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Biomedical Image Processing With Nonlinear Filters

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ABSTRACT :

Nonlinear filtering techniques are becoming increasingly important in image processing applications, and are often better than linear filters at removing noise without distorting image features. However, design and analysis of nonlinear filters are much more difficult than for linear filters. One structure for designing nonlinear filters is mathematical morphology, which creates filters based on shape and size characteristics. Morphological filters are limited to minimum and maximum operations that introduce bias into images. This precludes the use of morphological filters in applications where accurate estimation of the true gray level is necessary. This work develops two new filtering structures based on mathematical morphology that overcome the limitations of morphological filters while retaining their emphasis on shape. The linear combinations of morphological filters eliminate the bias of the standard filters, while the value-and-criterion filters allow a variety of linear and nonlinear operations to be used in the geometric structure of morphology. One important value-and-criterion filter is the Mean of Least Variance (MLV) filter, which sharpens edges and provides noise smoothing equivalent to linear filtering. To help understand the behavior of the new filters, the deterministic and statistical properties of the filters are derived and compared to the properties of the standard morphological filters. In addition, new analysis techniques for nonlinear filters are introduced that describe the behavior of filters in the presence of rapidly fluctuating signals, impulsive noise, and corners. The corner response analysis is especially informative because it quantifies the degree to which a filter preserves corners of all angles. Examples of the new nonlinear filtering techniques are given for a variety of medical images, including thermographic, magnetic resonance, and ultrasound images. The results of the filter analyses are important in deciding which filter to use for a particular application. For thermography, accurate gray level estimation is required, so linear combinations of morphological operators are appropriate. In magnetic resonance imaging (MRI), noise reduction and contrast enhancement are desired. The MLV filter performs these tasks well on MR images. The new filters perform as well or better than previously established techniques for biomedical image enhancement in these applications.

KEY WORDS: *MLV filter , LOCO filter , pseudomedian , Erosion and the Midrange Filter Averaging Filter , Midrange Filter, (MSE) of filtered noisy signals.*

I. INTRODUCTION

Nonlinear methods in signal and image processing have become increasingly popular over the past thirty years. There are two general families of nonlinear filters: the homomorphic and polynomial filters, and the order statisticand morphological filters [1]. Homomorphic filters were developed during the 1970's and obey a generalization of the superposition principle [2]. The polynomial filters are based on traditional nonlinear system theory and use Volterra series. Analysis and design of homomorphic and polynomial filters resemble traditional methods used for linear systems and filters in many ways. The order statistic and morphological filters, on the other hand, cannot be analyzed efficiently using generalizations of linear techniques. The median filter is an example of an order statistic filter, and is probably the oldest [3, 4] and most widely used order statistic filter. Morphological filters are based on a form of set algebra known as mathematical morphology. Most morphological filters use extreme order statistics (minimum and maximum values) within a filter window, so they are closely related to order statistic filters [5, 6].While homomorphic and polynomial filters are designed and analyzed by the techniques used to define them, order statistic filters are often chosen by more heuristic methods. As a result, the behavior of the median filter and other related filters was poorly understood for many years. In the early 1980's, important results on the statistical behavior of the median filter were presented [7], and a new technique was developed that defined the class of signals invariant to median filtering, the root signals [8, 9]. Morphological filters are derived from a more rigorous mathematical background [10-12], which provides an excellent basis for design but few tools for analysis. Statistical and deterministic analyses for the basic morphological filters were not published until 1987 [5, 6, 13]. The understanding of the filters' behavior achieved by these analyses is not complete, however, so further study may help determine when morphological filters are best applied. This dissertation investigates the use of morphology-based nonlinear filters to enhance biomedical images. Specifically, new filters based on mathematical morphology are developed, analyzed, and applied to a variety of medical images. The behavior of the standard morphological filters is undesirable for certain applications, and the new filters are designed to overcome these weaknesses. Some new analysis techniques are introduced, including a method to quantify the response of filters to two-dimensional features. These new analysis methods and the basic statistical and deterministic analyses are used to compare the new filters with the standard filters. Finally, the new nonlinear filters are used to enhance magnetic resonance, thermographic, and ultrasound images and their performance is compared to established filtering techniques for each of the imaging modalities.

II. ORGANIZATION

This dissertation begins with a review of mathematical morphology, including the statistical and deterministic properties of the morphological filters. These properties point out weaknesses (specifically, a bias problem) in the behavior of the standard morphological filters that motivate the development of new filters. Next, new filters that address the bias problem of the standard filters are introduced. Linear combinations of morphological operators are one of the new types of filters. This work develops the deterministic and statistical 4 proper ies of these filters and illustrates the potential advantages of these filters over the standard morphological filters. Another new type of filter introduced in this work is the value-andcriterion filter. This filter structure uses the shape-based organization of morphology, but expands the operations used for the filtering beyond just the maximum and minimum operators. Thus, any linear or nonlinear function can be used to determine the output value from values in a window, and to determine which window to use to get the output value. A promising application of this new structure is for designing filters that sharpen edges and smooth noise simultaneously. One of these new filters is the "Mean of Least Variance" filter, or MLV filter, which is a significant improvement over previously defined edgepreserving smoothing filters. The deterministic and statistical properties of the MLV filter are also investigated to contrast its behavior with other morphologybased filters. Since the usual statistical and deterministic analyses provide only an incomplete understanding of the behavior of nonlinear filters, new analysis methods are introduced here to gain further insight into the response of the filters.A technique to quantify the response of filters to periodic signals of various frequencies is outlined. This method is similar to Fourier analysis for linear filters, but is much more limited in scope because of the nonlinear nature of the filters examined. Nonetheless, this analysis gives valuable clues about the response of nonlinear filters to rapidly fluctuating signals. Another important property of many nonlinear filters is their resistance to outlying values and impulsive noise. The "breakdown point" is a measure of the robustness of filters in the presence of outliers. This method is another way to help explain differences among filters. The last analysis method developed in this dissertation furthers the understanding of the behavior of filters at two-dimensional structures. This technique, called "corner response analysis," quantifies the percentage of binary corners of various angles that is preserved by a filter. By plotting this information in polar format, the change in the response of a filter to corners of various angles is easily visualized. This method is a major improvement over previous analyses that focused on general characteristics like noise reduction or one-dimensional characteristics like edge preservation. The response of the filter to different rotations of the same feature is also explored using corner response analysis, indicating whether a filter acts similarly to different rotations of 2-D objects. The final portion of this work illustrates the use of the new nonlinear filters in biomedical image processing applications. The results for the various filters yield important information for selecting the proper filter for a given application. Among the considerations for selecting a filter are the signal and noise characteristics of the specific imaging modality and the type of information that is to be extracted from the data. The imaging modalities considered (thermography, magnetic resonance, and ultrasound) have a variety of different characteristics that call for different filters. The theoretical analyses in the earlier sections provide a solid basis for selecting appropriate filters for each modality.

III. MATHEMATICAL MORPHOLOGY

Mathematical morphology is a set algebra used to process and analyze data based on geometric shapes. The theory of mathematical morphology was introduced by Matheron [10] in 1974 and refined by Serra [11, 12] in the 1980's. The basic morphological operations are erosion and dilation. For binary signals, erosion is a Minkowski set subtraction (an intersection of set translations), and dilation is a Minkowski set addition (a union of set translations). These operators were extended to operate on non-binary signals by Serra [11] and others [5, 15, 16].

There are two main types of morphological filters [5]: set processing and function processing filters. Set processing filters accept binary input signals and give binary output signals, while function processing filters accept binary or nonbinary functions as input and yield non-binary functions as output. The interpretation of binary signals as sets and non-binary signals as functions is straightforward; more details are given in [5].

3.1.Basic Morphological Operators

Set Processing Operations.

Let *X* denote an *m*-dimensional set and *N* denote a compact *k*-dimensional Set (*k* <=*m*), and let **y** denote a point in $\Re k$ and **z** a point in $\Re m$. The set *X* is a binary signal or image to be filtered, and the set *N* is called the structuring element of the morphological filter. Define the symmetric set $\tilde{N} = \{-\mathbf{z}: \mathbf{z} \in N\}$, which is a reflection of *N* about the origin. The translation of a set to a point **z** is denoted by a subscript; for example, the set *N* translated to the point **z** is $N\mathbf{z}$. The set processing morphological erosion and dilation are defined by:

Erosion: $$X \ominus \tilde{N} = \{\mathbf{z}: N_{\mathbf{z}} \subseteq X\} = \bigcap_{\mathbf{y}\in N} X_{-\mathbf{y}} \quad (1)$$

Dilation: $$X \oplus \tilde{N} = \{\mathbf{z}: (N_{\mathbf{z}} \cap X) \neq \emptyset\} = \bigcup_{\mathbf{y}\in N} X_{-\mathbf{y}} \quad (2)$$

The symbols $\ominus$ and $\oplus$ denote Minkowski subtraction and Minkowski addition, respectively [11, 12]. The erosion of a set *X* is then the set of points to which the structuring element *N* may be translated while remaining entirely within the original set *X*. The dilation of *X* is the set of points to which *N* may be translated and still intersect *X* with at least one point. Examples of erosion and dilation of some simple discrete sets are shown in Figure 1 below. Clearly, erosion shrinks a set while dilation expands a set.

Figure 1. (a) Erosion example. (b) Dilation example. (Adapted from [5].)

Squares (■) denote origin of the plane; circles (●) denote other points in the set.

In most applications of mathematical morphology, the structuring elements are symmetric about the origin, so that $N = \tilde{N}$. When this is the case, there is no distinction between erosion and Minkowski subtraction nor between dilation and Minkowski addition. Erosion and dilation also are duals of each other with respect to set complementation. If a superscript *c* denotes set

$$X \oplus \tilde{N} = (X^c \ominus \tilde{N})^c \text{ and } X \ominus \tilde{N} = (X^c \oplus \tilde{N})^c.$$

complementation, then

3.2.Function Processing Operations.

Since most signal and image processing applications do not deal with binary data, mathematical morphology must extend to non-binary signals (functions) to be widely useful. This extension is performed by representing a function as an ordered set of binary signals [5, 11]. The cross section of a function at a particular level is a binary set, and the set of all such cross sections forms a complete representation of the function. This process of reducing a function to a set of binary signals is called threshold decomposition. The only restriction on threshold decomposition is that the function must be upper semicontinuous, which means that each cross section of the function must be a closed set. This is not a problem in most applications because all sampled functions are upper semicontinuous [5]. Sternberg [15, 16] used another technique to extend morphology to functions, but the resulting function processing filters are identical to those derived from threshold decomposition. Threshold decomposition is illustrated in Figure 2, which shows three cross sections of a one-dimensional function $f(\mathbf{x})$. Given all the cross sections $X(f, t)$ of a function *f*, the original signal $f(\mathbf{x})$ may be uniquely reconstructed by simply "stacking" the cross sections. The value of the original signal at a location **x** is equal to the highest value of *t* for which the location **x** is included in the set $X(f, t)$. For a quantized signal, there are a finite number of levels (*t*) where the cross sections of the signal are taken.

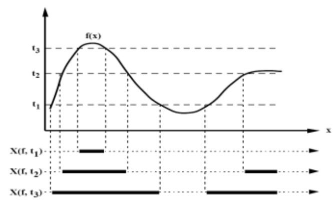

Figure 2. Example of threshold decomposition of a function into cross sections.(Adapted from [5].)

A set processing filter $f(X)$ is said to be "increasing" [5] if for any two sets A and B where $A \subseteq B$, the filtered sets maintain the same set relationship; that is, $f(A) \subseteq f(B)$. This property is also called the "stacking property" [17]. A discrete, binary set processing filter possesses the stacking property if and only if its output can be expressed as a Boolean function that does not contain the complement of any of the input variables [17]. Such expressions are called positive Boolean functions. Set processing filters that are increasing (possess the stacking property) may be converted to function processing filters by performing the set processing operations on the individual cross sections of a signal; the filtered signal is found by stacking the filtered cross sections in the manner described previously. This process of converting the cross sections back into a function is a supremum operation. Examples of filters that are increasing and therefore may operate on a threshold decomposition of a signal are all order statistic filters [18, 19], including the median filter, and the morphological filters. Function processing filters that operate in this manner on the cross sections of a function are called stack filters [6, 17, 20]. Note that not all function processing filters obey the stacking property; those that do are part of a subset of function processing filters called function and set processing (FSP) filters [5, 13]. FSP filters are useful because they can accept either sets or functions as input, and give the same type of output as the input they receive. The set processing morphological filters may be converted to a function processing operation by this stacking property. The resulting grayscale morphological filters are a subset of the stack filters. All stack filters are FSP filters, and so the morphological filters that use set structuring elements are FSP filters. The resulting expressions for the FSP morphological filters are [5, 13]:

$$\text{Erosion:} \qquad (f \ominus \tilde{N})(\mathbf{z}) = \inf\{f(\mathbf{y}) : \mathbf{y} \in N_{\mathbf{x}}\} \qquad (3)$$

$$\text{Dilation:} \qquad (f \oplus \tilde{N})(\mathbf{z}) = \sup\{f(\mathbf{y}) : \mathbf{y} \in N_{\mathbf{x}}\} \qquad (4)$$

where $f(\mathbf{y})$ denotes an m-dimensional upper semicontinuous function, N denotes a compact k-dimensional set ($k <= m$), and $\mathbf{y}$ and $\mathbf{z}$ denote points in $\Re k$ and $\Re m$, respectively. The infimum (inf) and supremum (sup) operations reduce to simple minimum and maximum operations, respectively, when acting on discrete signals.The structuring element of a morphological filter does not have to be a set. Like the signal the filter operates on, the structuring element may be a function. In this case, the output of the morphological filter is always a function, so such filters are not FSP, but are function processing. Instead of the structuring element set N, the structuring element is a k dimensional upper semicontinuous function $g(\mathbf{z})$ that is defined over a compact region of support, S. As for the set structuring element case, the erosion and dilation are defined as the Minkowski subtraction and addition of the signal with the reflection of the structuring element about the origin. Let $\tilde{g}(\mathbf{z}) = g(-\mathbf{z})$ denote this reflection. The morphological erosion and dilation of a function by a function are given by:

$$\text{Erosion:} \qquad (f \ominus \tilde{g})(\mathbf{z}) = \inf\{(f(\mathbf{z}+\mathbf{y}) - g(\mathbf{y})) : \mathbf{y} \in S\} \qquad (5)$$

$$\text{Dilation:} \qquad (f \oplus \tilde{g})(\mathbf{z}) = \sup\{(f(\mathbf{z}+\mathbf{y}) + g(\mathbf{y})) : \mathbf{y} \in S\} \qquad (6)$$

Note that if the function structuring element $g(\mathbf{z})$ is zero over its entire region of support S, then the above expressions are equivalent to the FSP expressions with structuring element $N = S$. Set structuring elements are far more widely used than function structuring elements in applications of mathematical morphology. In theory, however, function structuring elements are a significant addition to morphological filtering because they are able to process signals and image based on a specific intensity profile over a certain shape. Set structuring elements assume a flat intensity profile over their shapes.

3.3.Compound Morphological Operators

Erosion and dilation are complementary operations, one shrinking the size of objects in an image and the other expanding them. However, erosion and dilation are not inverses of each other. Some objects are completely removed by erosion, and therefore cannot be restored by dilation. Likewise, dilation often joins nearby objects which erosion cannot then separate. The compound morphological operations formed by performing the complementary operators in sequence are the morphological operations "opening" and "closing." Opening is defined as erosion followed by dilation, while closing is dilation followed by erosion. The structuring element used for the second operation is the reflection about the origin of the structuring element used for the first operation. The specific expressions for opening (denoted by a subscript) and closing (denoted by a superscript) for a set structuring element N acting on a function $f(\mathbf{z})$ are:

$$\text{Opening:} \qquad f_N(\mathbf{z}) = [\,(f \ominus \tilde{N}) \oplus N\,](\mathbf{z}) \qquad (7)$$

$$\text{Closing:} \qquad f^N(\mathbf{z}) = [\,(f \oplus \tilde{N}) \ominus N\,](\mathbf{z}) \qquad (8)$$

The structuring element rotation between the erosion and dilation creates an “effective” structuring element for opening and closing that is symmetric about the origin, even if the original structuring element is asymmetric. This is illustrated in Figure 3 below. In most applications, this rotation has no practical effect since the original structuring element is usually symmetric.

Figure 3. Illustration of an “effective” structuring element of opening and closing formed by rotation between successive erosions and dilations. Squares (■) denote the origin of the plane; circles (●) denote other points in the set. The opening and closing operations are also complementary, and when applied in sequence, they form the doubly compound morphological operators open-close (OC) and close-open (CO):

Open-Closing: $$OC(f; N) = (f_N)^N \quad (9)$$

Close-Opening: $$CO(f; N) = (f^N)_N \quad (10)$$

IV. RESULT

Figures 4–7 below illustrate the operation of the simple and compound morphological operators on a one dimensional signal (frequency modulated in this example). For finite-length signals, there is a problem defining the output of filters near the ends of the signal; in the following examples, the output near the ends is found by“padding” the signal at either end with as many repetitions of the first or last value in the signal as necessary to define the output. Figure .4 demonstrates the effect of erosion and opening. Notice especially how opening preserves“negative” features of the signal, but cuts off positive impulses narrower than the structuring element. Figure .5 compares dilation and closing of the same signal; the effect is exactly the opposite. Closing preserves positive features and cuts off negative impulses. The effects of cascaded operators, open-closing (OC) and close-opening (CO), are shown in Figures 6 and 7 respectively. Notice that as the signal frequency increases, the close-opening tends toward flat regions with high values, whereas the open-closing yields flat regions with low values.

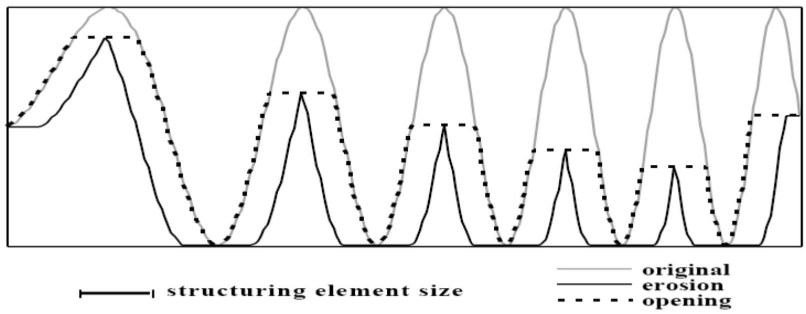

Figure 4. Effect of erosion and opening on a 1-D frequency-modulated signal.

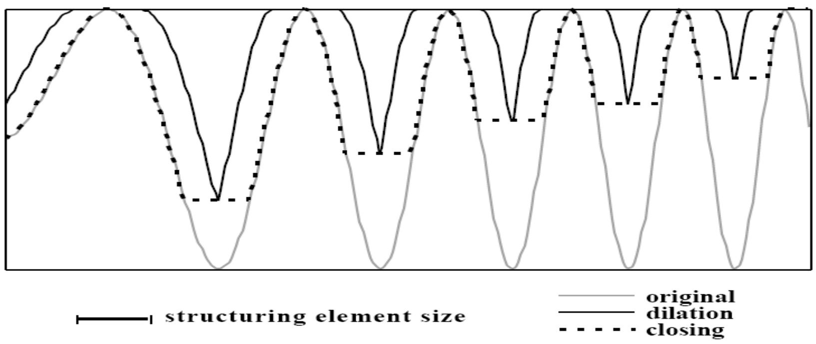

Figure 5. Effect of dilation and closing on a 1-D frequency-modulated signal.

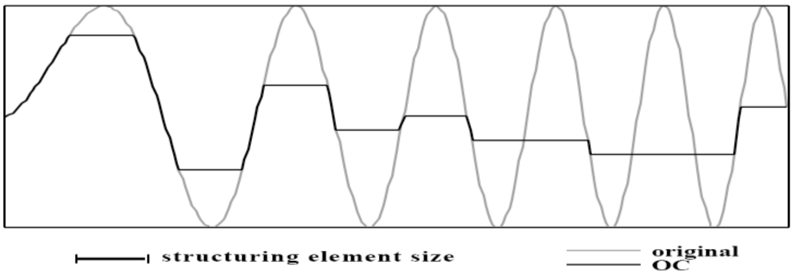

Figure 6. Effect of open-closing (OC) on a 1-D frequency-modulated signal.

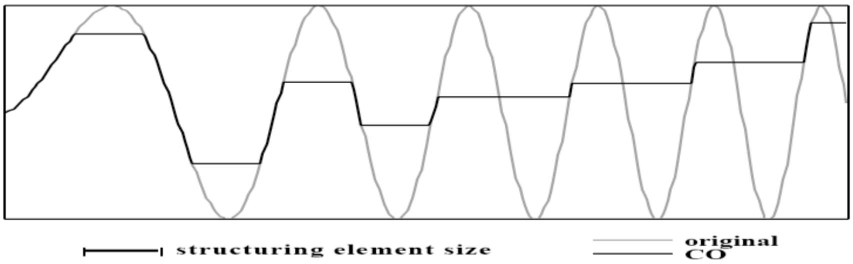

Figure 7. Effect of close-opening (CO) on a 1-D frequency-modulated signal.

Figures 8–14 below illustrate the effect of the morphological filters in two dimensions with a square structuring element. Figure 8 is the original 110 x 110 pixel binary image of several characters. In the applications given in Chapter 5, image features are considered to have high gray level values and background to have low values; this means that dilation expands image features and erosion shrinks them. However, in the binary examples given below, the characters (features) are black, and so black will be considered to take on the value 1 and white the value 0. This is inverted from the convention of 0 for black and maximum range (often 255) for white which will be used later in this work. Figures 9 and 10 show the erosion and dilation, respectively, of the original image in Figure 2.8 using a 5 x 5 square structuring element. Note that the thin parts of the characters disappear in the eroded image, while nearby portions of adjacent characters are merged in the dilated image. The opening and closing of the image with a 5 x 5 structuring element are illustrated in Figures 11 and 12, respectively. The opening, which expands the characters back from what was left after erosion, still shows many gaps where thin features were removed. The closing, however, preserves the thin features of the characters, but partially closes some of the enclosed white areas and leaves many of the characters touching. The doubly compound operators OC and CO, shown in Figures 13 and 14 respectively, are quite similar in appearance. There are only a few small differences between the OC and CO results, all of which are related to sharp corners in the original image (such as the angles in the “4”). One important trait to notice in Figures 2.9–2.14 is the effect of the structuring element shape on the filtered images. In every case, the square shape of the structuring element has a noticeable effect, preserving straight edges and 90° corners and converting other features to a “more square” appearance. The fact that the structuring element shape greatly influences the output of the filter is one of the most important features of mathematical morphology.

Figure .8. Original binary image.

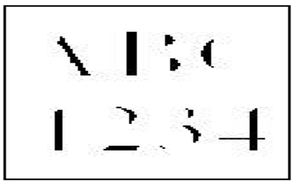

Figure 9. Image after erosion

Figure 10. Image after dilation.

Figure 11**.** Image after opening.

Figure 12**.** Image after closing.

Figure 13**.** Image after OC.

Figure 14. Image after CO.

V. CONCLUSION

This topic has demonstrated the utility of nonlinear filters for the enhancement of medical images, including thermography, MRI, and ultrasound. Morphology-based filters allow processing by shape and size, which can be particularly useful for feature extraction and segmentation problems in medical image analysis. For MRI, the desired processing usually includes contrast enhancement and noise reduction. These enhancements simplify the segmentation process on MR images. The morphology-based MLV filter, which is one of the new filters in the value-and-criterion structure introduced in topic 3, does an excellent job of sharpening edges and reducing noise in MR images. The MLV filter yields excellent results in only one pass, as opposed to anisotropic diffusion, which requires many iterations. The MLV filter is also computationally more efficient than similar edge-preserving smoothing filters. This work extends the theory of nonlinear image processing by introducing new filter structures and analysis methods, and demonstrates the utility of these new techniques on a variety of medical images. The filters are based on generalizations of mathematical morphology, which is itself a relatively recent development in image processing. One of these generalizations is the class of linear combinations of morphological operators. This filter class includes the previously defined midrange and pseudomedian filters and leads to the definition of the LOCO filter. The deterministic properties of the linear combinations are similar to those of the constituent morphological filters; however, the linear combinations are statistically unbiased, unlike the conventional morphological operators. This is important in applications like thermography, where the LOCO filter allows shape-based filtering as in standard morphology without introducing a statistical bias. The other new class of filters introduced here is the value-and-criterion filter structure. This structure is based on morphological opening and closing, but allows the use of both linear and nonlinear operators within the window structure. One useful filter designed with this structure is the Mean of Least Variance (MLV) filter, which takes the mean of the window with the smallest variance within a set of windows defined by the morphological structure. This filter reduces noise and enhances edges in images. Its statistical and deterministic properties resemble those of the averaging filter more than the morphological filters, with several important differences. The MLV filter is valuable in applications like MRI, where noise smoothing and contrast enhancement are the primary goals.

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