Analytical Solution Of Unsteady Flow Past An Accelerated Vertical Plate With Constant Temperature and Variable Mass Transfer

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ABSTRACT

Analytical solution of unsteady flow past an accelerated vertical plate has been carried out in the presence of variable temperature and mass transfer. The dimensionless governing equations are solved using Laplace transform technique. The results are obtained for velocity, temperature, and concentration were obtained for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number and time. It is observed that the velocity increases with increase in Sc, t, Gr, and Gc.

KEY WORDS: *Mass transfer,unsteady, accelerated vertical plate, variabletemperature.*

I. INTRODUCTION

Heat and mass transfer plays an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, space craft design, solar energy collectors, design of chemical processing equipment, satellites and space vehicles are example of such engineering applications. Mass transfer certainly occurs within the mantle and cores of planets of the size of or larger than the earth. It is therefore interesting to investigate this phenomenon and to study in particular, the case of mass transfer in the free convection flow.Several researches have been carried out to investigatedunsteady flow past an accelerated vertical plate with mass transfer effect.Singh (1983) have investigated the mass transfer effects on flow past an accelerated vertical plate with uniform heat flux.Jha (1990) studiedmass transfer effects and the flow past accelerated infinite vertical plate with heat sources. Mohammed and Karim (2000) have investigated the combined effects of transpiration and free convective current on the unsteady flow of viscous incompressible fluid past an exponential accelerated vertical permeable plate which is at a uniform temperature.

In their course of studies, expressions for the velocity -field, temperature- field, and Skin- friction were obtained in closed form by Laplace transform. Gupta *et al*. (2003) have analysed the flow in the Ekman layer on an oscillating plate. Mass transfer effects on unsteady flow plate an accelerated vertical plate with suction have been investigated by Das *et al*. (2006). A mass transfer effect on vertical oscillating plate with heat flux was studied by Muthucumaraswamy and Manivannan (2007). In their work, it shows that the temperature from the plate to the fluid at a uniform rate and the mass is diffused uniformly. They observed that the velocity increases with decreasing phase angle ωt. Muthucuraswamy *et al.* (2009) have studied the unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion. They analysed the velocity profiles and concentration for different physical parameters like the thermal Grashof number, mass Grashof number, Schmidt number and time. They also observed that the velocity increases with decreasing values of the Schmidt number.Okedoye and Lamidi, (2009) have investigated analytical solution of mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction.
This present study investigates mass transfer effects on unsteady flow past an accelerated vertical plate with wall temperature. The dimensionless governing equations are solved using Laplace – transform technique. The solutions are obtained in terms of exponential and error functions

II. FORMULATION OF THE PROBLEM

Analytical solution of Mass transfer effects on unsteady flow past an accelerated vertical plate with variable temperature has been considered.The x'-axis is taken along the plate in the vertically upward direction and also the y'-axis is taken normal to the plate. At t'>0,velocity of the plate raise with respect totime in its own plane, the temperature from the plate raise to T_{ω} , and the concentration is raised linearly withrespect to time.

2.1 THE GOVERNING EQUATION

Then under the usual Boussinesq's approximation the unsteady flow equations are momentum equation, energy equation, and mass equation respectively.

$$\frac{\partial u}{\partial t'} = g\beta\left(T - T_\infty\right) + g\beta^*\left(C' - C'_\infty\right) + \nu\frac{\partial^2 u}{\partial y^2} \qquad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = \kappa\frac{\partial^2 T}{\partial y^2} \qquad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \qquad (3)$$

where u is the velocity of the fluid, T is the fluid temperature, C' is the concentration, g is the

gravitational constant, β and β* are the thermal expansions of fluid and concentration respectively, t' is thetime,

ρ is the fluid density, C_p is the specific heat capacity, v is the viscosity of the fluid, k is the thermal conductivity.

The initial and boundary conditions are:

$$\left.\begin{array}{lllll} & u = 0 & T = T_\infty & C' = C'_\infty & for\ all\ y, t' \le 0 \\ t' > 0: & u = u_0 t & T = T_\omega & C' = C'_\infty + \left(C'_\omega - C'_\infty\right)At'\ at\ y = 0 \\ & u \to 0 & T \to T_\infty & C' \to C'_\infty & at\ y \to \infty \end{array}\right\} \qquad (4)$$

where $A = \left(\frac{u_0^2}{\nu}\right)^{\frac{1}{3}}$

The non-dimensional quantities are:

$$\left.\begin{array}{l} U = \frac{u}{\left(\nu u_0\right)^{\frac{1}{3}}}, t = t'\left(\frac{u_0^2}{\nu}\right)^{\frac{1}{3}}, Y = y\left(\frac{u_0}{\nu}\right)^{\frac{1}{3}} \\ \theta = \frac{T - T_\infty}{T_\omega - T_\infty}, Gr = \frac{g\beta\left(T_\omega - T_\infty\right)}{u_0}, C = \frac{C' - C'_\infty}{C'_\omega - C'_\infty} \\ Gc = \frac{g\beta^*\left(C'_\omega - C'_\infty\right)}{u_0}, \Pr = \frac{\mu c_\rho}{\kappa}, Sc = \frac{\nu}{D} \end{array}\right\} \qquad (5)$$

The non-dimensional quantities of equation (5) which analysed (1) to (4),and they lead tothe dimensionless equations as follows;

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \qquad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} \qquad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \qquad (8)$$

Where Sc is the Schmidt number, Pr is prandtl number, and Gr and Gc are the Grashof numbers

The initial and boundary conditions are reduces to:

$$\left.\begin{array}{ll} & U = 0,\ \theta = 0,\ C = 0,\ for\ all\ Y, t \le 0 \\ t > 0: & U = t,\ \theta = 1,\ C = t\ \ at\ Y = 0 \\ & U \to 0, \theta \to 0, C \to 0\ as\ Y \to \infty \end{array}\right\} (9)$$

III. SOLUTION TO THE PROBLEM

Equations (6) – (8), are solved subjected to the boundary conditions of (9), and the solutions are obtained for concentration, temperature and velocity flow in terms of exponential and complementary error function using the Laplace- transform technique as follows;

$$C = t\left[\left(1+2\eta^2 Sc\right) erfc\left(\eta\sqrt{Sc}\right) - \frac{2\eta\sqrt{Sc}}{\sqrt{\pi}}\exp\left(-\eta^2 Sc\right)\right] \quad (10)$$

$$\theta = erfc\left(\eta\sqrt{\Pr}\right),$$

(11)

$$U = \left[\left(1+2\eta^2\right) erfc(\eta) - \frac{2\eta}{\sqrt{\pi}} e^{-\eta^2}\right] + \frac{Grt}{\Pr-1}\left[\left(1+2\eta^2\right) erfc(\eta) - \frac{2\eta}{\sqrt{\pi}} e^{-\eta^2}\left(1+2\eta^2 \Pr\right) erfc\left(\eta\sqrt{\Pr}\right)\right.$$

$$\left. + \frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}} e^{-\eta^2 \Pr}\right] + \frac{Gc \cdot t^2}{6(Sc-1)}\left[\left(3+12\eta^2+4\eta^4\right) erfc(\eta) - \frac{\eta}{\sqrt{\pi}}\left(10+4\eta^2\right)e^{-\eta^2} - \left(3+12\eta^2 Sc+4\eta^4 (Sc)^2\right)\right.$$

$$\left. erfc\left(\eta\sqrt{Sc}\right) + \frac{\eta\sqrt{\Pr}}{\sqrt{\pi}}\left(10+4\eta^2 Sc\right)e^{-\eta^2 Sc}\right] \quad (12)$$

where $\eta = \frac{y}{\sqrt{2t}}$

IV. RESULTS AND DISCUSSION

The problem of unsteady flow past an accelerated vertical plate with variable temperature and mass transfer has here been formulated, analyzed and solved analytically. In other to point out the effects of different physical parameters like Gr, Gc, Sc, Pr, and t on the flow, the computations of this parameter are carried out, and the following discussions are made. The value of the Prandtl number (Pr=0.71) is chosen to represent air. The value of the Schmidt number (Sc=0.6) is chosen to represent the presence of species by water vapour. Figures 4.1 to 4.7 represent the solutions of the problem, and is investigated for the physical parameters namely thermal Grashof number Gr, mass Grashof number Gc, Schmidt number Sc, Prandtl number Pr, and time t.

The effect of velocity for different values of time (Sc =0.16, 0.3, 0.6, 2.01) is presented in Figure 4.1. It shows that velocity increases with increasing values of Sc.

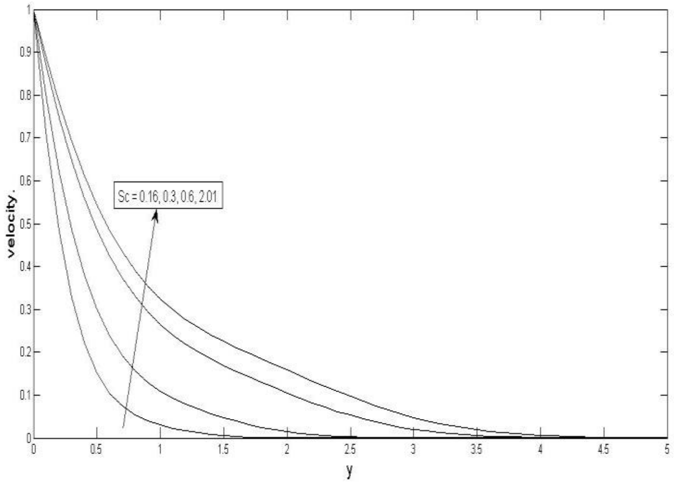

Figure 4.1 Velocity profiles for different values of Sc

The effect of velocity for different values of time (t=0.1, 0.4, 0.8, 1.2) is presented in Figure 4.2. It shows that velocity increases with increasing values of t.

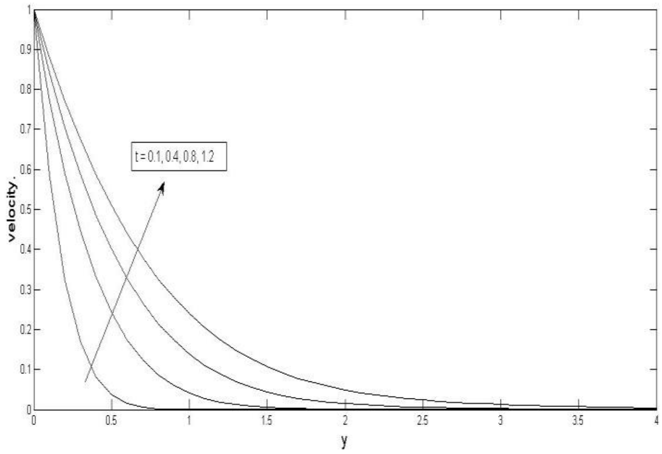

Figure 4.2 Velocity profiles for different values of t

The velocity profiles for different values of mass Grashof number (Gc = 0.2, 0.4, 0.6, 0.8) is presented in Figure 4.3It is observed that velocity increases with increasing Gc.

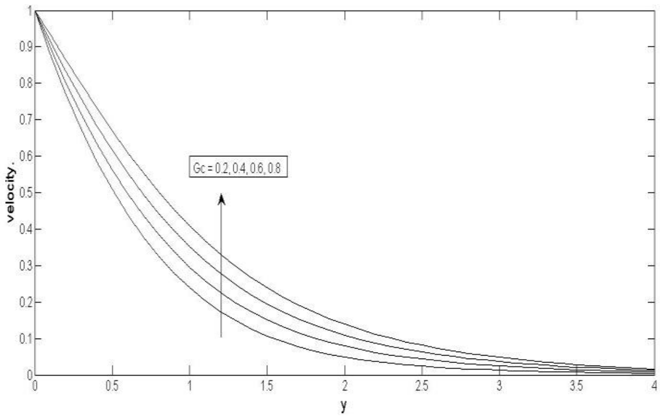

Figure 4.3 Velocity profiles for different values of Gc

The velocity profiles for different values of thermal Grashof number (Gr =0, 0.2, 0.4, 0.6) is seen in Figure 4.4. It is observed that velocity increases with increasing Gr.

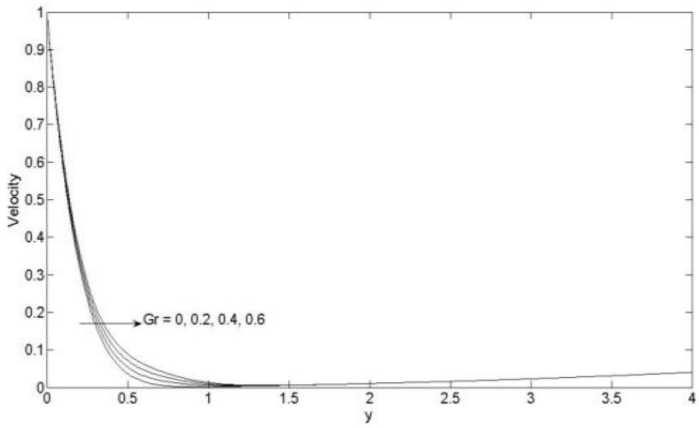

Figure 4.4 Velocity profiles for different values of Gr

The effect of temperature for different values of time (t=0.1, 0.4, 0.8, 1.2) is presented in Figure 4.5. It shows that temperature rises with increasing t

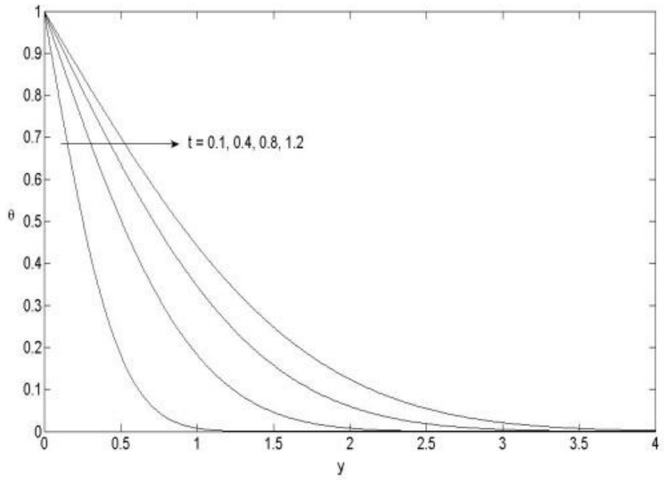

Figure 4.5 Temperature profiles for different values of t

The temperature profiles for different values Prandtl number (Pr= 1.71,1, 0.85, 0.71) is presented in figure 4.6. It is observed that increases in Prandtl number Pr with decreases the temperature.

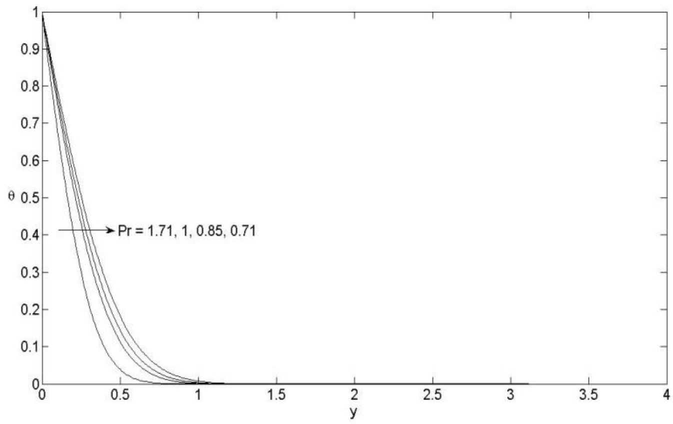

Figure 4.6 Temperature profiles for different values of Pr

The effect of concentration for different values of time (t = 0.1, 0.2, 0.3, 0.4) is presented in Figure 4.7. It is observed that time increases with increase in concentration.

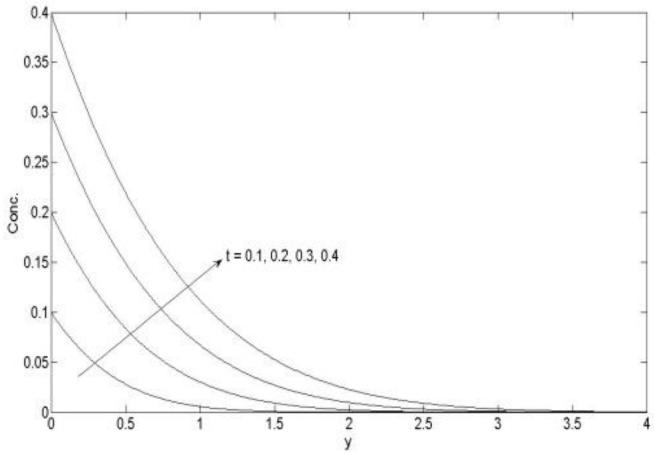

Figure 4.7 Concentration profiles for different values of t

The concentration profiles for different values of Schmidt number (Sc = 0.16, 0.3, 0.6, 2.01) is presented Figure 4.8. It is observed that the concentration increase with decreasing S**c.**

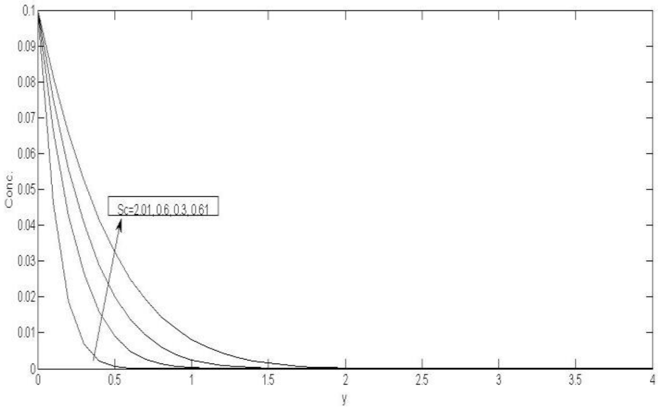

Figure 4.8 Concentration profiles for values of Sc

V. SUMMARY AND CONCLUSION

Analytical solutions of unsteady flow past an accelerated vertical plate with variable temperature and mass transfer have been studied. The dimensional governing equations are solved by Laplace transform technique and computed for different parameters using MATLAB. The effect of different parameters like Schmidt number, prandtl number, mass Grashof number, thermal Grashof number, and time are presented graphically. It is observed that velocity profile increases with increasing parameter like Sc, t, Gc, and Gr. It also observed that temperature increases with increasing t, but increases with decreasing Pr. The concentration profiles is observed also that rise in t increases the concentration profile.

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