ANALYSIS OF WAVELET AND CURVELET IMAGE
DENOISING FOR DIFFERENT KINDS OF ADDITIVE
NOISES

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I. Introduction

Image denoising refers to the recovery of a digital image that has been contaminated by
Additive white Gaussian Noise (AWGN). AWGN is a channel model in which the only impairment to
communication is a linear addition of wideband or white noise with a constant spectral density (expressed as
watts/Hz of bandwidth) and a Gaussian distribution of amplitude. On a daily basis, hospitals are witnessing a
large inflow of digital medical images and related clinical data. The main hindrance is that an image gets often
corrupted by noise in its acquisition and transmission [1]. Image denoising is one of the classical problems in
digital image processing, and has been studied for nearly half a century due to its important role as a pre-
processing step in various electronic imaging applications. Its main aim is to recover the best estimate of the
original image from its noisy versions [2]. Wavelet transform enable us to represent signals with a high degree
of scarcity. This is the principle behind a non-linear wavelet based signal estimation technique known as
wavelet denoising. In this paper we explore wavelet denoising of images using several thresholding techniques
such as SURE SHRINK, VISU SHRINK and BAYES SHRINK. Moreover, the curvelet reconstruction, offering visually sharper images and in particular, higher quality recovery of edges and of
faint linear and curvilinear features. The empirical results reported here are in encouraging agreement.

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In this paper, we also describe approximate of new mathematical transforms, namely as curvelet
transform for image denoising [4] and wavelet transform for image denoising. Our implementations offer exact
reconstruction, stability against perturbations, ease of implementations and low computational complexity. A
central tool is Fourier domain computation of an approximate digital random transform. In a curvelet transform,
we will use sparsity and its applications [5]. In the past, we have proposed a work on novel image denoising
method which is based on DCT basis and sparse representation [6]. To achieve a good performance in these
aspects, a denoising procedure should adopt to image discontinuities. Therefore, a comparative study on
mammographic image denoising technique using wavelet, and curvelet transform [7]. Therefore, multi
resolution analysis [8] is preferred to enhance the image originality. The transform domain denoising typically
assumes that the true image can be well approximated by a linear combination of few basis elements. That is,
the image is sparsely represented in the transform domain. Hence, by preserving the few high magnitude
transform coefficients that convey mostly the original image property and discarding the rest which are mainly due to noise, the original image can be effectively estimated [9]. The sparsity of the representation are critical for compression of images, estimation of images and its inverse problems. A sparse representation for images with geometrical structure depends on both the transform and the original image property.

In the recent years, there has been a fair amount of research on various denoising methods like wavelet, curvelet contourlet and various other multi resolution analysis tools. Expectation - Maximization (EM) algorithm introduced by Figueirodo and Robert [10] for image restoration based on penalized livelihood formulated in wavelet domain. State-of-art Gaussian Scale Mixture (GSM) algorithms employs modelling of images according to the activity within neighbourhoods of wavelet coefficients and attaching coefficients heavily in inactive regions [11]. Coif man and Donoho [12] pioneered in wavelet thresholding pointed out that wavelet algorithm exhibits visual artefacts’. Curvelet transform is a multi scale transform with strong directional character in which elements are highly anisotropic at fine Scales. The developing theory of curvelets predict that, in recovering images which are smooth away from edges, curvelets obtain smaller asymptotic mean square error of reconstruction than wavelet methods [13]. The fundamental quality of curvelet transform is that it can easily converge for high frequency component due to which in curvelet transform we get a better performance as compare to wavelet transform.

MULTIRESOLUTION TECHNIQUES: An image can be represented at different scales by multi resolution analysis. It preserves an image according to certain levels of resolution or blurring in images and also improves the effectiveness of any diagnosis system [14].

A. WAVELET: Wavelet transform can achieve good scarcity for spatially localized details, such as edges and singularities. For typical natural images, most of the wavelet coefficients have very small magnitudes, except for a few large ones that represent high frequency features of the image such as edges. The DWT (Discrete wavelet transforms) is identical to a hierarchical sub band system. In DWT, the original image is transformed into four pieces which is normally labelled as A1,H1,V1 and D1 as the schematic depicted in fig.1. The A1 sub-band called the approximation, can be further decomposed into four sub-bands. The remaining bands are called detailed components. To obtain the next level of decomposition, sub-band A1 is further decomposed.

Figure 1. DWT based Wavelet decomposition to various levels

Many wavelet’s are needed to represent an edge(number depends on the lengh of the edge,not the smoothness).In this,m-term approximation error would be occur.

\[ \left( \sum \left| f - f_m \right| \right)^2 \approx m^p \]
Wavelets and its Geometry: The basis function of wavelets is isotropic. They cannot “adapt” to geometrical structure. In this we need more refined scaling concepts.

B.CURVELET: Curvelets are a non-adaptive technique for multi-scale object representation. Being an extension of the wavelet concepts, they are becoming popular in similar fields, namely in image processing and scientific computing. Curvelet transform is a multi-scale geometric wavelet transforms, can represent edges and curves singularities much more efficiently than traditional wavelet. Curvelet combines multiscale analysis and geometrical ideas to achieve the optimal rate of convergence by simple thresholding. Multi-scale decomposition captures point discontinuities into linear structures. Curvelets in addition to a variable width have a variable length and so a variable anisotropy. The length and width of a curvelet at fine scale due to its directional characteristics is related by the parabolic scaling law:

\[ \text{Width} \times (\text{length})^2 \]

Curvelets partition the frequency plan into dyadic coronae that are sub partitioned into angular wedges displaying the parabolic aspect ratio as shown in fig.2. Curvelets at scale \( 2^k \), are of rapid decay away from a ‘ridge’ of length \( 2^{-k/2} \) and width \( 2^{-k} \) and this ridge is the effective support. The discrete translation of curvelet transform is achieved using wrapping algorithm\[15\]. The curvelet coefficients \( C_k \) for each scale and angle is defined in Fourier domain by

\[ C_k(r, \theta) = 2^{3k/2} R(2^{-k/2}) A(2^{k/2}/2\pi \theta) \]

Where \( C_k \) in this equation represents polar wedge supported by the radial (R) and angular (A) windows.

Figure 2. Curvelets in Frequency Domain
Digital Curvelet Transform can be implemented in two ways (FDCT via USFFT and FDCT via wrapping), which differ by spatial grid used to translate curvelets at each scale and angle.[16].

II. PROPOSED WORK

In this paper, we report initial efforts at image denoising based on a recently introduced family of transforms- Wavelet transform and Curvelet transform. In this paper, we compare the results from wavelet transform and curvelet transform and we will see which transform is better for the image denoising. Our main objective is to decrease a mean square error (MSE) and to increase a peak signal to noise ratio (PSNR) in db. by adding a white noise like Gaussian noise, Poisson noise and Speckle noise. During this configuration, we will use Threshold estimator like heursure, rigrsure, sqtwolog, and minimaxi. We can adjust decomposition level from 1 to 5 and we use Thresholding [17]. Thresholding is the simplest method of image segmentation. From a greyscale image, thresholding can be used to create binary images. Thresholding is a simple non-linear technique, which operates on one wavelet coefficient at a time. In its most basic form, each coefficient is threshold by comparing against threshold. If the coefficient is smaller than threshold, set to zero, otherwise it is kept or modified. On replacing the small noisy coefficients by zero and inverse wavelet transform. In both case (Soft thresholding and Hard thresholding) the coefficients that are below a certain threshold are set to zero. In hard thresholding, the remaining coefficients are left unchanged. In soft thresholding, the magnitudes of the coefficients above threshold are reduced by an amount equal to the value of the threshold. In both cases, each wavelet coefficient is multiplied by a given shrinkage factor, which is a function of the magnitude of the coefficient. In our thesis, we will use a curvelet transform as well as wavelet transform for removing a additive noise which is present in our images.

III. MATERIALS & METHODS

Image from MIAS database was denoised using wavelet and curvelet transforms. Various types of noise like the Random noise, Gaussian noise, Salt&Pepper and speckle noise were added to this image.

A. Algorithm

Denoising procedure followed here is performed by taking wavelet/curvelet transform of the noisy image (Random, Salt and Pepper, Poisson, Speckle and Gaussian noises) and then applying hard thresholding technique to eliminate noisy coefficients. The algorithm is as follows:

Step1: Computation of threshold
Step2: Apply wavelet/curvelet/contourlet transform to image
Step3: Apply computed thresholds on noisy image
Step4: Apply inverse transform on the noisy image to transform image from transform domain to spatial domain.
IV. EXPERIMENTAL RESULTS

The Experiment was done on several natural images like lena, Barbara, baboon, cameraman etc. using multiple denoising procedures for several noises. In our experiment, we have considered a image of A cricketer Mahendra Singh dhoni. In this image we have used a different additive noises like Gaussian noise, poisson noise, and speckle noise with different noise levels $\sigma=10,15,20,25,30,35$ etc. And before adding a noise, mean value is always be 0.

<table>
<thead>
<tr>
<th>NOISES</th>
<th>NOISY IMAGES PSNR/db</th>
<th>WAVELET PSNR/db</th>
<th>CURVELET PSNR/db</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson</td>
<td>27.7344</td>
<td>28.0602</td>
<td>33.8397</td>
</tr>
<tr>
<td>Gaussian</td>
<td>24.9825</td>
<td>26.2889</td>
<td>32.4896</td>
</tr>
<tr>
<td>Speckle</td>
<td>30.2455</td>
<td>32.4944</td>
<td>38.8447</td>
</tr>
<tr>
<td>Salt &amp; pepper</td>
<td>33.2355</td>
<td>34.7823</td>
<td>35.8442</td>
</tr>
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</table>

**TABLE.A:** COMPARISON OF WAVELET AND CURVELET WITH DIFFERENT NOISE IN PSNR.

![Graph indicating comparative results of the PSNR values of wavelet and curvelet based thresholding for image denoising](image)

Table A. shows the comparison of wavelet and curvelet with different noises like poisson noise, Gaussian noise, speckle noise, and salt & pepper noise. and we measures the peak signal to noise ratio in Db and Fig.A shows a graph which indicates a comparative results of the PSNR values of wavelet and curvelet based thresholding (soft/hard) for image denoising and there is, we apply a different types of threshold estimators like rigsure, heursure, sqtwolog, mini-maxi. And different decomposition levels like 1, 2, 3, 4, 5 & so on.

<table>
<thead>
<tr>
<th>NOISES</th>
<th>NOISY IMAGES MSE</th>
<th>WAVELET MSE</th>
<th>CURVELET MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson</td>
<td>109.5562</td>
<td>88.9571</td>
<td>26.8605</td>
</tr>
<tr>
<td>Gaussian</td>
<td>207.5685</td>
<td>152.8252</td>
<td>36.6541</td>
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<tr>
<td>Speckle</td>
<td>61.4507</td>
<td>25.7913</td>
<td>23.3111</td>
</tr>
<tr>
<td>Salt &amp; Pepper</td>
<td>81.4220</td>
<td>22.5612</td>
<td>20.2213</td>
</tr>
</tbody>
</table>

**TABLE.B:** COMPARISON OF WAVELET AND CURVELET WITH DIFFERENT NOISE IN MSE.

![Graph indicating comparative results of the MSE values of wavelet and curvelet based thresholding for image denoising](image)
Table B. shows the comparison of wavelet and curvelet with different noises like poisson noise, Gaussian noise, speckle noise, and salt & pepper noise and we measures the mean square error (MSE) and Fig. B shows a graph which indicates a comparative results of the MSE values of wavelet and curvelet based thresholding (soft/hard) for image denoising and there is, we apply a different types of threshold estimators like rigsure, heursure, sqtwolog, mini-maxi. And different decomposition levels like 1, 2, 3, 4, 5 & so on.

V. CONCLUSION

The comparison of wavelet transform and curvelet transform technique is rather a new approach. The fundamental quantity of curvelet transform is that it can easily and fastly converged for high frequency components. It has a big advantages over the other techniques that it less distorts spectral characteristics of the image denoising. The experimental results show that the curvelet transform gives better results/performance than wavelet transform method. That why the curvelet transform is more efficient or better technique for image denoising for different additive noises.

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