443755
4.0	1.7461687	1.5133699

From table – 1, It is observed that the Hartmann number M decreases the skin friction τ, irrespective of whether the plate is heated or cooled, both in the presence and absence of Hall current. This is because the effect of Hartmann number is to decrease the resistance offered by the Lorentz force.It is noticed that, both in the presence and absence of Hall current, due to magnetic field, the rate of heat transfer Nu increases when the plate is heated and it decreases when the plate is cooled.However, the effect of Hall current is to increases the skin friction in both the cases of heating and cooling of the plate.This is because the effect of transverse magnetic field is to retard the flow by offering additional resistance called magnetic viscosity.Finally, the effect of Hall current is to decrease the rate of heat transfer when the plate is heated and increase the same when the plate is cooled.The influence of the thermal radiation parameter R on the skin friction and rate of heat transferare shown in the table – 2. The radiation parameter R defines the relative contribution of conduction heat transfer to thermal radiation transfer. It is obvious that an increase in the radiation parameter results in decreasing the skin friction and rate of heat transferwithin the boundary layer.

CONCLUSIONS:

At the outset, our numerical results are in good agreement with those of [73] in the absence of Hall current, thermal radiation, when the viscous and Joule's dissipations are neglected. In the non – magnetic case, our results reduce to that of [78]. Qualitatively and quantitatively our results are in good agreement with the earlier analytical results reported in [74] and [78]. The effects of magnetic field, thermal radiation and Hall current on the flow and heat transfer are analyzed and physical interpretations or justifications of the results are provided as and when possible. The results obtained can be summarized as follows.

1. Applied magnetic field retards the primary flow along the plate and supports the secondary flow induced by the Hall current.
2. Hall current promotes the flow along the plate. The secondary flow is supported when the Hall parameter is increased up to unity. If the Hall parameter is increased beyond unity, the secondary flow is retarded. These results are true for both cooling and heating of the plate.
3. Both primary and secondary velocities are found to be greater in the case of cooling of the plate than in the case of heating of the plate.
4. Flow reversal is observed in both primary and secondary velocity components in the case of greater heating of the plate.
5. Magnetic field andHall current modify only the slope of the temperature profile in the narrow region close to the plate called the thermal boundary layer. Otherwise, their effect on temperature is not significant.
6. Increasing of thermal radiation parameter values, there is a reduction in both primary and secondary velocities.
7. The presence of radiation effects caused reductions in the fluid temperature.
8. Skin friction is increased by the magnetic field.
9. The effect of magnetic field is to increase the heat transfer rate when the plate is heated and decrease it when the plate is cooled.
10. Hall current decreases the skin friction.
11. Due to Hall current the heat transfer rate decreases when the plate is heated and it increases when the plate is cooled.
12. The radiation effect leads to enhance the skin friction and heat transfer rate.

To improve upon the present work, it is suggested that one may consider the effect of rotation. Further, in order to investigate supersonic flows one may consider the effects of compressibility. It is also suggested that the atmosphere may be considered as a stratified fluid. Obviously, these suggestions will lead to more complex problems but are worth investigating.

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