Reliability of Thermal Stresses in Bars When Stress Follows Half-Logistic Distribution

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Abstract
This paper deals with the reliability of Thermal stresses in simples bars whose body may be expand or contract due to some increase or decrease in the temperature of the body when stress follows the Half-logistic distribution . For such thermal stresses in simple bars we are particularly interested in investigating the reliability by using the Half-logistic distribution function .It is also compared the results between the reliability of thermal stresses in bars if the ends of the bars are fixed to rigid supports and if the supports yield by an amount equal to Δ.it is observed that the reliability is depending on the coefficient of linear expansion $\propto$.This coefficient of linear expansion is different for each material.

Keywords: Thermal stresses, Temperature, Reliability, Coefficient of linear expansion, Simple bars,Half- logistic distribution, Hazard Rate.

1. Introduction:
Whenever there is some increase or decrease in the temperature of a body, it causes the body to expand or contract. A little consideration will show that if the body is allowed to expand or contract freely, with the rise or fall of the temperature, no stresses are induced in the body. But if the deformation of the body is prevented, some stresses are induced in the body. Such stresses are called thermal stresses or temperature stresses. The corresponding strains are called the thermal strains or temperature strains. In probability theory and statistics, the half-logistic distribution is a continuous probability distribution. The distribution of the absolute value of a random variable following the logistic distribution.Reliability is used for developing the equipment manufacturing and delivery to the user. A reliable system is one which operates according to our expectations. Reliability of a system is the probability that a system perform its intended purpose for a given period of time under stated environment conditions. In some cases system failures occur due to certain type of stresses acting on them. These types of system are called stress dependent models of reliability. These models nowadays studied in many branches of science such as Engineering, Medicine, and Pharmaceutical Industries etc [1]. In assessing system reliability it is first necessary to define and categorize different modes of system failures. It is difficult to define failure in unambiguous forms. However a system's performance can deteriorate gradually over time and sometimes there is only a fine line between systems success and system failure. Once the system function and failure modes are explicitly stated reliability can be precisely quantified by probability statements.

2. Statistical Methodology:
The probability of failure as a function of time can be defined by

$$F(t) = P(T \leq t), \quad t \geq 0 \tag{1.1}$$

Where T is a random variable denoting the failure time. **Reliability** function is defined as the probability of success for the intended time t

$$R(t) = 1 - F(t) = P(T > t) \tag{1.2}$$

The Hazard function $h(t)$ is defined as the limit of the failure rate as the interval approaches zero. Thus the hazard function is the instantaneous failure rates is defined as

$$z(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1 - F(t)} \qquad where\ f(t) = \frac{dF(t)}{dt} \tag{1.3}$$

2.1. Stress Dependent Hazard Models: Basically, the reliability of an item is defined under stated operating and nvironmental conditions. This implies that any change in these conditions can effect. The failure rate of almost all components is stress dependent. A component can be influenced by more than one kind of stress. For such cases, a power function model of the form [2]

$$h(t) = z(t)\sigma_1{}^a\sigma_2{}^b \tag{1.4}$$

where a, b are positive constants, σ_1 and σ_2 are stress ratios for two different kinds of stresses, and $z(t)$ is the failure rate at rated stress conditions.

3. Thermal Stresses In Simple Bars:

The thermal stresses or strains bar may be found out as discussed below:
The thermal stresses or strains may be found out first by finding out amount of deformation due change in temperature, and then by finding out thermal strain due to the deformation. The thermal stress may now be found out from the thermal strain as usual. Now consider a body subjected to an increase in temperature [3].

Let l=original length of the body,
θ=increase of temperature and
α=coefficient of linear expansion
E=modulus of elasticity (young's modulus)

We know that the increase in length due to increase of temperature

$$\delta l = l.\alpha.\theta, \qquad (1.5)$$

If the ends of the bar are fixed to rigid supports, so that its expansion is prevented, then compressive strain induced in the bar.

$$\varepsilon = \frac{\delta l}{l} = l.\alpha.\frac{\theta}{l} = \alpha.\theta \qquad (1.6)$$

Stress $\boldsymbol{\sigma = \varepsilon.E = \alpha.E.\theta}$ (1.7)

If the supports yield by an amount equal to Δ,then the actual expansion that has taken place,

$$\delta l = l.\alpha.\theta - \Delta \qquad (1.8)$$

And strain, $\varepsilon = \frac{\delta l}{l} = \frac{l.\alpha.\theta - \Delta}{l} = (\alpha\theta - \frac{\Delta}{l})$ (1.9)

Stress, $\boldsymbol{\sigma = \varepsilon.E = (\alpha\theta - \frac{\Delta}{l})E}$

3.1. The value of a coefficient of linear expansion of materials in day use is given below in table:

S.NO	MATERIAL	COEFICIENT OF LINEAR EXPANSION/^{0}C(α)
1	STEEL	11.5×10^{-6} *to* 13×10^{-6}
2	WROGHT IRON,CAST IRON	11×10^{-6} *to* 12×10^{-6}
3	ALUMINIUM	23×10^{-6} *to* 24×10^{-6}
4	COPPER,BRASS,BRONZE	17×10^{-6} *to* 18×10^{-6}

3.2. When Stress Follows half -logistic Distribution:

A continuous Random variable X having the probability density function

$$f(x) = \frac{2e^{-x}}{(1+e^{-x})^2}, \quad x \geq 0 \qquad (1.10)$$

is said to have Half Logistic Distribution.

Then $F(t) = \frac{2}{1+e^{-t}} - 1$

Hazard Rate is $Z(t) = \frac{f(t)}{1-F(t)} = \frac{1}{1+e^{-t}}$ (1.11)

3.2.1. When σ=stress in the bar if the ends of the bar are fixed to rigid supports:

Stress in the bar $\sigma = \alpha.\theta.E$ (1.12)

Therefore Reliability of the bar $R(t) = exp\left[-\int_0^t h(t)dt\right] = exp\left[-\int_0^t (z(t)\times\sigma)dt\right]$

$$= exp\left[-\int_0^t \frac{1}{1+e^{-t}} \times \alpha.\theta.Edt\right] = exp\left[-\alpha.\theta.E \times \int_0^t \frac{e^t}{1+e^t}dt\right] \qquad (1.13)$$

$$\text{Put } e^t + 1 = s$$

$$e^t dt = ds$$

$$= exp\left[-\alpha.\theta.t\int_0^t \frac{ds}{s}\right] == exp[-\alpha.\theta.E \times [\log(s)]_2^t] = exp[-\alpha.\theta.E \times (\log(e^t+1) - \log(2))]$$

$$= exp[-\alpha.\theta.E\times(\log(\frac{e^\wedge t+1}{2})] = exp[-\alpha.\theta.E\times(\log(\frac{e^\wedge t+1}{2})] = exp[\alpha.\theta.E\times(\log(\frac{2}{e^t+1})]$$

$$= \exp\ \log(2/(e^\wedge t+1))^{\alpha.\theta.E} = (2/(e^\wedge t+1))^{\alpha.\theta.E}$$

3.2.2. When σ=stress in the bar if the supports yield by an amount equal to Δ:

Stress in the bar $\sigma = (\alpha\theta - \frac{\Delta}{l})E$ (1.14)

Therefore Reliability of the bar R(t)= $exp\left[-\int_0^t h(t)dt\right] = exp\left[-\int_0^t (z(t)\times\sigma)dt\right]$

$$= exp\left[-\int_0^t \frac{1}{1+e^{-t}} \times (\alpha.\theta - \frac{\Delta}{l})Edt\right] = exp\left[-(\alpha.\theta - \frac{\Delta}{l})E \times \int_0^t \frac{e^t}{1+e^t}dt\right]$$

$$\text{Put } e^t + 1 = s$$

$$e^t dt = ds$$

$$= exp\left[-(\alpha.\theta - \frac{\Delta}{l})E\int_0^t \frac{ds}{s}\right] == exp\left[-(\alpha.\theta - \frac{\Delta}{l})E \times [\log(s)]_2^t\right]$$

$$= exp\left[-(\alpha.\theta - \frac{\Delta}{l})E \times (\log(e^t+1) - \log(2))\right] = exp[-(\alpha.\theta - \frac{\Delta}{l})E \times (\log(\frac{e^\wedge t+1}{2})]$$

$$= exp[-(\alpha.\theta - \frac{\Delta}{l})E \times (\log(\frac{e^\wedge t+1}{2})] = exp[(\alpha.\theta - \frac{\Delta}{l})E \times (\log(\frac{2}{e^t+1})]$$

$$= exp[(\log(2/(e^\wedge t+1))].^{(\alpha.\theta - \frac{\Delta}{l})E}$$

$$= (2/(e^\wedge t+1))^{(\alpha.\theta - \frac{\Delta}{l})E} \quad (1.15)$$

3.3. Reliability computations for simple bars when σ=Stress in the rod if the ends do not yield:

Table-1[Aluminium alloy]

when $\alpha=24\times 10^{-6}$				
t	θ	E	σ	R(t)
0.01	20	80000	38.4	0.824910818
0.02	20	80000	38.4	0.679824931
0.03	20	80000	38.4	0.559719318
0.04	20	80000	38.4	0.460390845
0.05	20	80000	38.4	0.378326103
0.06	20	80000	38.4	0.310591291
0.07	20	80000	38.4	0.25473915
0.08	20	80000	38.4	0.2087304
0.09	20	80000	38.4	0.170867499
0.1	20	80000	38.4	0.139738842

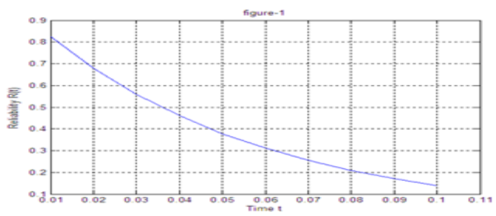
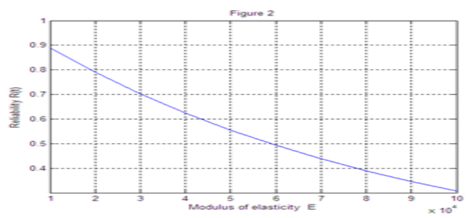


Table-2[Steel]

when t=0.1				
α	θ	E	σ	R(t)
11.5×10^{-6}	20	10000	2.3	0.888808211
11.5×10^{-6}	20	20000	4.6	0.789980035
11.5×10^{-6}	20	30000	6.9	0.702140741
11.5×10^{-6}	20	40000	9.2	0.624068456
11.5×10^{-6}	20	50000	11.5	0.554677168
11.5×10^{-6}	20	60000	13.8	0.493001621
11.5×10^{-6}	20	70000	16.1	0.438183888
11.5×10^{-6}	20	80000	18.4	0.389461438
11.5×10^{-6}	20	90000	20.7	0.346156524
11.5×10^{-6}	20	100000	23	0.30766676



3.4.Reliability Computations For Simple Bars When σ=Stress In The Rod If The Ends Yield By An Amount Equal

Table-4[Iron]

when $\alpha=12\times 10^{-6}$, l=6000, Δ=1				
t	θ	E	σ	R(t)
0.01	40	200000	62.7	0.730311775
0.015	40	200000	62.7	0.623745142
0.020	40	200000	62.7	0.532519934
0.025	40	200000	62.7	0.454458668
0.030	40	200000	62.7	0.387688349
0.035	40	200000	62.7	0.330598561
0.040	40	200000	62.7	0.281805218
0.045	40	200000	62.7	0.240119251
0.050	40	200000	62.7	0.20451955
0.055	40	200000	62.7	0.174129596

Table-5[Iron]

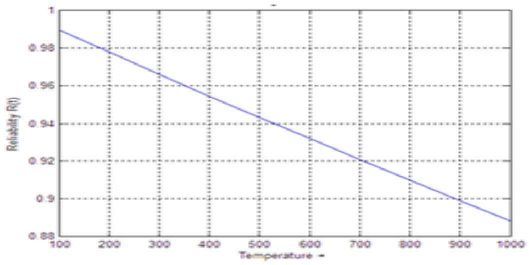
when t=.001, Δ=1, l=6000				
α	θ	E	σ	R(t)
12×10^{-6}	100	200000	207	0.901652692
12×10^{-6}	200	200000	447	0.799670209
12×10^{-6}	300	200000	687	0.709222574
12×10^{-6}	400	200000	927	0.629005125
12×10^{-6}	500	200000	1170	0.557024391
12×10^{-6}	600	200000	1410	0.494021495
12×10^{-6}	700	200000	1650	0.438144616
12×10^{-6}	800	200000	1890	0.388587756
12×10^{-6}	900	200000	2130	0.344636083
12×10^{-6}	1000	200000	2370	0.305655616

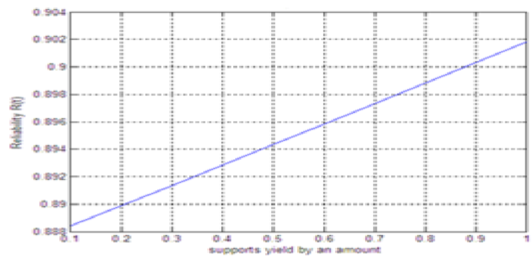
Table-6[Iron]

when $t = 0.001$, $\alpha=12\times 10^{-6}$, l=6000				
Δ	θ	E	σ	R(t)
1	100	200000	207	0.901652692
0.9	100	200000	210	0.900300889
0.8	100	200000	213	0.898951113
0.7	100	200000	217	0.897154559
0.6	100	200000	220	0.8958095
0.5	100	200000	223	0.894466458
0.4	100	200000	227	0.892678866
0.3	100	200000	230	0.891340517
0.2	100	200000	233	0.890004175
0.1	100	200000	237	0.888225502

Table-7[Copper]

when t=0.1, Δ=1, 1$\alpha=17\times 10^{-6}$				
l	θ	E	σ	R(t)
10000	40	200000	116	0.559087106
20000	40	200000	126	0.531753633
30000	40	200000	129	0.523817209
40000	40	200000	131	0.518592176
50000	40	200000	132	0.515999237
60000	40	200000	133	0.513419262
70000	40	200000	133	0.513419262
80000	40	200000	134	0.510852187
90000	40	200000	134	0.510852187
100000	40	200000	134	0.510852187





4. Conclusions:

Reliability of the Thermal stresses or strains are found out first by finding out amount of deformation due to change in temperature, and then by finding out thermal strain due to the deformation. Reliability computations are obtained for simple bars when stress (σ)in the Rod i) if the ends do not yield and ii) if the ends yield by an equal to amount Δ for various Materials. When the temperature increases Reliability decreases. Also when the amount Δ is increases, Reliability increases. It is also compared the results between the reliability of thermal stresses in bars if the ends of the bars are fixed to rigid supports and if the supports yield by an amount equal to Δ.It is observed that the reliability is depending on the coefficient of linear expansion $\propto$.This coefficient of linear expansion is different for each material. Also the Reliability is depending on Time in this paper.

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