Nonsplit Dom Strong Domination Number Of A Graph

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Abstract

A subset D of V is called a dom strong dominating set if for every $v \in V - D$, there exists $u_1, u_2 \in D$ such that u_1v, $u_2v \in E(G)$ and $\deg(u_1) \geq \deg(v)$. The minimum cardinality of a dom strong dominating set is called dom strong domination number and is denoted by γ_{dsd}. In this paper, we introduce the concept of nonsplit dom strong domination number of a graph. A dom strong dominating set D of a graph G is a nonsplit dom strong dominating set (nsdsd set) if the induced subgraph <V-D> is connected. The minimum cardinality taken over all the nonsplit dom strong dominating sets is called the nonsplit dom strong domination number and is denoted by $\gamma_{nsdsd}(G)$. Also we find the upper bound for the sum of the nonsplit dom strong domination number and chromatic number and characterize the corresponding extremal graphs.

1. Introduction

Let G = (V, E) be a simple undirected graph. The degree of any vertex u in G is the number of edges incident with u and is denoted by d(u). The minimum and maximum degree of G is denoted by $\delta(G)$ and $\Delta(G)$ respectively. A path on n vertices is denoted by P_n. The graph with $V(B_{n,n})=\{u_1,u_2,u_3,\ldots.u_n,v_1,v_2,v_3,\ldots v_n\}$ and $E(B_{n,n}) =\{uu_i,vv_i,uv: 1 \leq i \leq n\}$ is called the n-bistar and is denoted by $B_{n,n}$. The graph with vertex set $V(H_{n,n}) = \{v_1,v_2,v_3,\ldots v_n,u_1,u_2,u_3\ldots.u_n\}$ and the edge set $E(H_{n,n}) = \{ v_i, u_j, 1\leq i \leq n, n-i+1 \leq j \leq n\}$ is denoted by $H_{n,n}$. The corona of two graphs G_1 and G_2 is the graph $G = G_1 o\, G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

The Cartesian graph product $G = G_1 \square G_2$ is called the graph product of graphs G_1 and G_2 with disjoint vertex sets V_1 and V_2 and edge set X_1 and X_2 is the graph with the vertex set $V_1 \times V_2$ and $u = (u_1,u_2)$ adjacent with $v = (v_1,v_2)$ whenever [$u_1 = v_1$ and u_2 adjacent to v_2] or [$u_2 = v_2$ and u_1 adjacent v_1]. The book graph B_m is defined as the graph cartesian product $S_{m+1} \times P_2$, where S_m is a star graph and P_2 is the path graph on two nodes. The friendship graph or (Dutch windmill graph) F_n is constructed by joining n copies of the cycle C_3 with a common vertex. The ladder graph can be obtained as the Cartesian product of two path graphs, one of which has only one edge. A graph G is a called a (n x m) flower graph if it has n vertices which form an n-cycle and n-sets of m-2 vertices which form m-cycles around the n-cycle so that each m-cycle uniquely intersects with n-cycle on a single edge.

A (n, k)- banana tree is defined as a graph obtained by connecting one leaf of each of n copies of an k-star graph root vertex that is distinct from all the stars. Recently many authors have introduced some new parameters by imposing conditions on the complement of a dominating set. For example, Mahadevan et.al [14] introduced the concept of complementary perfect domination number.

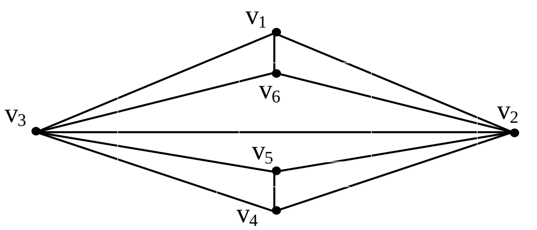
A subset S of V of a non-trivial graph G is said to be an complementary perfect dominating set if S is a dominating set and <V-S> has a perfect matching. The concept of nonsplit domination number of a graph was defined by Kulli and Janakiram [5]. A dominating set D of a graph G is a nonsplit dominating set if the induced subgraph <V-D> is connected. The non split domination number $\gamma_{ns}(G)$ of G is minimum cardinality of a nonsplit dominating set. The concept of dom strong domination number of the graph is defined in [16]. Double domination introduced by Haynes[18] serves as a model for the type of fault tolerance where each computer has access to atleast two fileservers and each of the fileservers has direct access to atleast one backup fileserver. Sampathkumar and Pushpalatha [15] have introduced the concept of strong weak domination in graphs. A combination of the concepts of double domination and strong weak domination is the concept of domination strong domination where in for every vertex outside the dominating set, there are two vertices inside the dominating set, one of which dominates the outside vertex and the other strongly dominates the outside vertex. In this paper we introduce the concept of non split dom strong domination number of a graph.

2. Non Split Dom Strong Domination Number

Definition 2.1

A dom strong dominating set D of a graph G is a nonsplit dom strong dominating set (nsdsd set) if the induced subgraph <V-D> is connected. The minimum cardinality taken over all the nonsplit dom strong dominating sets is called the non split dom strong domination number and is denoted by γ_{nsdsd} (G).

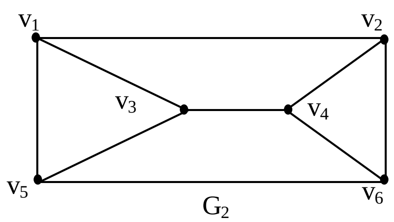
Examples 2.2



G_1

Figure 2.1

In the figure 2.1, D_1 = { v_1, v_2, v_5 } form the nonsplit dom strong dominating set of G_1.



G_2

Figure 2.2

In the figure 2.2, D_1 = { v_1, v_2, v_5, v_6}and D_2 = { v_1, v_2, v_4, v_5, v_6} form the nonsplit dom strong dominating set of G_2. The minimum cardinality is taken as the nonsplit dom strong domination number for G_2 is 4.

Basic Observations 2.3

The nonsplit dom strong domination number of some of the standard classes of graphs are given below

1. $\gamma_{nsdsd}(P_n)$ = n-1 for $n \geq 4$,where P_n is a path on n vertices.
2. $\gamma_{nsdsd}(C_n)$ = n-1 for $n \geq 4$,where C_n is a cycle on n vertices
3. $\gamma_{nsdsd}(K_n)$ = 2 for $n \geq 3$, where K_n is a complete graph on n vertices.
4. $\gamma_{nsdsd}(K_{1,n})$ = n+1, where $K_{1,n}$ is a star graph.
5. $\gamma_{nsdsd}(K_{m,n})$ = m + n -1 for $m \neq n$ where $K_{m,n}$ is a bipartite graph on m+n vertices
6. $\gamma_{nsdsd}(K_{m,n})$ = 4 for m = n where $K_{m,n}$ is a bipartite graph on m+n vertices
7. $\gamma_{nsdsd}(P)$ = 8, where P is the Peterson graph.
8. $\gamma_{nsdsd}(W_n)$ = n-1 where W_n is a wheel whose outer cycle has n vertices.
9. $\gamma_{nsdsd}(H_n)$ = n +1 where H_n is a Helm graph.
10. $\gamma_{nsdsd}(B_{m,n})$ = m+n+1 where $B_{m,n}$ is a bistar.
11. If G is the corona C_no K_1, then $\gamma_{nsdsd}(G)$ = 2n-2 for $n \geq 3$
12. If G is the corona K_no K_1, then $\gamma_{nsdsd}(G)$ = n+1 for $n \geq 3$
13. $\gamma_{nsdsd}(B_m)$ = n - 1, where B_m is a book graph.
14. $\gamma_{nsdsd}(F_n)$ = n -1, where F_n is a friendship graph.
15. $\gamma_{nsdsd}(L_n)$ = 2n-2, where L_n is a ladder graph.
16. $\gamma_{nsdsd}(F_{nxm})$ = n(m-1)-2, where F_{nxm} is a flower graph.
17. $\gamma_{nsdsd}(B_{n,k})$ = nk, where $B_{n,k}$ is a banana tree.

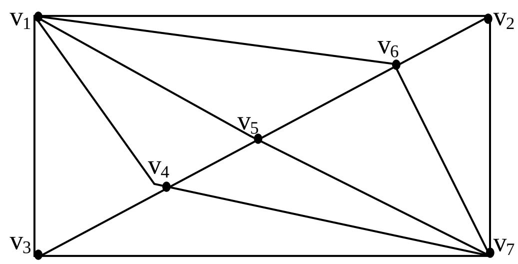
Theorem 2.4 Let G be a graph with no isolates. Then $2 \leq \gamma_{nsdsd}(G) \leq n$ and the bounds are sharp.

Proof Since any dom strong dominating set has at least two elements and at most n elements. Hence for any nonsplit dom strong dominating set has at least two elements and at most n elements. For a star $\gamma_{nsdsd}(K_{1,n}) = n+1$ and for K_n, $\gamma_{nsdsd}(K_n) = 2$.Therefore the bounds are sharp.

Theorem 2.5 In a graph G, if a vertex v has degree one then v must be in every nonsplit dom strong dominating set of G. That is every nonsplit dom strong dominating set contains all pendant vertices.

Proof Let D be any nonsplit dom strong dominating set of G. Let v be a pendant vertex with support say u. If v does not belong to D, then there must be two points say x,y belong to D such that x dominates v and y dominates v. Therefore x and y are adjacent to v and hence deg $v \geq 2$ which is a contradiction. Since v is a pendant vertex, so v belongs to D.

Observation 2.6 $\gamma(G) \leq \gamma_{dsd}(G) \leq \gamma_{nsdsd}(G)$ and the bounds are sharp for the graph G_3 figure 2.3

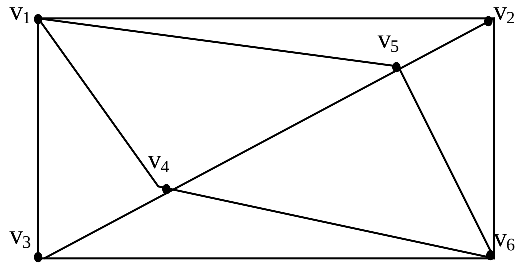


G_3

Figure 2.3

Observation 2.7 For any graph G, $\gamma_{nsdsd}(G) \geq \lceil n/(\Delta+1) \rceil$ and the bound is sharp.

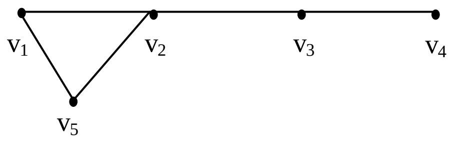
Proof For any graph G, $\lceil n/(\Delta+1) \rceil \leq \gamma$ and also by observation 2.6, the theorem follows. The bound is sharp for the graph G_4 in figure 2.4.



G_4

Figure 2.4

Remark 2.8 Support of a pendant vertex need not be in a nonsplit dom strong dominating set. For the graph G_5 in figure 2.5, $\gamma_{nsdsd}(G_5) = 4$. Here $D_1 = \{ v_1, v_2\ v_4, v_5 \}$is a nonsplit dom strong dominating set which does not contains the support v_3.



G_5

Figure 2.5

Observation 2.9 If H is any spanning subgraph of a connected graph G and $E(H) \subseteq E(G)$ then $\gamma_{nsdsd}(G) \leq \gamma_{nsdsd}(H)$.

Theorem 2.10 Let $G \cong C_n$ $(n \geq 5)$. Let H be a connected spanning subgraph of G, then $\gamma_{nsdsd}(G) = \gamma_{nsdsd}(H)$.

Proof We have $\gamma_{nsdsd}(G) = n - 1$ and also a connected spanning sub graph of G is a path. Hence <H> is a path so that $\gamma_{nsdsd}(H) = n - 1$.

Observation 2.11 For any cycle C_n and any $v \in V(C_n)$,

$$\gamma_{nsdsd}(C_n - v) = \begin{cases} 2 & \text{if } n = 3, \\ 3 & \text{if } n = 4, \\ n-2 & \text{if } n > 4. \end{cases}$$

Proof Follows from theorem 2.10.

Observation 2.12 If $G \cong K_n \circ K_1$, for any complete graph K_n, then $\gamma_{dsd}(G) = \gamma_{nsdsd}(G)$.

Theorem 2.13 For any connected graph G, $\gamma_{nsdsd}(G) = n$ if and only if G is a star.

Proof If G is a star then V is the only nonsplit dom strong dominating set so that $\gamma_{nsdsd}(G) = n$. Conversely, assume that $\gamma_{nsdsd}(G) = n$. We claim that G is a star. Suppose not, let u be a vertex of a maximum degree Δ with $N(u) = \{ u_1, u_2, \ldots, u_\Delta \}$. If $< N(u) >$ has an edge $e = u_iu_j$, then $V - \{ u_i \}$ is a nonsplit dom strong dominating set of cardinality n - 1, which is a contradiction. If $< N(u) >$ has no edge then G has an edge e = xy which is not incident with u such that u is adjacent to x. then $V-\{u\}$ is a nonsplit dom strong dominating set of cardinality n-1 which is a contradiction. Hence G is a star.

Theorem 2.14 For any connected graph G, $\gamma_{nsdsd}(G) = 2$ if and only if there exist u and v such that deg u = deg v = Δ, then deg u and deg v $\geq$ n-2.

Proof Let there exist u and v satisfying the hypothesis. Let $D = \{u,v\}$. Let $x \in V-D$, then x is adjacent to both u and v. Since deg u = deg v = Δ, we have deg x $\leq$ deg u and deg x $\leq$ deg v, therefore D is the nonsplit dom strong dominating set. Conversely, let $D = \{u, v\}$ be a nonsplit dom strong dominating set. Every point $x \in V-D$ is adjacent to both u and v. Therefore deg u $\geq$ n-2, deg v $\geq$ n-2. Also deg x $\leq$ deg u or deg v. Suppose deg u and deg v < Δ then there exists $x \in V - D$ of deg Δ. Therefore D is not a nonsplit dom strong dominating set, which is a contradiction. Hence deg u = deg v = Δ. If deg u is not equal to deg v then deg u = n-1 and deg v = n-2, which is impossible. Therefore deg u = deg v = Δ.

Theorem 2.15 Let G be a graph without isolates and let there exists a γ_{nsdsd} set which is not independent. Then $\gamma(G) + 1 \leq \gamma_{nsdsd}(G)$.

Proof Let D be a γ_{nsdsd} set which is not independent. Let $x \in D$ be such that x is adjacent to some point of D. If $N(x) \cap (V-D) = \Phi$, then as G has no isolates $N(x) \cap D \neq \Phi$. Hence $D - \{ x \}$ is a dominating set. Therefore $\gamma(G) \leq |D-\{ x \}| = \gamma_{nsdsd}(G) - 1$. If $N(x) \cap (V-D) \neq \Phi$. Then for any $y \in N(x) \cap (V-D)$ there exists $z \in D$ such that z is adjacent to y. As x is adjacent to some point of D, $D - \{ x \}$ is a dominating set. Therefore $\gamma(G) \leq |D-\{ x \}| \leq \gamma_{nsdsd}(G) - 1$.The bound is sharp. $\gamma(K_n) = 1$and $\gamma_{nsdsd}(K_n) = 2$.

Theorem 2.16 $\gamma_{nsdsd}(G) \geq \lceil 2n/(\Delta + 2) \rceil$

Proof Every vertex in V-D contributes two to degree sum of vertices of D. Hence $2|V-D| \leq \sum_{u \in D} d(u)$ where D is a nonsplit dom strong dominating set, so that $2|V-D| \leq \gamma_{nsdsd} \Delta$ which implies $2(|V| - |D|) \leq \gamma_{nsdsd} \Delta$. Therefore $2n - 2\gamma_{nsdsd} \leq \gamma_{nsdsd} \Delta$, which implies $\gamma_{nsdsd}(\Delta+2) \geq 2n$. Hence $\gamma_{nsdsd} \geq \lceil 2n/(\Delta + 2) \rceil$. The bounds are sharp. For K_4, $\gamma_{nsdsd}(K_4) = 2$. $\gamma_{nsdsd}(G) = \lceil 2n/(\Delta + 2) \rceil = 2$.

3. Relation Between The Nonsplit Dom Strong Domination Number And Chromatic Number :

Recently many authors have studied the problem of obtaining an upper bounds for the sum of the one domination parameter and graph theory parameter and characterize the corresponding extremal graphs. In [11], Paulraj Joseph J and Arumugam S proved that $\gamma + k \leq p$. In [12], Paulraj Joseph J and Arumugam S proved that $\gamma + \chi \leq p + 1$. They also characterized the class of graphs for which the upper bound is attained. They also proved similar results for γ and γ_t. In [13], Paulraj Joseph J and Mahadevan G proved that $\gamma_{cc} + \chi \leq 2n-1$ and characterized the corresponding extremal graphs. In [6], Mahadevan G, proved that $\gamma_{pr} + \chi \leq 2n-1$ and characterized the corresponding extremal graphs. He also proved that $\gamma_{ipr} + \chi \leq 2n - 2$ and characterized the corresponding extremal graphs. In [14], Paulraj Joseph J, Mahadevan G and Selvam A. introduced the concept of complementary perfect domination number γ_{cp} and proved that $\gamma_{cp} + \chi \leq 2n-2$, and characterized the corresponding extremal graphs. They also obtained the similar results for the induced complementary perfect domination number and chromatic number of a graph. We find the upper bound for the sum of the nonsplit dom strong domination number and chromatic number and characterize the corresponding extremal graphs

Notations 3.1

$P_k(m_1,m_2)$ where $k \geq 2$, $m_1,m_2 \geq 1$ be the graph obtained by identifying centers of the stars $K_{1,m1}$ and $K_{1,m2}$ at the ends of P_K respectively. The graph $C_3(m_1, m_2, 0)$ is obtained from C_3 by identifying the centers of stars $K_{1,m1}$ and $K_{1,m2}$ at any two vertices of C_3. The graph $K_n(m_1, m_2, m_3, m_4, m_5, \ldots m_n)$ denote the graph obtained from K_n by pasting m_1 edges to any one vertex u_i of K_n, m_2 edges to any vertex u_j of K_n, for $i \neq j$, m_3 edges to any vertex u_k for $i \neq j \neq k$, m_4 edges to u_l $i \neq j \neq k \neq l, \ldots\ldots m_n$ edges to all the distinct vertices of K_n. $C_n(P_k)$ is the graph obtained from C_n by attaching the end vertex of P_k to any one vertices of C_n. $K_n(P_k)$ is the graph obtained from K_n by attaching the end vertex of P_k to any one vertices of K_n.

Theorem 3.2 For any graph G, $\gamma_{nsdsd}(G) \leq n$.

Theorem 3.3 For any connected graph G, $\chi(G) \leq \Delta(G) + 1$.

Theorem 3.4 For any graph, $\gamma_{nsdsd}(G) + \chi(G) \leq 2n$ and equality holds if and only if $G \cong K_2$.

Proof By theorem 3.2 and 3.3, it follows that $\gamma_{nsdsd}(G) + \chi(G) \leq n + \Delta + 1 \leq n + n - 1 + 1 \leq 2n$. Now we assume that $\gamma_{nsdsd}(G) + \chi(G) = 2n$. This is possible only if $\gamma_{nsdsd}(G) = n$ and $\chi(G) = n$. Since $\chi(G) = n$, G is complete. But for complete graph, $\gamma_{nsdsd}(G) = 2$. Hence $G \cong K_2$. Converse is obvious.

Theorem 3.5 For any graph G, $\gamma_{nsdsd}(G) + \chi(G) = 2n-1$ if and only if $G \cong P_3, K_3$.

Proof If G is either P_3 or K_3, then clearly $\gamma_{nsdsd}(G) + \chi(G) = 2n-1$. Conversely, assume that $\gamma_{nsdsd}(G) + \chi(G) = 2n-1$. This is possible only if $\gamma_{nsdsd}(G) = n$ and $\chi(G) = n-1$ (or) $\gamma_{nsdsd}(G) = n-1$ and $\chi(G) = n$.

Case (i) $\gamma_{nsdsd}(G) = n$ and $\chi(G) = n-1$. Since $\gamma_{nsdsd}(G) = n$, G is a star. Therefore n=3. Hence $G \cong P_3$. On increasing the degree we get a contradiction.

Case (ii) $\gamma_{nsdsd}(G) = n-1$ and $\chi(G) = n$. Since $\chi(G) = n$, G is complete. But for K_n, $\gamma_{nsdsd}(G) = 2$, so that n = 3. Hence $G \cong K_3$.

Theorem 3.6 For any graph G, $\gamma_{nsdsd}(G) + \chi(G) = 2n-2$ if and only if $G \cong K_{1,3}, K_3(P_2), K_4$.

Proof If G is any one of the following graphs $K_{1,3}, K_3(P_2), K_4$, then clearly $\gamma_{nsdsd}(G) + \chi(G) = 2n-2$. Conversely, assume that $\gamma_{nsdsd}(G) + \chi(G) = 2n-2$. This is possible only if $\gamma_{nsdsd}(G) = n$ and $\chi(G) = n-2$ (or) $\gamma_{nsdsd}(G) = n-1$and $\chi(G) = n-1$ (or) $\gamma_{nsdsd}(G) = n-2$ and $\chi(G) = n$

Case (i) $\gamma_{nsdsd}(G) = n$ and $\chi(G) = n-2$. Since $\gamma_{nsdsd}(G) = n$, G is a star. Therefore n = 4. Hence $G \cong K_{1,3}$. On increasing the degree we get a contradiction.

Case (ii) $\gamma_{nsdsd}(G) = n-1$ and $\chi(G) = n-1$. Since $\chi(G) = n-1$, G contains a clique K on n-1 vertices. Let S = {v} be the vertex other than the clique K_{n-1}.Then v is adjacent to u_i for some i in K_{n-1}. Then $\{v_1, u_i, u_j\}$ is a γ_{nsdsd} set. Hence n = 4. Therefore $K = K_3$. If $d(v_1) = 1$ then $G \cong K_3(P_2)$. On increasing the degree of v_1, no graph exists.

Case (iii) $\gamma_{nsdsd}(G) = n-2$ and $\chi(G) = n$. Since $\chi(G) = n$, $G \cong K_n$. But for K_n, $\gamma_{nsdsd}(G) = 2$. Therefore n = 3. Hence $G \cong K_4$.

Theorem 3.7 For any graph G, $\gamma_{nsdsd}(G) + \chi(G) = 2n-3$ if and only if $G \cong K_{1,4}, K_3(P_3), K_3(2), K_3(P_2, P_2, 0), K_5$, or any one of the graphs in the figure 3.1

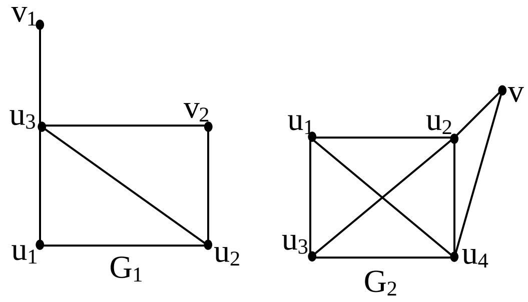


Figure 3.1

Proof If G is any one of the above graphs then clearly $\gamma_{nsdsd}(G) + \chi(G) = 2n-3$. Conversely, assume that $\gamma_{nsdsd}(G) + \chi(G) = 2n-3$. This is possible only if $\gamma_{nsdsd}(G) = n$, $\chi(G) = n-3$ (or) $\gamma_{nsdsd}(G) = n-1$, $\chi(G) = n-2$ (or) $\gamma_{nsdsd}(G) = n-2$, $\chi(G) = n-1$ (or) $\gamma_{nsdsd}(G) = n-3$, $\chi(G) = n$.

Case (i) γ_{nsdsd} (G) = n and χ (G) = n-3. Since γ_{nsdsd} (G) = n, G is a star. Therefore n = 5. Then G $\cong K_{1,4}$. On increasing the degree no new graph exists.
Case (ii) γ_{nsdsd} (G) = n-1 and χ (G) = n-2. Since χ (G) = n-2, G contains a clique K on n-2 vertices. Let S = {v_1, v_2} be the vertices other than the clique K_{n-2} then the possible cases are <S> = K_2 or $\overline{K_2}$.
Subcase (i) Let <S> = K_2. Since G is connected, either v_1 or v_2 is adjacent to u_i for some i in K_{n-2}, then {v_1, v_2, u_i, u_j} is a γ_{nsdsd} set so that n = 5. Hence K = K_3. If d (v_1) = 2 and d (v_2) = 1, then G $\cong K_3(P_3)$. On increasing the degree, no graph exists.
Subcase (ii) Let <S> = $\overline{K_2}$. Since G is connected, v_1 and v_2 is adjacent to u_i for some i in K_{n-2}. Then γ_{nsdsd} (G) = 4, so that K = K_3. If d(v_1) = d (v_2) = 1, then G $\cong K_3(2)$. If d(v_1)=1 and d(v_2) = 2 then G $\cong G_1$. If v_1 is adjacent to u_i and v_2 adjacent to u_j for some i $\neq$ j in K_{n-2} then γ_{nsdsd}(G) = 4. Hence K = K_3. If d (v_1) = d (v_2) = 1, then G $\cong K_3$ (P_2, P_2, 0). On increasing the degree, no graph exists.

Case (iii) γ_{nsdsd} (G) = n-2 and χ (G) = n-1. Since χ (G) = n-1, G contains a clique K on n-1 vertices. Let S = {v} be the vertex other than the clique K_{n-1}.If v is adjacent to u_i for some i in K_{n-1}, then γ_{nsdsd} (G) = 3. Hence n = 4. Therefore K= K_4. If d (v) = 1, then G $\cong K_4(P_2)$. If d(v) = 2, then G $\cong G_2$. On increasing the degree, no new graph exists.
Case (iv) γ_{nsdsd} (G) = n-3 and χ (G) = n. Since χ (G) = n, G $\cong K_n$. But for complete Graph K_n, γ_{nsdsd} (G) = 2 so that n = 5. Therefore G $\cong K_5$.

Theorem 3.8 For any graph G, γ_{nsdsd} (G) + χ (G) = 2n - 4 if and only if G $\cong K_{1,5}$, K_3 (3), $C_4(P_2)$, $S(K_{1,3})$, K_3 (P_3), C_3 (1,1,1), K_3 (2, 1, 0) , K_4 (2), K_4 (P_2, P_2, 0,0), K_5 (P_2). K_6, or any one of the graphs given in the figure 3.2

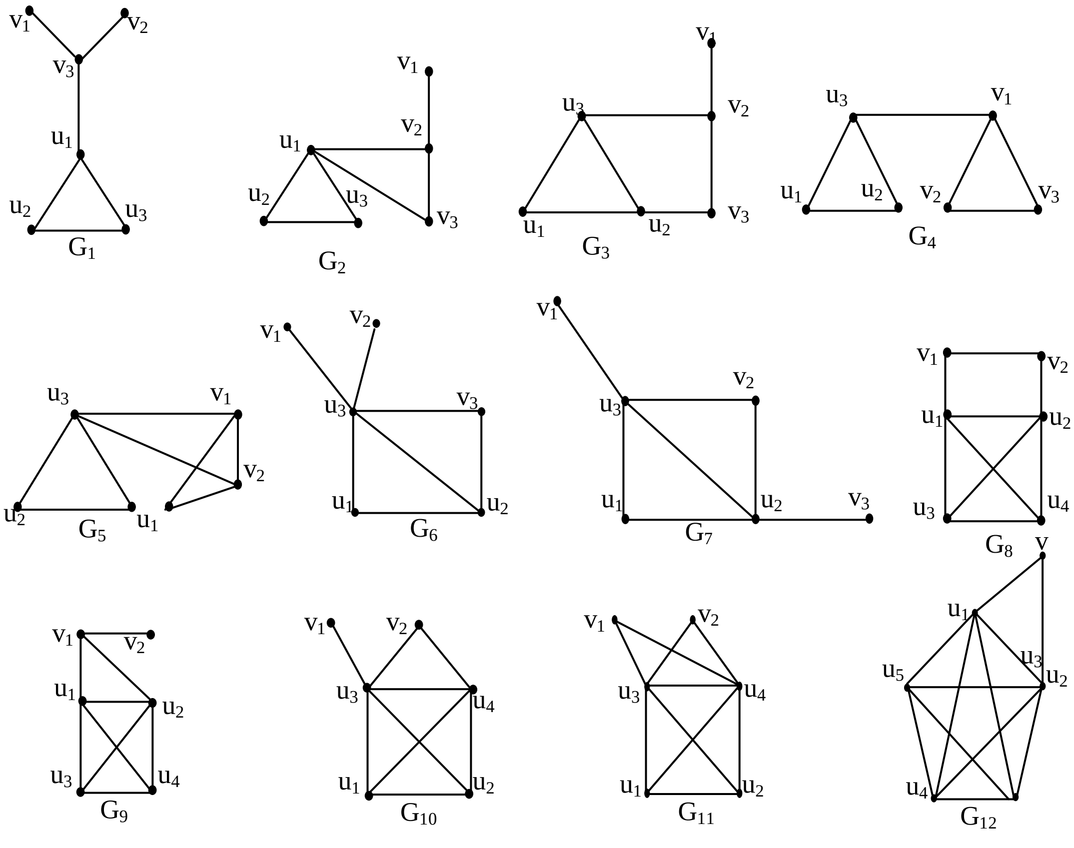


Figure 3.2

Proof Assume that γ_{nsdsd} (G) + χ (G) = 2n-4. This is possible only if γ_{nsdsd} (G) = n and χ(G) = n-4 (or) γ_{nsdsd} (G) = n-1 and χ(G) = n-3 (or) γ_{nsdsd} (G) = n-2 and χ(G) = n-2 (or) γ_{nsdsd} (G) = n-3 and χ(G) = n-1 (or) γ_{nsdsd} (G) = n-4 and χ(G) = n.

Case (i) γ_{nsdsd} (G) = n and χ (G) = n-4. Since γ_{nsdsd} (G) = n, G is a star. Therefore n = 6. Then G ≅ $K_{1,5.}$ On increasing the degree, we get a contradiction.

case (ii) γ_{nsdsd}(G) = n-1 and χ (G) = n-3.

Since χ (G) = n-3, G contains a clique K on n-3 vertices Let S = {$v_{1,}$ $v_{2,}$ v_3} be the vertices other than the clique K_{n-3} then <S> = P_3, K_3, $\overline{K_3}$, $K_2 U K_1$

Subcase (i) Let < S > = P_3. Since G is connected, the following are the possible cases (i) there exist a vertex u_i of K_{n-3} which is adjacent to any one of end vertices (ii) there exist a vertex u_i of K_{n-3} which is adjacent to other than end vertices. If there exist a vertex u_i of K_{n-3} which is adjacent to any one of end vertices, then γ_{nsdsd} (G) = 5. Hence n = 6. Therefore K = K_3. If d(v_1) = 2 and d(v_2) = d(v_3) = 1 then G ≅ $K_3(P_4)$. If u_i is adjacent to v_2 which is not a pendant vertices then γ_{nsdsd} (G) = 5. Hence n = 6. Therefore K = $K_{3.}$ If d(v_1) = d(v_3) = 1 and d(v_2) = 3 then G ≅ G_1. If d(v_3) = 2 and d(v_1) = 1 and d(v_2) = 3 then G ≅ G_2. If d(v_1) = 1 and d(v_2) = 3 and d(v_3) = 2 then G ≅ $G_{3.}$

Subcase (ii) Let < S > = K_3. Since G is connected, there exist a vertex u_i of K_{n-3} adjacent to anyone of {$v_{1,}$ v_2 ,v_3}. Without loss of generality let v_1 be adjacent to $u_{i,}$ then γ_{nsdsd} (G) = 5. Therefore K=$K_{3.}$ If d(v_1) =3 and d(v_2) = d(v_3) = 2 then G ≅ $G_{4.}$ If d(v_1) = 3 and d(v_2) = 3 and d(v_3) = 2 then G ≅ $G_{5.}$ On increasing the degree we get a contradiction.

Subcase (iii) Let < S > = $\overline{K_3}$. Since G is connected, let all the vertices of $\overline{K_3}$ be adjacent to vertex u_i . Then γ_{nsdsd} (G) = 5. Hence n = 6. Therefore K = $K_{3.}$ Let $u_{1,}$ $u_{2,}$ u_3 be the vertices of $K_{3.}$ Let all the three vertices of $\overline{K_3}$ adjacent to $u_{1.}$ Then G ≅ K_3 (3). If d(v_3) = 2 and d(v_1) = 1 and d(v_2) = 1 then G ≅ G_6.On increasing the degree, we get a contradiction. If two vertices of $\overline{K_3}$ are adjacent to u_i and the third vertex adjacent to u_j for some i≠j, then γ_{nsdsd}(G) = 5. Hence n = 6. Therefore K = $K_{3.}$ Let $u_{1,}$ $u_{2,}$ u_3 be the vertices of $K_{3.}$ Then G ≅ K_3 (2, 1, 0). If d(v) = 1 and d(v_2) = 2 and d(v_3) = 1 then G ≅ G_7. On increasing the degree, we get a contradiction. If all the three vertices of $\overline{K_3}$ are adjacent to three distinct vertices of K_{n-3} say $u_{i,}$ $u_{j,}$ u_k for i ≠ j ≠ k, then γ_{nsdsd} (G) = 5. Hence n = 6. Therefore K = $K_{3.}$ Let $u_{1,}$ $u_{2,}$ u_3 be the vertices of $K_{3.}$ Then G ≅ K_3 (1,1,1). On increasing the degree, we get a contradiction.

Subcase (iv) Let < S > = $K_2 \cup K_1$. Since G is connected, there exist a vertex u_i of K_{n-3} which is adjacent to anyone of {v_1, v_2} and v_3. Then γ_{nsdsd}(G) = 4. Hence n = 6. Therefore K = $K_{2,}$ so that G ≅ S($K_{1,3}$). On increasing the degree, we get a contradiction. Let there exist a vertex u_i of K_{n-3} be adjacent to any one of {v_1, v_2} and u_j for some I ≠ j in K_{n-3} adjacent to v_3. Hence γ_{nsdsd}(G) = 4, so that n = 5. Therefore K = $K_{2,}$ which is a contradiction.

If G does not contain a clique K on n-3 vertices, then it can be verified that no new graph exist.

Case (iii) γ_{nsdsd} (G) = n-2 and χ (G) = n-2. Since χ (G) = n-2, G contains a clique K on n-2 vertices. Let S = { $v_{1,}$ $v_{2,}$ $v_{3,}$ v_4 } be the vertices other than the clique K_{n-2} then the possible cases are <S> = K_2, $\overline{K_2}$.

Subcase (i) Let <S> = $K_{2.}$ Since G is connected, either v_1 or v_2 is adjacent to u_i for some i in K_{n-2}. Then γ_{nsdsd} (G) = 4 so that n = 6. Therefore K= $K_{4.}$ Let $u_{1,}$ $u_{2,}$ u_3 be the vertices of K_3. Therefore G ≅ $K_4(P_3)$. On increasing the degree, then G ≅ $G_{8,}$ $G_{9.}$

Subcase (ii) Let <S> = $\overline{K_2}$. Since G is connected, both v_1 and v_2 adjacent to u_i for some i in $K_{n-2.}$ Then γ_{nsdsd} (G) = 4 so that n = 6.Therefore K= $K_{4.}$ Let $u_{1,}$ $u_{2,}$ u_3, u_4 be the vertices of K_4. Therefore G ≅ K_4 (2). If d(v_1) = 1 and d(v_2) = 2 then G ≅ $G_{10.}$ On increasing the degree, we get a contradiction.. If the two vertices are adjacent to two distinct vertices of $K_{n-2,}$ then γ_{nsdsd} (G) = 4. Hence n = 6.Theefore K = $K_{4.}$ Then G ≅ K_4 ($P_{2,}$ $P_{2,}$ 0, 0). If d(v_1) = 2 and d(v_2) = 1 then G ≅ $G_{11.}$ If d(v_1) = 2 and d(v_2) = 2 then G ≅ $G_{12.}$ On increasing the degree, we get a contradiction.

Case (iv) γ_{nsdsd} (G) = n-3 and χ(G) = n-1.Since χ(G) = n-1, G contains a clique K on n-1 vertices. Let the vertex v_1 is adjacent to u_i for some i in $K_{n-1.}$ Therefore γ_{nsdsd} (G) = 3, hence n = 6. Therefore K = K_5. Then G ≅ K_5 (P_2). If d (v) = 2 then G ≅ $G_{15.}$On increasing the degree, we get a contradiction.

Case (v) Let γ_{nsdsd}(G) = n-4 and χ (G) = n. Since χ (G) = n, G ≅ K_n. But for K_n, γ_{nsdsd} (G) = 2, so that n = 6. Therefore G ≅ $K_{6.}$

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