Mathematical Modeling of Class B Amplifire Using Natural and Regular Sampled Pwm Moduletion

[1,] N. V. Shiwarkar, [2,] K. G. Rewatkar[*]

Department of Electronics, Dr. Ambedkar College, Deeksha Bhoomi, Nagpur, India

**Department of Physics, Dr. Ambedkar College, Deeksha Bhoomi, Nagpur India*

ABSTRACT

Class-D amplifiers operate by converting an audio input signal into a high-frequency square wave output, whose lower-frequency components can accurately reproduce the input. Their high power efficiency and potential for low distortion makes them suitable for use in a wide variety of electronic devices. By calculating the outputs from a classical class-D design implementing different sampling schemes we demonstrate that a more advance method, over the double Fourier series method, which is the traditional technique employed for this analysis. This paper shows that when natural sampling is used the input signal is reproduced exactly in the low-frequency part of the output, with no distortion. Although this is a known result, our calculations present the method and notation that develops the classical class-D design is prone to noise, and therefore negative feedback is often included in the circuit. Subsequently we incorporate the Fourier transform/Poisson Re-summation method into a formulised and analysis of a feedback amplifier. Using perturbation expansions we derive the audio-frequency part of the output, demonstrating that negative feedback introduces undesirable distortion.

KEYWORDS: *class D, natural sampling, regular sampling, Fourier analysis method re-summation*

I. INTRODUCTION

Amplifiers are used increasingly in our every day appliances. In many of the applications efficiency is highly desirable to reduce power consumption. This is important not only from an environmental and cost perspective, but also to maximize battery life on portable devices. Traditional audio amplifiers can achieve efficiencies only in the region of 65-70%, whereas class-D amplifiers can achieve over 90% efficiency [1, 2]. Their high power efficiency, and dissipation less energy is dissipate, there is no need for a large heat sink, means they are suited for use in very small devices, or those where a long battery life is essential, e.g. mobiles, laptops, hearing aids and MP3 players, as well as home sound systems. The key feature of class-D amplifiers that provides such high efficiency is that they are switching amplifiers. This means that their output is a high-frequency square wave that alternates between two voltages. While efficiency is desirable, it is also vital that the amplifier output has low distortion. Theoretically a classical class-D amplifier is able to reproduce an input signal with no distortion at all. It has long been known that this is the case if a sinusoidal signal is input [3], and has been shown more recently for a general input signal [4]. Class-D amplifiers have been implemented commercially. Since the transistors were readily available in the early 1990s [5].

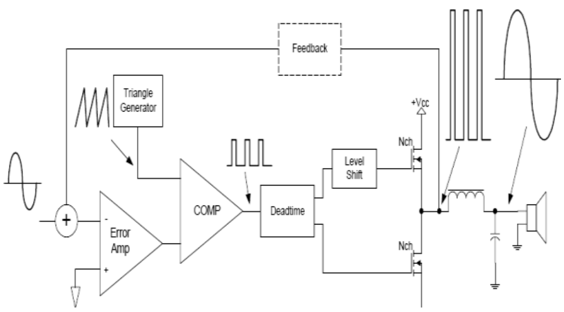

Fig:1 block diagram of class D amplifire

Counter-intuitive though it may seem, the square wave output from class-D amplifiers can reproduce a sound free from distortion, and in a highly efficient manner. A high frequency square wave is the most efficient output, much more efficient than a slowly varying output signal where a lot of energy would be dissipated as heat.
In order to understand how the square wave output from the amplifier provides low distortion, it is important to examine the performance of amplifier is used. Class-D amplifiers are used in the output stage (see figure 1). A pre-amplifier first increases the amplitude of the low-amplitude analogue audio signal. The signal, now at the required amplitude for playback, then passes through a class-D amplifier, which converts the signal into a more efficient form (a square wave) for playback. The square wave then passes through a filter and a loudspeaker, which plays the final output signal in its amplified form. Therefore, rather than to increase amplitude, the aim of a class-D amplifier is to convert the input signal into a square wave that represents the input signal. To do this, a class-D amplifier creates a square wave whose width varies according to the input signal, via a process called pulse width modulation (PWM)[3,7]. The way this process is carried out is important because after filtering, the output should ideally equal the signal input to the class-D amplifier. When PWM is used, a relatively low-frequency input signal is compared with a carrier wave of much higher frequency to create a high-frequency square wave that switches between voltages $+V$ and $-V$. The widths of the pulses in this resulting

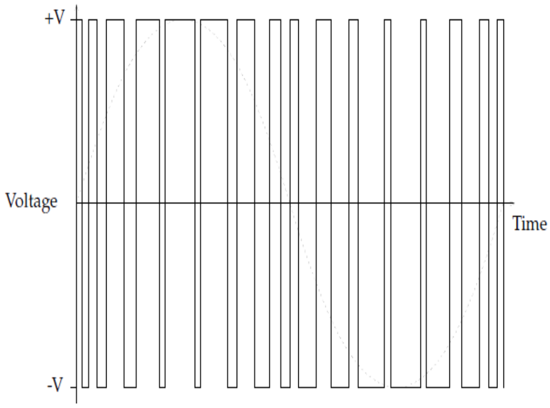

Fig: 2 PWM cycle swing between +v to –v

Thus the pulse width modulated square wave is composed of low-frequency components related to the input signal, and high-frequency components related to the carrier wave. The square wave is then passed through a filter where the high-frequency components are attenuated, while the low-frequency components are allowed to pass through relatively unchanged. These low-frequency components constitute the final output, which is as close to the original input signal as possible. The duty cycle of this square wave is defined as the ratio between the length of time the wave is at $+V$ and the period of the carrier wave. An ideal square wave, which is at $+V$ for half of the period and $-V$ for the remaining half, has a 50% duty cycle. Therefore we see that when PWM is used it is the duty cycle of the square wave output which varies according to the input signal. The typical frequency ranges are 80-250 kHz for the carrier wave [1], and20Hz-20 kHz for the input signal [6]

Experimentation

The classical class- D Amplifier design is known to reproduce the input signal exactly in the low-frequency part of the output with no distortion [1 7]. However, this simple design is susceptible to noise, for example due to non-ideal components, or variation in the carrier wave [8]. For this reason, negative feedback is often implemented in class-D designs. Negative feedback allows the output to be “fed back” into the circuit, in order to achieve a final output that is closer to the input signal. While negative feedback reduces noise in the circuit, it does however introduce distortion [7] In order to investigate class-D amplifiers, it is therefore essential to analyse the square wave output produced by PWM. However, it is not straightforward to achieve this because, even for a periodic or quasi-periodic input, the output is only quasi-periodic. we calculate the pulse width modulated output from a classical class-D amplifier, and show that the input signal can be reproduced exactly in the theoretical output[8,9]. The method of calculating output, in order to highlight the advantages of the second. The first method we implement is the double Fourier series method [3]. Natural sampling and regular sampling (which is sometimes referred to as uniform sampling) are two commonly used methods of sampling[11] which is the most commonly used. However, the approach is unnecessarily complex and it is circuitous, though possible, to extend the method to more advanced modulation schemes [12, 13]. which is simpler and can be adapted easily to investigate other modulation schemes.

II. ANALYSIS

Natural sampling and regular sampling (which is sometimes referred to as uniform sampling) are two commonly used methods of sampling. We now show how these methods can be implemented to create a pulse width modulated square wave output, $g^*(t^*)$, that alternates between $+V$ and $-V$. The switching times of $g^*(t^*)$ are determined by the intersection of the input signal $s^*(t^*)$ with a high-frequency carrier wave $v^*(t^*)$ of period T and (angular) frequency $\omega^*c = 2\pi/T$ the carrier wave can be defined in different ways, according to the type of modulation required, as we shall see below.When natural sampling is used, the switching occurs when $s^*(t^*) + v^*(t^*) = 0$. When regular sampling is used, the input signal is sampled at a fixed time in each carrier wave period, and the switching occurs -when this sample equal-minus the carrier wave. For example, if the input signal is sampled at the beginning of each carrier wave period, when $t^* = nT$, the switching occurs at a time t^* later in that carrier wave period when $s^*(nT) + v^*(t^*) = 0$.

It is possible to use either single-edge or double-edge modulation. When single- edge modulation is used, only one edge of the square wave output is determined by the input signal, the other edge occurs at a fixed time. The leading edge of the square wave is defined as the one that switches from $-V$ to $+V$, and the trailing edge is defined as the one that switches from $+V$ to $-V$.

If leading-edge modulation used, the leading edge is determined by the input signal and the trailing edge remains fixed. If trailing-edge modulation is used, then its edge is determined by the input signal and it remains fixed[14]. For single-edge modulation the carrier wave is a sawtooth wave with period T, where for leading-edge modulation it is defined to be

$$v^*(t^*) = -V + \frac{2V}{T}(t^* - nT) \text{ for } nT < t^* < (n+1)T, \quad \text{---------------------(1)}$$

$$v^*(t^*) = V - \frac{2V}{T}(t^* - nT) \text{ for } nT < t^* < (n+1)T, \quad \text{--------------------------(2)}$$

Equation for leading edge and trailing edge input siganal.

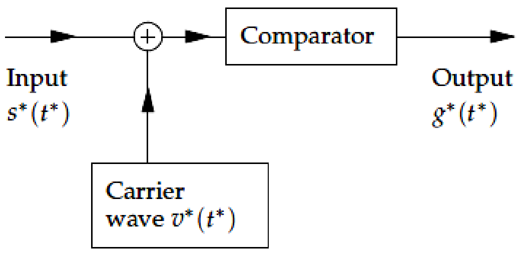

Fig.3: modeling for traling edge or leading edge class d amplifire

4.1 Equation basis analysis

A classical model of class-D amplifier, as depicted in figure 3. The input signal $s^*(t^*)$ is first added to a carrier wave $v^*(t^*)$. The resulting voltage, $s^*(t^*) + v^*(t^*)$, is fed into a comparator that produces a square wave output, $g^*(t^*)$, defined by equation (3) as

$$g^*(t^*) = \begin{cases} -V & \text{for } s^*(t^*) + v^*(t^*) < 0 \\ +V & \text{for } s^*(t^*) + v^*(t^*) > 0. \end{cases} \quad \text{----------------------(3)}$$

Where appropriate you should overtype the different fields When natural sampling is used, the switching occurs when $s^*(t^*) + v^*(t^*) = 0$. When regular sampling is used, the input signal is sampled at a fixed time in each carrier wave period, and the switching occurs when this sample equals minus the carrier wave. For example, if the input signal is sampled at the beginning of each carrier wave period, when $t^* = nT$, the switching occurs at a time t^* later in that carrier wave period when $s^*(nT) + v^*(t^*) = 0$.

Natural and regular sampling has been investigated extensively, and it is well documented that natural sampling produces less distortion than regular sampling. This has been shown for a general input signal [4], and has also been verified for particular input signals, [15–19]. investigate this classical class-D amplifier design for a sinusoidal input signal, defined to be

$S^*(t^*) = S_0 V \sin\omega^* a\, t^*$-------------------(4)

The carrier wave is therefore a sawtooth wave defined by equation(4) and figure 3, and we apply natural and regular sampling as depicted in figure 3. The square wave $g^*(t^*)$ switches from $-V$ to $+V$ at times $t^* = nT + \beta^* n$ and from $+V$ to $-V$ at times $t* = nT$, and therefore we may write

$$g^*(t^*) = \begin{cases} -V & \text{for } nT < t^* < nT + \beta_n^* \\ +V & \text{for } nT + \beta_n^* < t^* < (n+1)T. \end{cases}$$

----------------------------------(5)

When natural sampling is used, the leading-edge switching occurs when $s(t) + v(t) = 0$. Thus for natural sampling we have

$$\beta_n = \frac{1}{2}(1 - s(n + \beta_n)).$$

--------------------(6)

When regular sampling is used, the input signal is sampled at the beginning of each carrier wave period, $t = n$. The leading-edge switching occurs when $s(n) + v(t) = 0$. Thus for regular sampling we have

$$\beta_n = \frac{1}{2}(1 - s(n)).$$

---------------------(7)

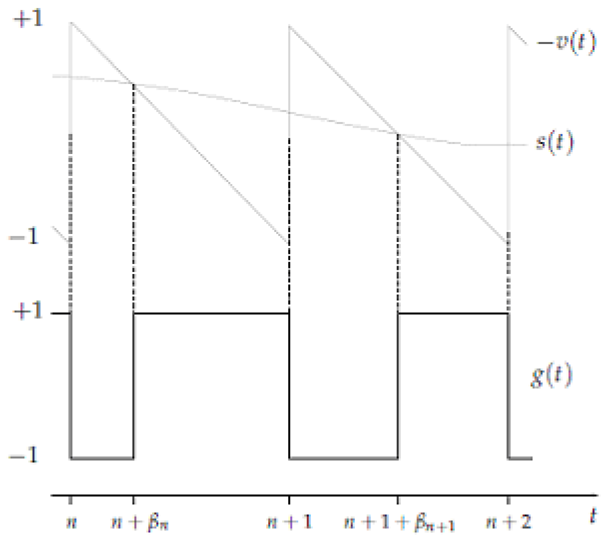

Fig 4: wave shaped and applied technology

An effective way to analyses the output resulting from particular sampling scheme is to plot the spectrum, i.e. plot the magnitude of the amplitude of each coefficient *Gmn* in the output against its frequency ω. By plotting and comparing the spectra of different sampling schemes we can see clearly what the components of each output are, and their magnitudes, and so determine which sampling scheme produces the output with lower distortion. The spectrum for regular sampling is plotted in figure 4 next to the spectrum for natural sampling. Note that to plot both spectra on the same graph we have shifted the spectrum for regular sampling to the right by 0.05, so that, for example, the black peak that appears at $\omega = 0.3$ is actually the peak that corresponds to $\omega = 0.25$.

It is seen from figure 4 that the only component in the low-frequency part of the natural sampling spectrum is exactly the input signal, whereas for regular sampling the input signal harmonics of the input signal appear in the low-frequency part of the output. These harmonics can be seen more clearly in figure 4 where we plot only the low-frequency part of the output. Thus comparing the low-frequency parts of the spectra for natural and regular sampling, it is obvious that the output resulting from regular sampling contains much more distortion than that from natural sampling, which contains no distortion. Outside the low-frequency part of the spectrum, the outputs from both sampling schemes comprise peaks at multiples of the carrier wave frequency as well as lower amplitude peaks (called side-bands) concentrated around multiples of the carrier wave frequency. Note that we have chosen to plot the spectra up to $\omega = 16$ in figure 5 merely so that the low-frequency part of the spectra, as well as the peaks at $\omega = 2\pi$ and $\omega = 4\pi$ (i.e. at the carrier wave frequency and at twice the carrier wave frequency) and their corresponding sidebands, can be seen clearly. In addition there are peaks at, and sidebands around, all larger multiples of the carrier wave frequency, as can be determined from the natural and regular sampling output formulae. There are minor differences in the amplitudes of these peaks outside the low-frequency part of the spectrum for regular sampling compared with natural sampling, but these are irrelevant as they will be attenuated by a low-pass filter.

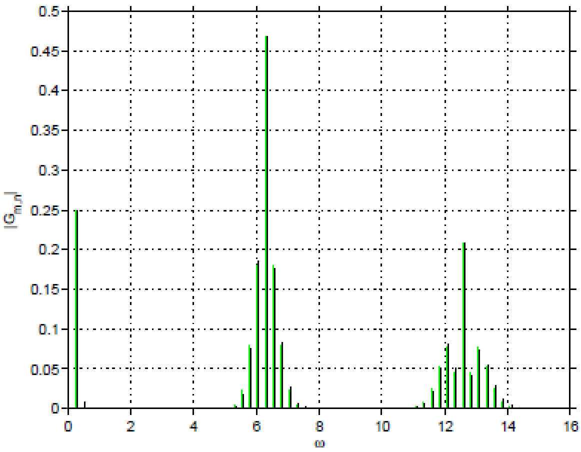

Fig. 5 spectrum of low frequency PWM

III. DISCUSSION

The response of the design is determined the outputs from a classical class-D amplifier when a sinusoidal signal is input and leading-edge natural or regular sampling PWM is used to create the square wave output. For natural sampling, the input is reproduced exactly in the output, and there are no other terms in the low- frequency part of the output, and so the input signal can be reproduced with no distortion. For regular sampling, the input signal is reproduced with distortion and harmonics of the input signal appear in the low-frequency part of the output, and so the amplifier output is a distorted version of the input signal. Although natural sampling provides better distortion performance than regular sampling, it is only suited to some applications. The equations for the natural sampling switching times are implicit, and so natural sampling is often used in analogue applications, but is difficult to implement digitally [16].

The equations for the regular sampling switching times are explicit and so this sampling scheme is commonly used in digital applications. This motivates us to investigate sampling schemes that aim to provide low distortion, like natural sampling, whilst being simple to use in digital applications, like regular sampling. The calculated outputs for the two sampling schemes first using the commonly used double Fourier series method, and then repeated the calculations using the Fourier transform/Poisson Re-summation method, in order to illustrate the advantages of the latter method. When the comparison of the two methods for natural sampling, it is easy to see that the latter method is simpler and quicker to implement. Not needing to introduce two separate timescales to the problem and using the Poisson resumption formula shortens the calculation considerably. In addition, it is possible to demonstrate via the Fourier transform/Poisson resumption method for a general input signal that the low-frequency part of the output for natural sampling is exactly the input signal, which is not possible via the double Fourier series method. Both methods require adaptation to examine the output resulting from regular sampling. However, using the double Fourier series method, an additional change of variables is required to solve the problem, making the method unnecessarily complex. The alteration to the Fourier transform/Poisson re-summation method is to take the Fourier transform, which is simple to invert later. This change ensures that the equations for the switching times are used in their explicit form.Each method requires separate consideration of particular frequency components, though using the Fourier transform/Poisson re-summation method this can be done quickly, especially in the natural sampling case. The Fourier transform/Poisson re-summation method has considerable advantages over the double Fourier series method. It is shorter and simpler to use, as well as being more easily adaptable to different sampling schemes.

It enables easy comparisons between existing modulation and sampling techniques, as well as mathematical analysis of new or complex strategies that so far have not been tackled.

IV. CONCLUSION

With the detailed result analysis considered , the PWM process and investigated the methods by which pulse width modulated square waves can be analyzed. The two different approaches to analyzing the pulse width modulated output created by a classical class-D amplifier. The double Fourier series method, which is the conventional technique, was shown to be unnecessarily complex. We demonstrated that the Fourier transform/Poisson resumption method is simpler and quicker to implement, as well as being easier to adapt to different sampling schemes, in the analysis of the classical class-D amplifier, it determined that, if natural sampling is used, the low-frequency part of the output is exactly the input signal. However, the classical design is susceptible to noise, and negative feedback is often included in the circuit to counter this problem. We devote the rest of the investigation of negative feedback designs, and incorporate the Fourier transform/Poisson resumption method into each model.

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