Analyzing Of Low Altitude Mimo Radarbeampattern Design

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Abstract:
A multiple input multiple output (MIMO) radar is an emerging field which has attracted many researchers recently. In this type of radar, unlike an standard phased array radar, one can choose transmitted probing signals freely to maximize the power around the locations of the targets of interest, or more generally to approximate a given transmit beam-pattern, or to minimize the cross-correlation of the signals reflected back to the radar by the targets of interest. These signals can be correlated to each other or non-correlated. Many papers have investigated beampattern design and waveforming in MIMO radars. One of the most famous approaches for beampattern design is named covariance based method which design cross-correlation matrix of transmitted signals instead of the own signalsdirectly. Many papers have investigated the problem of MIMO radar beampattern design in an ideal operational environment. But one of the important operational situations which causes faults and errors to the radar systems is low altitude track, detection and etc. It is because of in low altitude operations beside of the desire reflected signals multipath signals will be received too. In this article it is desirable to study the effect of multipath of a MIMO radar system which is employing cross-correlation matrix of transmitted signals to designing the transmitted beampattern. MATLAB software is being employed for simulations.

Keywords: Multipath, MIMO radar, low altitude; beampatterndesign, covariance based method.

1. Introduction

Multiple input multiple output (MIMO) radar is an emerging radar field which has attracted many researchers from all over the world recently. In this type of radar, unlike an standard phased array radar, one can choose transmitted probing signals freely to maximize the power around the locations of the targets of interest, or more generally to approximate a given transmit beam-pattern, or to minimize the cross-correlation of the signals reflected back to the radar by the targets of interest [1,2]. These signals can be correlated to each other or non-correlated. This feature provides extra degrees of freedom in the design of radar system. Generally MIMO radar systems can be classified into two main categories:

1. MIMO radars with widely separated antennas [3]
2. MIMO radars with colocated antennas [4]In the case of widely separated antennas, the transmitting antennas are far from each other such that each views a different aspect of a target of interest. In this type of MIMO radars, the concept of MIMO can be used to increase the spatial diversity of the radar system [5,6,7,8]. this spatial diversity which has been achieve from RCS diversity gain, can improve the performance of detection [7], finding slow moving targets [5] and angle estimation [8]. Useful references about MIMO radar with widely separated antennas can be found in [3].In the case of MIMO radar with colocated antennas, the transmitting antennas are close to each other such that the aspects of a target of interest observed by antenna elements, are identical. In this type of MIMO radars, the concept of MIMO can be used to increase the spatial resolution. Many papers have demonstrated the advantages of this type of MIMO radars, including high interference rejection capability [9,10], improved parameter identifiability [11], and enhanced flexibility for transmit beampattern design [12,13]. useful references about MIMO radars with colocated antennas can be found in [4].Generally MIMO radar waveform design methods can be classified into three main categories [12] - [23]:

1. Covariance matrix based design methods [12] – [16]
2. Radar ambiguity function based design methods [17] – [20]
3. Extended targets based design methods [21] – [23]

In the covariance matrix based design methods, the cross correlation matrix of transmitted signals is taken into consideration instead of entire waveform. Then these types of waveform design methods affect only the spatial domain. In references [12,14] the cross correlation matrix of transmitted signals is design such that the power can be transmitted to a desire range of angles. In [13] the cross correlation matrix of transmitted signals is design such that one can control the spatial power.

In addition in [13] the cross correlation between the transmitted signals at a number of given target locations is minimized which can improved the spatial resolution in the radar receiver. In [15] the authors have optimized the covariances between waveforms based on the Cramer – Rao bound matrix. And finally in [16], given the optimize covariance matrix, the corresponding signal waveforms are designed to further achieve low peak to average power ratio (PAR) and higher range resolution.The radar ambiguity based design methods optimize the entire signal instead of matrix of cross correlation of the signals. Then these design methods involve not only the spatial domain but also the range domain. Actually in these design methods it is desirable to find a set of signals which satisfied a desire radar ambiguity function. Of course these design methods are more complicated than the covariance design methods. Angle, Doppler and range resolution of a radar system can be characterized by the MIMO radar ambiguity function [24] – [26]. In [18] – [20] the authors goal is to sharpen the radar ambiguity function. For achieving this goal the authors have minimized the sidelobe of the autocorrelation and the cross correlation between the signals. In [17] the signals are directly optimized so that a sharper radar ambiguity function can be obtained. Then the spatial and range resolution of point targets can be improved.In the extended target based design methods also, the entire signals is considered as in the radar ambiguity function design based methods, however, unlike the radar ambiguity function design based methods which considered only the resolutions of point targets, these methods considered the detection and estimation of extended targets. These methods require some prior information about the target or clutter impulse response. The extended target based design methods have been studied in [17] – [20].

Many papers have investigated beampattern design and waveforming in MIMO radars. One of the most famous approaches for beampattern design (as mentioned before) is named covariance based method which design cross-correlation matrix of transmitted signals instead of the signals. On the other hand, many papers have investigated beamforming of an standard phased array radar in different ways. In this paper it is desirable to design beampattern of a phased array radar by means of the MIMO radar covariance based beampattern design method, as a special case of general MIMO radar.

The present study has six sections as follow:

Section 1 presents a brief introduction of MIMO radars. In section 2 the covariance based MIMO radar beamforming is reviewed. Utilization a model of maximizing the transmitted power around the locations of the targets of interest is discussed in section 3. Utilization a model of beampattern design in presence of multipath is discussed in section 4. Numerical examples are provided in section 5. Section 6 is focused on conclusion and references are presented at the end of the paper.

2. Covariance Based Method For MIMO Radar Beampattern Design

Assume that we have a collection of N transmitter antenna which are located at known coordinates $x_i = (x_{1,i}, x_{2,i}, x_{3i}) = (x, y, z)$ in some spherical coordinate along the z-axis as shown in Figure. 1. In the presented study and in all of the examples and formulas of current paper, it is assume that these transmitter antennas are along the z-axis. It is assume that each transmitter antenna is driven by a specific signal on the carrier frequency of f_c or with wavelength of λ and complex envelope of $s_i(t), i = 1, \dots, N$. At a specific point in space with distance of r and direction of $k(\theta, \phi)$ from the transmitter antenna, each radiated signal $s_i(t)$ gives rise to a "signal" the far field at radius r, with complex envelope given by

$$y_i(t, r, \theta, \phi) = \frac{1}{\sqrt{4\pi r}} s_i\left(t - \frac{r}{c}\right) e^{j\left(\frac{2\pi}{\lambda}\right) x_i^T k(\theta,\phi)} \qquad (1)$$

Where, in this equation, k is a unit vector in the (θ, ϕ)direction.

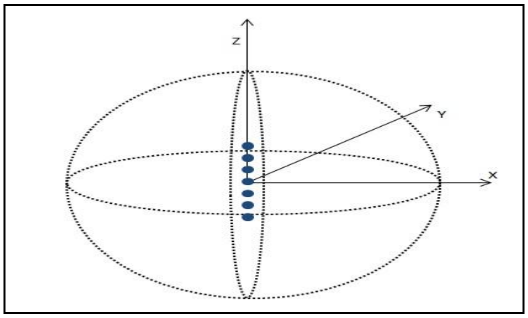


Figure. 1. T/R modules and spherical coordinate system

At the far field, these signals add linearly and the radiated powers P_i add linearly as well. At this point assume that the i-th element location is on the z-axis at coordinate z_i. The signal at position (r, θ, ϕ) resulting from all of the transmitted signals at far field will be:

$$y(t, r, \theta, \phi) = \sum_{i=1}^{N} y_i(t, r, \theta, \phi)$$

$$= \frac{1}{\sqrt{4\pi r}} \sum_{i=1}^{N} s_i\left(t - \frac{r}{c}\right) e^{j\left(\frac{2\pi z_i}{\lambda}\right)\sin(\theta)}$$

(2)

The power density of the entire signals then given by

$$P_y(r, \theta, \phi)$$

$$= \frac{1}{4\pi r^2} \sum_{k=1}^{N} \sum_{l=1}^{N} < s_k(t) s_l^*(t) > e^{j\left(\frac{2\pi(z_k - z_l)}{\lambda}\right)\sin(\theta)}$$

(3)

And it is known that the complex signal cross-correlation is defined by

$$R_{kl} = < s_k(t) s_l^*(t) > \qquad (4)$$

With defining the direction vector as below

$$a(\theta) = \left[e^{j\left(\frac{2\pi z_1}{\lambda}\right)\sin(\theta)}, \ldots, e^{j\left(\frac{2\pi z_N}{\lambda}\right)\sin(\theta)}\right]^T \qquad (5)$$

The normalized power density $P(\theta, \phi)$ of signals, in (W/ster), would be:

$$P(\theta, \phi) = \frac{1}{4\pi} \sum_{k=1}^{N} \sum_{l=1}^{N} R_{kl} e^{\frac{j2\pi}{\lambda}(z_k - z_l)\sin(\theta)} \qquad (6)$$

Recognizing that (6) is quadratic form in the Hermitian matrix R which is the cross-correlation matrix of signals, this can be written compactly as

$$P(\theta, \phi) = \frac{1}{4\pi} a^*(\theta) R a(\theta) \qquad (7)$$

This normalized power density $P(\theta, \phi)$ is exactly the beampattern which is desirable to find [3]. In the following some examples of beampatterns produce from such a cross-correlation matrix has been shown. Figure. 2 shows the beampattern produced by signal cross-correlation matrix of (8), (9) and (10) respectively. It is noticeable that these figures are beam-patterns of 10-element uniform linear array (ULA) with half-wavelength spacing.

$$\begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix} \qquad (8)$$

$$\begin{bmatrix} 0.8^0 & \cdots & 0.8^9 \\ \vdots & \ddots & \vdots \\ 0.8^9 & \cdots & 0.8^0 \end{bmatrix} \qquad (9)$$

$$\begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} (10)$$

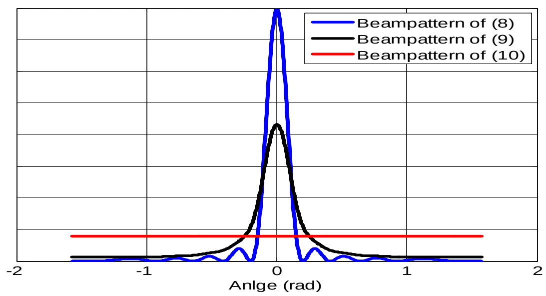


Figure. 2. Beampattern respect to (7). The blue one is corresponds to cross-correlation matrix of (8), The black one iscorresponds to cross-correlation matrix of (9) and the red one is corresponds to cross-correlation matrix of (10)

In general case the elements of the signal cross-correlation matrix are complex values except the diagonal elements that are real. This general case is related to MIMO radars but in the case of phased array radar, all the transmitter signals are correlated with each other and then absolute value of all the elements in R, are equal to 1(blue one at Figure. 2).

3. Maximizing the Transmitter Power Around The Locations Of Targets Of Interest

In [13] the authors have introduced a cost function to maximize the transmitter power around some interest location for MIMO radars which is written bellow:

$$\max_{\mathrm{R}} \quad \mathrm{tr}(\mathrm{R}\hat{\mathrm{B}}) \quad \text{subject to}$$

$$\mathrm{tr}(\mathrm{R}) = \text{const}$$

$$\mathrm{R} \geq 0$$

(11)

Where in these equations R is the cross correlation matrix of transmitted signals and $\hat{\mathrm{B}}$ is an estimation of B, the location of the targets, which is shown below:

$$\mathrm{B} = \sum_{k=1}^{K} \mathrm{a} \quad (\theta_k)\mathrm{a}^*(\theta_k) \qquad (12)$$

Where in general case it is assumed that there is total number of K targets with the locations which is denoted by θ_k.In [13] the authors have solved the above problem in a general case of MIMO which its transmitted signals can be arbitrary chosen. This answer would be of the form bellow:

$$R = uu^* \qquad (13)$$

Where in this equation u is eigenvector related to maximum eigenvalue of $\hat{B}$. In this article it is desirable to investigate the effect of multipath on this approach.

4. Beampattern Signal model in presence of multipath

Form (7) it is understood that if multipath exists, the equation of (7) will be rewritten as bellow:

$$P(\theta,\phi) = \frac{1}{4\pi} a^*(\theta) \sum_{i=1}^{M} R_i \, a(\theta) \qquad (14)$$

Where in this equation M is the number of multipath which the transmitted signals will be arrived to the receiver by them and $R_i$denotes the corresponding cross-correlation matrix of transmitted signals throw each paths. It is known that for $i = 1$it will show the cross correlation matrix of transmitted signals throw the main or line of sight path.For $i > 1$ each element of $R_i$can be modeled by complex number of amplitude with normal distribution of zero mean and some variancesrespect to the operational environment, and the phase with uniform distribution between 0 and 2π.In the section of numerical examples the effect of multipath is compared to the ideal situation in the sense of (13).

5. Numerical examples

In this section it is desirable to examine the effect of multipath with some numerical examples.

5.1. Linear array

In this subsection it is assumed that there is a linear array with 20 elements which are placed along the z-axis with the center of the origin with half wave length spacing between its elements, and it is assumed that there are total numbers of three targets at the space with locations of [-50 10 60] degrees. Figure 3 shows the beampattern of such array for ideal and two paths situation. In this figure it is assumed that the false path has a variable amplitude of normal distribution with zero mean and variance of 0.1 and inclined a random phase with normal distribution between 0 and 2π to the signal.

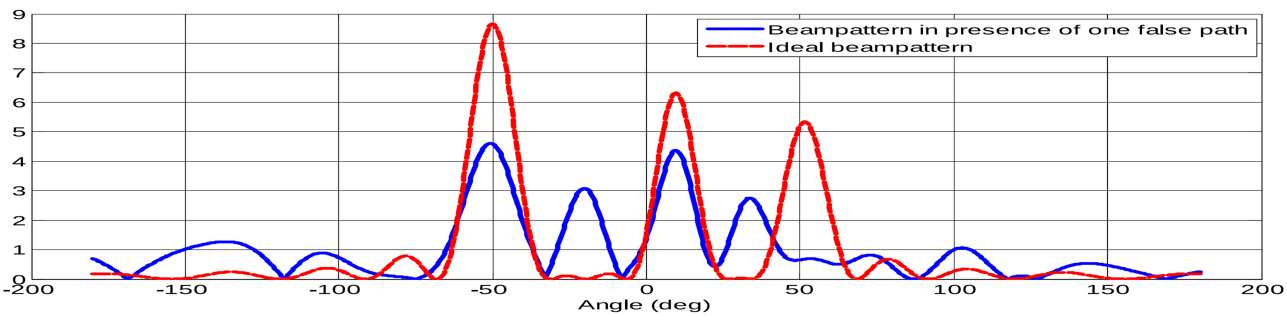


Figure.3. Comparison between ideal beampattern and beampattern resulted in the case with one false path

As it is seen from this figure, beside some decreases in power around the desire directions,only one false path can change the exact location of the desire peak power and can cause producing extra and false sidelobes in the transmitted beampattern which can cause errors in the suitable results.Figure 4 shows the beampattern of an array with the above characteristics for ideal and four paths situation. In this figure it is assumed that the false pathshave variable amplitudes of normal distribution with zero mean and variance of 0.1 and inclined a random phase with normal distribution between 0 and 2π to the signal.

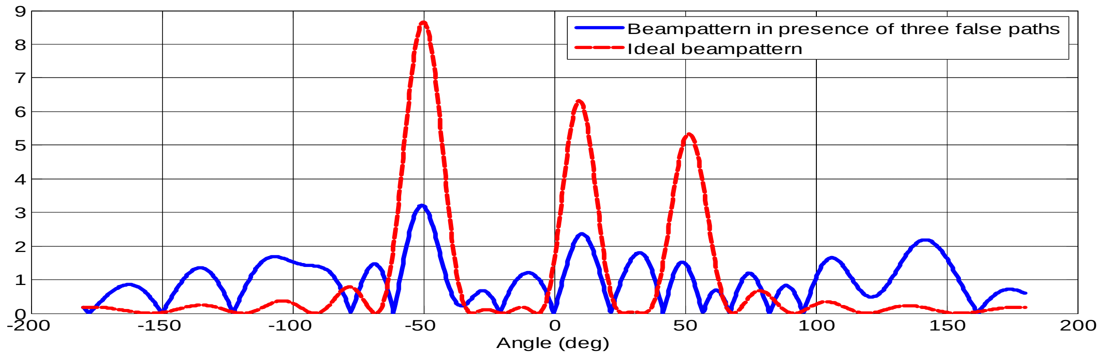


Figure.4. Comparison between ideal beampattern and beampattern resulted in the case with three false paths

As it is seen from this figure, beside some decreases in power around the desire directions, only one false path can change the exact location of the desire peak power and can cause producing extra and false sidelobes in the transmitted beampattern which can cause errors in the suitable results. It is noticeable that in the case of three false paths compare to one false path the transmitted beampattern is almost irrelevant to the desire beampattern, which denotes that as false paths increase the resulting beampattern get worse as well.

5.2. Planar array

In this subsection it is assumed that there is a planar array with 20 by 20 elements which are placed along the z-axis and y-axis with the center of the origin with half wave length spacing between its elements, and it is assumed that there are total numbers of three targets at the space with locations of which has been shown in Figure 5. Figure.6shows the beampattern of such array for ideal situation without any multipath, Figure. 7 shows the beampattern of such an array of two paths situation and Figure. 8 shows the transmitted beampattern of three false path situation. In these figures it is assumed that the false paths havevariable amplitude of normal distribution with zero mean and variance of 0.1 and inclined a random phase with normal distribution between 0 and 2π to the signal.

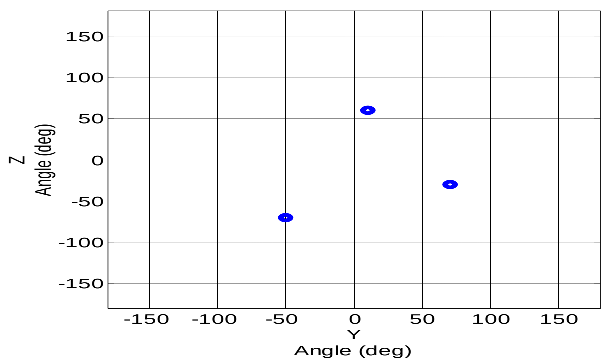
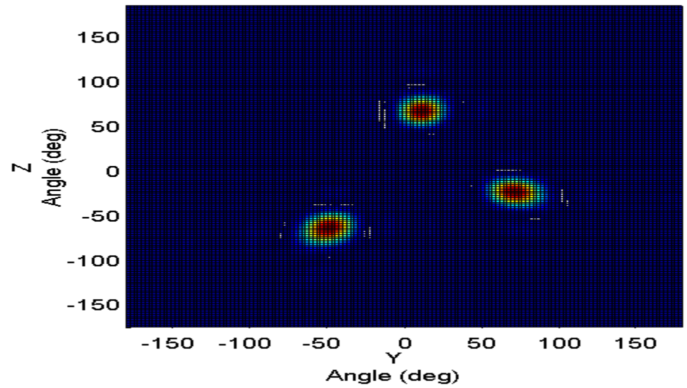
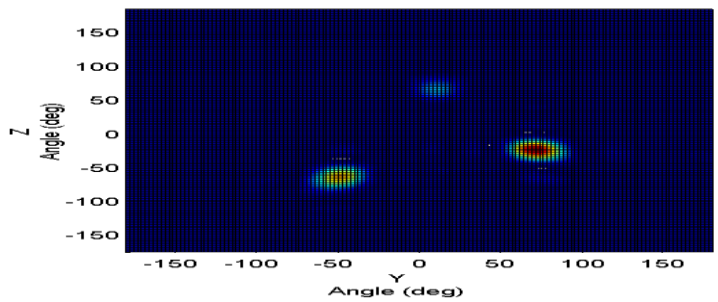


Figure.5. Locations of targets of interest which it is desirable to maximize the transmitted



power around these locations

Figure.6. Maximizing the transmitted power around the targets of interest in an ideal



case

Figure.7. Maximizing the transmitted power around the targets of interest in the case of one false path

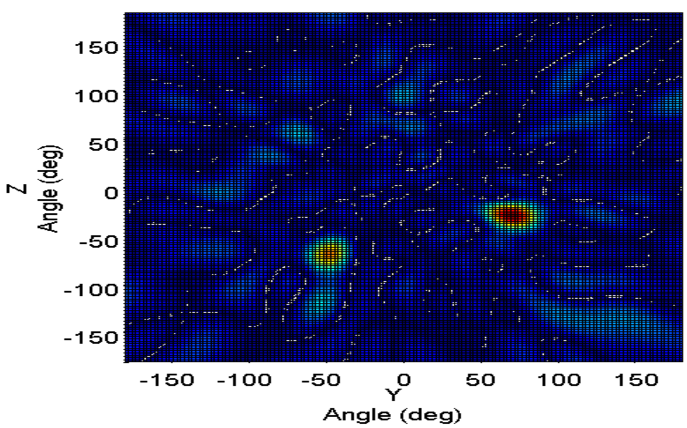


Figure.8. Maximizing the transmitted power around the targets of interest in the case of three false paths

Just like the case of linear array it is seen from these figures as the paths of multipath increase the resulting beampattern getting worst.

6. Conclusion

Many papers and authors have investigated the problem of designing the beampattern of MIMO radars with covariance based methods. In this article the effects of multipath for low altitude radar systems in an operational environment had been considered and showed that this effect can be very vital in some cases and must be taken into consideration.

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