A New Geometric Method to Plotting a Family of Functions Logarithm

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Abstract:

In this paper, from the study of the family of logarithmic function, we derive a new method to construct the curves: $y = kx+ln(x), k \in IR$. This method will be a new learning situation to present the logarithm function at high school.

Keywords: Algorithm, Family of functions, Function, Logarithm, Register

1. Introduction

The visualization is very important in the teaching of analysis [8]. The notion of function as an object of analysis can intervene with many frames [4] and it is related to other objects (real numbers, numerical sequences ...). This concept also requires the use of multiple registers [5], that are, algebraic Register (representation by formulas); numerical register (table of values), graphical register (curves); symbolic register (table of variations); formal register (notation *f, f (x), fog ...)* and geometrical register (geometrical variables).In addition, Balacheff and Garden [1] have founded two types of image conception among pairs of students at high school, that are, conception curve-algebraic, i.e., functions are seen as particular cases of curves, and a conception algebraic-curve, i.e., functions are first algebraic formulas and will be translated into a curve.The authors Coppe et al. [3] showed that students had more difficulties to translate the table of variations from one function to a graphical representation which shows that students have difficulties to adopt a global point of view about the functions. They have also shown that the algebraic register is predominant in textbooks of the final year at high school. They also noted that the study of functions is based on the algebraic calculation at the final year in high school (limits, derivatives, study of variations...). According to Raftopoulos and Portides [7], the graphical representations make use of point of global and punctual point of view of functions; on the contrary, the properties of the functions are not directly visible from the algebraic formulas. Bloch [2] highlighted that students rarely consider the power of the graphics at the global level and propose teaching sequences supported by a global point of view of the graphical register. The students do not know how to manipulate the functions that are not given by their algebraic representations. And they do not have the opportunity to manipulate the families of functions depending on a parameter.

To study the logarithm three methods are available:

a- From the properties of exponential functions.

b- Put the problem of derivable functions on IR^{+*} such as $f(xy) = f(x)+f(y)$ and admit the existence of primitive for the function $x \longrightarrow 1/x \ (x \neq 0)$.

c- Treat the log after the integration. In this paper, we propose a new method of tracing the family of functions $f_k(x) = kx + ln(x),\ k \in IR$, without going through the study of functions (boundary limits, table of variations, infinite branches ...) based only on algorithms for tracing tangents.

This method will be used in particular to plot the curve of the logarithm function and the student can from the graphical representation find the properties such as domain of definition, limits, monotony… etc. The idea of this new method is based on our work presented in [6].

2. Description of the method

Let f_k be a family of functions defined the interval $]0, +\infty[$ by

$$f_k(x) = kx + ln(x)$$

With $k \in IR$. The function f_k is strictly increasing on $]0, +\infty[$ when $k \geq 0$. If $k < 0$, then f_k is strictly increasing on $]0, \frac{-1}{k}[$ and is strictly decreasing on $]\frac{-1}{k}, +\infty[$. Moreover, the line $y = kx$ is an asymptotic direction of graph of f_k, and the equation of the tangent at any arbitrary point x_0 is

$$y = (k + \frac{1}{x_o})x - 1 + \ln(x_o)$$

2.1. Tangents of the family of functions f_k at $x_0 = \frac{-1}{k}$

The equation of the tangent at point $x_0 = \frac{-1}{k}$ is

$$y = -1 - \ln(-k)$$

Then, this tangent is line parallel to the axis *(OX)* passing through the point $N(\frac{-1}{k}, -1-\ln(-k))$ that is the intersection of the line $x = \frac{-1}{k}$ and the curve $y = -1 + \ln(x)$.

Hence, we obtain the following algorithm to plot geometrically the tangent at $x_0 = \frac{-1}{k}$

<u>Algorithm:</u>

- Plot the line $y = kx$ and the line $y = -1$;
- Mark the point M which is the intersection of the last two lines;
- Plot the parallel to *(OY)* passing through the point M;
- Mark the point N which is the intersection of this parallel with the curve $y = -1 + \ln(x)$.

Note that if $k < 0$, then the tangent at $x_0 = \frac{-1}{k}$ is the line passing through the point N and the parallel to *(OX)* (see figure 1), and the function f_k admits an extremum at the point N. However, if $k \geq 0$ we have the intersection of the parallel to *(OY)* passing through the point M with the curve $y = -1 + \ln(x)$ is empty, and the function f_k admits no extremum.

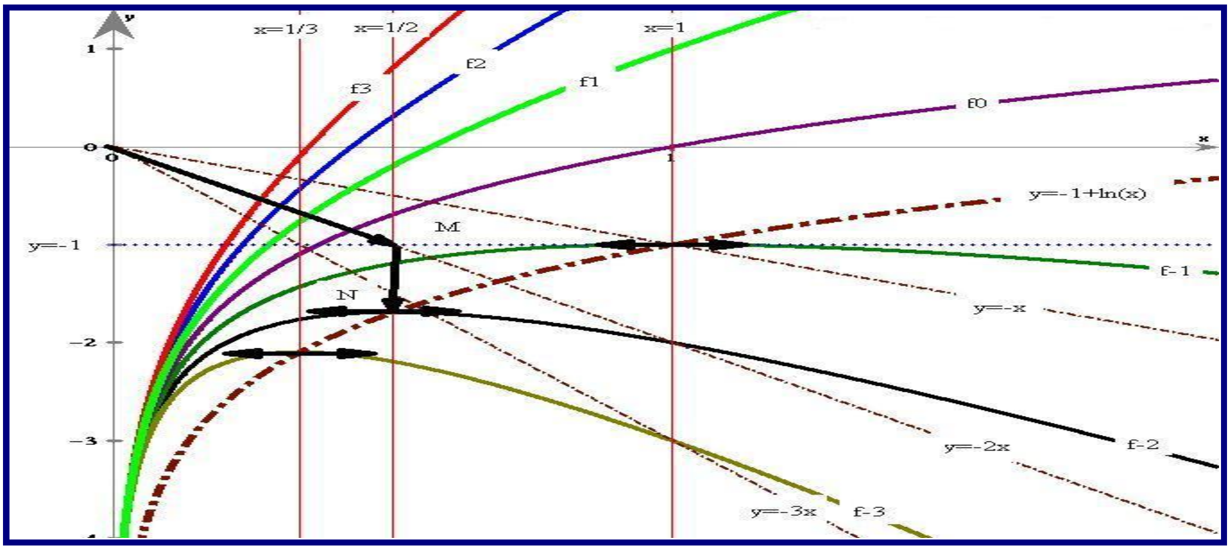

Figure 1. Tangents of the family $f_k(x) = kx + \ln(x)$ at $x_0 = \frac{-1}{k}$

2.2. Tangents of the family of functions f_k at $x_0 = 1$

The equation of the tangent at point $x_0 = 1$ is

$$y = (k + 1)x - 1$$

Then, this tangent is line passing through the point $M(1, k)$ and the point $N(0, -1)$.

Hence, we obtain the following algorithm to plot geometrically the tangent at $x_0 = 1$

<u>Algorithm:</u>

- Plot the line $y = kx$ and the line $x = 1$;
- Mark the point M which is the intersection of the last two lines.

In this case, the tangent at $x_0 = 1$ is the line *(MN)* (see figure 2).

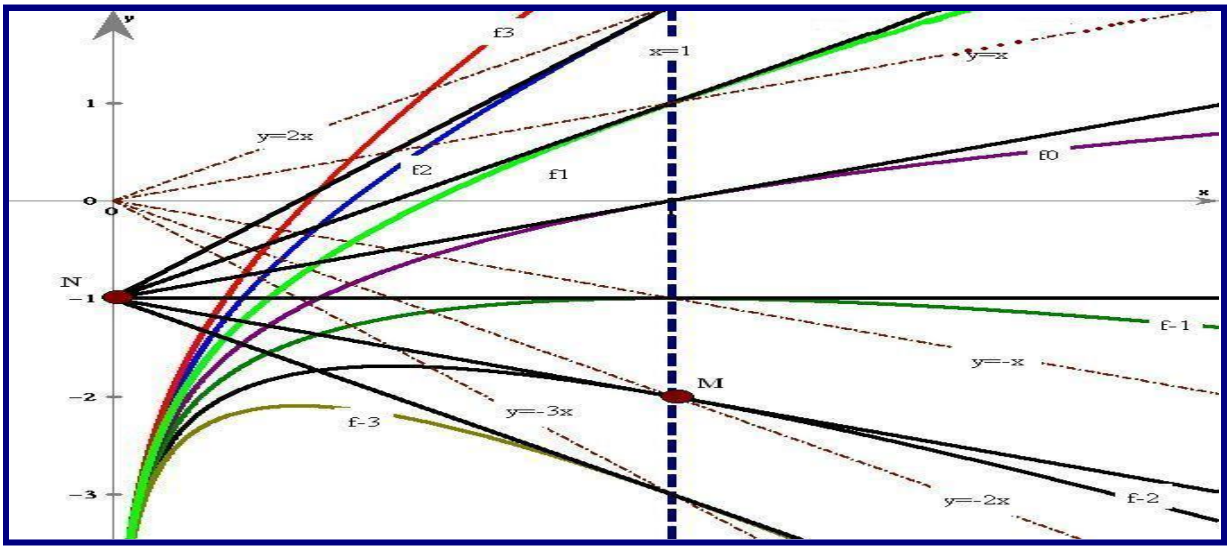

Figure 2. Tangents of the family $f_k(x) = kx + ln(x)$ at $x_0 = 1$

2.3 Tangents of the family of functions f_k at $x_0 = e$

The equation of the tangent at point $x_0 = e$ is

$$y = (k + \frac{1}{e})x$$

Then, this tangent is line passing through the point $N\,(e, ke + 1)$ and the point $O\,(0, 0)$.
Hence, we obtain the following algorithm to plot geometrically the tangent at $x_0 = e$

Algorithm:

- Plot the line y = kx and the line x = e;
- Mark the point M which is the intersection of the last two lines;
- Plot the point N which is image of the point M by the translation of vector $t\,\vec{j}$.

In this case, the tangent at $x_0 = e$ is the line (ON) (see figure 3).

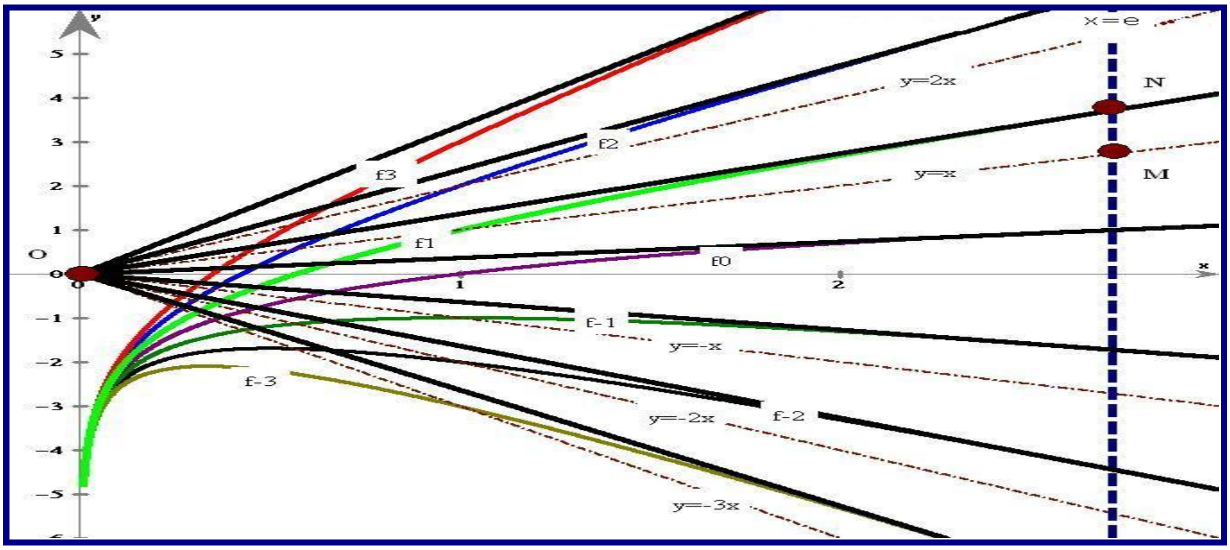

Figure 3. Tangents of the family $f_k(x) = kx + ln(x)$ at $x_0 = e$

2.4 Tangents of the family of functions f_k at $x_0 = \frac{1}{e}$

The equation of the tangent at point $x_0 = \frac{1}{e}$ is

$$y = (k + e)x - 2$$

Then, this tangent is line passing through the point $N(\frac{1}{e}, \frac{k}{e} - 1)$ and the point $P(0, -2)$.

Hence, we obtain the following algorithm to plot geometrically the tangent at $x_0 = \frac{1}{e}$

Algorithm:

- Plot the line $y = kx$ and the line $x = \frac{1}{e}$;
- Mark the point M which is the intersection of the last two lines;
- Plot the point N which is image of the point M by the translation of vector $t - \vec{j}$.

In this case, the tangent at $x_0 = \frac{1}{e}$ is the line (PN) (see figure 4).

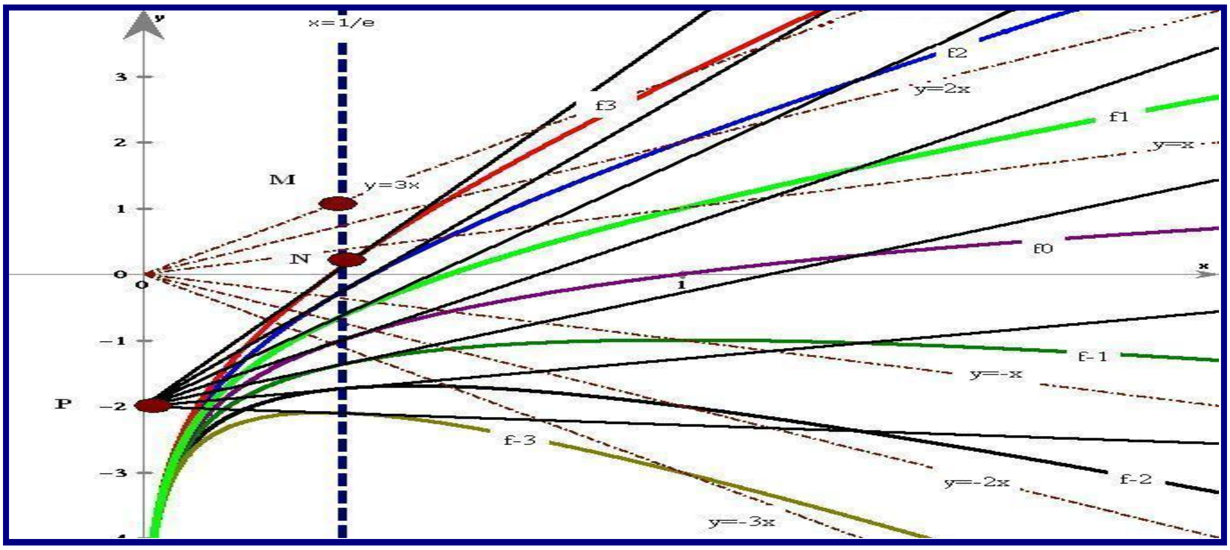

Figure 4. Tangents of the family $f_k(x) = kx + ln(x)$ at $x_0 = \frac{1}{e}$

2.5 Tracing the curves $y = kx + ln(x)$ from bundles of tangents

In this new learning situation, we give to student the algorithms of the tangents of a family of functions f_k , $k \in IR$, at the following points $x_0 = \frac{-1}{k}$, $x_0 = 1$, $x_0 = e$ and $x_0 = \frac{1}{e}$. And we demand to student to plot the curves of f_k , $k \in IR$. In the second part of this situation, the student is informed that this family of function is $f_k(x) = kx + ln(x)$, $k \in IR$, and we demand him to focus on the graphical representation of the function f_0 in order to find the properties of this function (domain of definition, convexity, monotony, boundary limits,...). Thus, this new learning situation can present the logarithm function at high school.

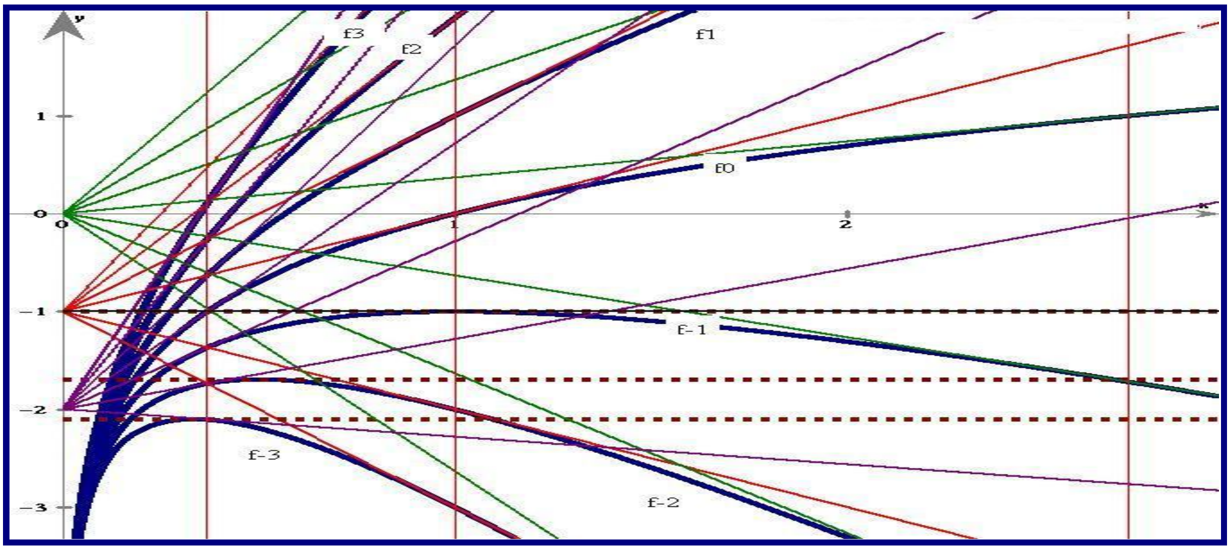

Figure 5. Bundles of tangents of the family $f_k(x) = kx + \ln(x)$, $k \in IR$

3. Conclusion and perspectives

In this paper, we have proposed a new method to present the logarithm function to the students at high school. A quick reading of current and past textbooks shows the absence of this manner of teaching the notion "numerical function" (example: the geometry of the logarithm function). This method is very motivating and after the experiments we did in class, the results were encouraging and the student will be able to find the graphical representation of the family of functions $f_k(x) = kx + ln(x)$, $k \in IR$, and in particular the logarithm function and from this representation, the student deduces the properties of the logarithm function such as domain of definition, convexity, monotony, limits,..., etc.

Other situations learning situations such as the exponential function and the function $x \longrightarrow arctan(x)$ will be the object of the future researches.

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