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Adjusment of a Braced Quadrilatral by Rigorous Method in Tabular Form

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Abstract
Adjusting a braced quadrilateral by rigorous method is a tedious and laborious job. This paper presents the step-by-step computations of adjustment in a simplified manner by making use of a table designed by the author for the purpose.

1. Introduction
A braced quadrilateral being the strongest triangulation figure is preferred in any triangulation scheme unless field conditions prohibit. When the work requires accuracy in results, the adjustment of the quadrilateral has to be done by rigorous method. By manual computations in rigorous method of adjustment being tedious and laborious, one is liable to make mistakes in computations and, therefore, the rigorous method is avoided unless the conditions demand. This paper presents a tabular form of step-by step computations involved in the adjustment of a braced quadrilateral. The advantage of computations using a table is that computations proceed mechanically without feeling any difficulty in remembering the steps of computations. Some new notations have been used to make the method look simpler.

2. Rigorous method of adjustment
A braced quadrilateral has eight observed angles as shown in Fig. 1. There are four conditions which must be satisfied to adjust the angles, excluding the one imposed by the least squares theory.

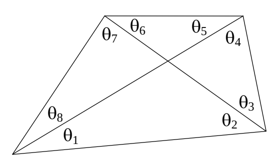

Fig. 1 Braced quadrilateral

Condition-1 $360° - (\theta_1 + \theta_2 + \ldots + \theta_8) = C_1$
Condition-2 $(\theta_5 + \theta_6) - (\theta_1 + \theta_2) = C_2$
Condition-3 $(\theta_7 + \theta_8) - (\theta_3 + \theta_4) = C_3$
Condition-4 $[\text{log sin (Left angles)} - \text{log sin (Right angles)}] \times 10^7 = C_4$

where C_1, C_2, C_3, and C_4 are the total corrections given by each condition equation.
If $c_1, c_2, \ldots c_8$ are the individual corrections to the observed angles $\theta_1, \theta_2, \ldots \theta_8$, respectively, then we have

$$c_1 + c_2 + \ldots + c_8 = C_1 \quad \ldots(1)$$
$$(c_1 + c_2) - (c_5 + c_6) = C_2 \quad \ldots(2)$$
$$(c_3 + c_4) - (c_7 + c_8) = C_3 \quad \ldots(3)$$
$$c_1 f_1 + c_2 f_2 + \ldots + c_8 f_8 = C_4 \quad \ldots(4)$$

where $f_1, f_2, \ldots f_8$ are log sin differences for 1″ in the values of the respective angles multiplied by 10^7.

The additional condition from the theory of least squares to be satisfied is

$$\phi = c_1^2 + c_2^2 + \ldots + c_8^2 = \text{a minimum.} \qquad \ldots(5)$$

Since we have four condition equations (1) to (4) excluding equation (5), there will be four correlates $-\lambda_1$, $-\lambda_2$, $-\lambda_3$, and $-\lambda_4$ which are multiplied to the differentiated form of equations (1) to (4), respectively, and the results are added to the differentiated form of equation (5). The resulting equation is

$$\begin{aligned}&(c_1 - \lambda_1 - \lambda_2 - f_1\lambda_4)\,\partial c_1 + (c_2 - \lambda_1 - \lambda_2 + f_2\lambda_4)\,\partial c_2 + (c_3 - \lambda_1 - \lambda_3 - f_3\lambda_4)\,\partial c_3\\ &+ (c_4 - \lambda_1 - \lambda_3 + f_4\lambda_4)\,\partial c_4 + (c_5 - \lambda_1 + \lambda_2 - f_5\lambda_4)\,\partial c_5 + (c_6 - \lambda_1 + \lambda_2 + f_6\lambda_4)\,\partial c_6\\ &+ (c_7 - \lambda_1 + \lambda_3 - f_7\lambda_4)\,\partial c_7 + (c_8 - \lambda_1 + \lambda_3 + f_8\lambda_4)\,\partial c_8 = 0\end{aligned}$$

Now equating the coefficients of ∂c_1 , ∂c_2 , etc., to zero, we get

$$\left.\begin{aligned}c_1 &= \lambda_1 + \lambda_2 + f_1\lambda_4\\ c_2 &= \lambda_1 + \lambda_2 - f_2\lambda_4\\ c_3 &= \lambda_1 + \lambda_3 + f_3\lambda_4\\ c_4 &= \lambda_1 + \lambda_3 - f_4\lambda_4\\ c_5 &= \lambda_1 - \lambda_2 + f_5\lambda_4\\ c_6 &= \lambda_1 - \lambda_2 - f_6\lambda_4\\ c_7 &= \lambda_1 - \lambda_3 + f_7\lambda_4\\ c_8 &= \lambda_1 - \lambda_3 - f_8\lambda_4\end{aligned}\right\} \qquad (6)$$

Substituting the values of the above corrections in equations (1) to (4), we have

$$\begin{aligned}8\lambda_1 + F\lambda_4 - C_1 &= 0\\ 4\lambda_2 + (F_{12} - F_{56})\lambda_4 - C_2 &= 0\\ 4\lambda_3 + (F_{34} - F_{78})\lambda_4 - C_3 &= 0\\ (F_{12} + F_{34} + F_{56} + F_{78})\lambda_1 + (F_{12} - F_{56})\lambda_2 + (F_{34} - F_{78})\lambda_3 + F^2\lambda_4 - C_4 &= 0\end{aligned}$$

where $F = f_1 + f_2 + \ldots + f_8$

$F_{12} = f_1 - f_2$

$F_{34} = f_3 - f_4$

$F_{56} = f_5 - f_6$

$F_{78} = f_7 - f_8$

$F^2 = f_1^2 + f_2^2 + \ldots + f_8^2$.

Now taking

$$\begin{aligned}F_{12} - F_{56} &= B\\ F_{34} - F_{78} &= C\\ F_{12} + F_{34} + F_{56} + F_{78} &= A\end{aligned}$$

we have

$$8\lambda_1 + F\,\lambda_4 - C_1 = 0$$
$$4\lambda_2 + B\lambda_4 - C_2 = 0$$
$$4\lambda_3 + C\lambda_4 - C_3 = 0$$
$$A\lambda_1 + B\lambda_2 + C\lambda_3 + F^2\lambda_4 - C_4 = 0\,.$$

The solution of the above four equations yields the values of the correlates λ_1, λ_2, λ_3, and λ_4. The corrections c_1, c_2, ….c_8 to the angles are calculated from equations (6) by substituting the values of the correlates.

By adopting some new notations above and putting the entire calculations in tabular form as in Table-1, the author has tried to make the above steps of calculations simpler and straight forward. It also gives various checks to have a check on the computations.

To explain the use of Table-1, a braced quadrilateral shown in Fig. 1 having the following observed angles, has been adjusted in Table-2.

θ_1 = 40°08′17.9″, θ_2 = 44°49′14.7″
θ_3 = 53°11′23.7″, θ_4 = 41°51′09.9″
θ_5 = 61°29′34.3″, θ_6 = 23°27′51.2″
θ_7 = 23°06′37.3″, θ_8 = 71°55′49.0″

Reference

Chandra A. M.: ‘Higher Surveying’, A text book published by New Age International Pvt. Ltd., Publishers, New Delhi, (2002).

Table-1: Chandra’s table for adjustment of a braced quadrilateral by rigorous method

<table>
<tr><th colspan="2">Angle</th><th rowspan="2">Correct-ion</th><th rowspan="2">f</th><th rowspan="2">F's</th><th rowspan="2">Coefficients</th><th rowspan="2">Equations in λ</th><th rowspan="2">λ</th><th rowspan="2">Corrections</th><th rowspan="2">Corrected angle</th></tr>
<tr><th>Left</th><th>Right</th></tr>
<tr><td>θ_1 =</td><td></td><td rowspan="2">C_1 =</td><td>f_1 =</td><td rowspan="8">F =
F_{12} =
F_{34} =
F_{56} =
F_{78} =
F^2 =</td><td rowspan="8">F=
$F_{12}-F_{56}=B$ =
$F_{34}-F_{78}=C$ =
$(F_{12}+F_{34}+F_{56}+F_{78})=A$ =
F^2=</td><td rowspan="2">$8\lambda_1+F\lambda_4=C_1$</td><td rowspan="2">$\lambda_1$ =</td><td>$c_1=\lambda_1+\lambda_2+f_1\lambda_4$ =</td><td>θ_1+c_1 =</td></tr>
<tr><td></td><td>θ_2 =</td><td>f_2 =</td><td>$c_2=\lambda_1+\lambda_2-f_2\lambda_4$ =</td><td>θ_2+c_2 =</td></tr>
<tr><td>θ_3 =</td><td></td><td rowspan="2">C_2 =</td><td>f_3 =</td><td rowspan="2">$4\lambda_2+B\lambda_4=C_2$</td><td rowspan="2">$\lambda_2$ =</td><td>$c_3=\lambda_1+\lambda_3+f_3\lambda_4$ =</td><td>θ_3+c_3 =</td></tr>
<tr><td></td><td>θ_4 =</td><td>f_4 =</td><td>$c_4=\lambda_1+\lambda_3-f_4\lambda_4$ =</td><td>θ_4+c_4 =</td></tr>
<tr><td>θ_5 =</td><td></td><td rowspan="2">C_3 =</td><td>f_5 =</td><td rowspan="2">$4\lambda_3+C\lambda_4=C_3$</td><td rowspan="2">$\lambda_3$ =</td><td>$c_5=\lambda_1-\lambda_2+f_5\lambda_4$ =</td><td>θ_5+c_5 =</td></tr>
<tr><td></td><td>θ_6 =</td><td>f_6 =</td><td>$c_6=\lambda_1-\lambda_2-f_6\lambda_4$ =</td><td>θ_6+c_6 =</td></tr>
<tr><td>θ_7 =</td><td></td><td rowspan="2">C_4 =</td><td>f_7 =</td><td rowspan="2">$A\lambda_2+B\lambda_2+C\lambda_3+F^2\lambda_4=C_4$</td><td rowspan="2">$\lambda_4$ =</td><td>$c_7=\lambda_1-\lambda_3+f_7\lambda_4$ =</td><td>θ_7+c_7 =</td></tr>
<tr><td></td><td>θ_8 =</td><td>f_8 =</td><td>$c_8=\lambda_1-\lambda_3-f_8\lambda_4$ =</td><td>θ_8+c_8 =</td></tr>
<tr><td colspan="10">$C_1=360°-\sum_{i=1}^{8}\theta_i$; $C_2=(\theta_5+\theta_6)-(\theta_1+\theta_2)$; $C_3=(\theta_7+\theta_8)-(\theta_3+\theta_4)$; $C_4=[\Sigma\log\{\sin(\text{Left angles})\}-\Sigma\log\{\sin(\text{Right angles})\}]\times10^7$
$f_i=[\log\{\sin(\theta_i+1'')\}-\log\{\sin(\theta_i)\}]\times10^7$, $i=1$ to 8; $F=\sum_{i=1}^{8}f_i$; $F_{12}=f_1-f_2$; $F_{34}=f_3-f_4$; $F_{56}=f_5-f_6$; $F_{78}=f_7-f_8$; $F^2=\sum_{i=1}^{8}f_i^2$</td></tr>
</table>

<table>
<tr><td>Checks :</td><td>(1) $\sum_{i=1}^{8}c_i$; $=C_1$
(2) $(c_1+c_2)-(c_5+c_6)=C_2$
(3) $(c_3+c_4)-(c_7+c_8)=C_3$</td></tr>
</table>

Table-2: Chandra's table for adjustment of a braced quadrilateral by rigorous method

Angle Left	Angle Right	C	f	F's	Coefficients	Equations in λ	λ	Corrections	Corrected angle
θ_1 = 40°08'17.9"		C_1 = 2"	f_1 = 25			$8\lambda_1+F\lambda_4=C_1$	λ_1	$c_1=\lambda_1+\lambda_2+f_1\lambda_4$ = −2.671"	θ_1+c_1 = 40°08'15.23"
	θ_2 = 44°49'14.7"		f_2 = 21	F = 0.0		$8\lambda_1+0.0\lambda_4=2$	= 0.25	$c_2=\lambda_1+\lambda_2-f_2\lambda_4$ = +0.963"	θ_2+c_2 = 44°49'15.66"
θ_3 = 53°11'23.7"		C_2 = −7.1"	f_3 = 16	F_{12} = 4	F = 0.0	$4\lambda_2+B\lambda_4=C_2$	λ_2 = −0.946	$c_3=\lambda_1+\lambda_3+f_3\lambda_4$ = −3.827"	θ_3+c_3 = 53°11'19.87"
	θ_4 = 41°51'09.9"		f_4 = 24	F_{34} = −8	$F_{12}-F_{56}=B$ = 42; $F_{34}-F_{78}=C$ = −50	$4\lambda_2+42\lambda_4=-7.1$		$c_4=\lambda_1+\lambda_3-f_4\lambda_4$ = −0.667"	θ_4+c_4 = 41°51'09.23"
θ_5 = 61°29'34.3"		C_3 = −7.3"	f_5 = 11	F_{56} = −38	$(F_{12}+F_{34}+F_{56}+F_{78})=A$ = 0	$4\lambda_3+C\lambda_4=C_3$	λ_3 = −2.813	$c_5=\lambda_1-\lambda_2+f_5\lambda_4$ = +0.327"	θ_5+c_5 = 61°29'34.63"
	θ_6 = 23°27'51.2"		f_6 = 49	F_{78} = 42	F^2 = 6870	$4\lambda_3-50\lambda_4=-7.3$		$c_6=\lambda_1-\lambda_2-f_6\lambda_4$ = +5.067"	θ_6+c_6 = 23°27'56.27"
θ_7 = 23°06'37.3"		C_4 = −437.984	f_7 = 49	F^2 = 6870		$A\lambda_2+B\lambda_2+C\lambda_3+F^2\lambda_4=C_4$	λ_4 = −0.079	$c_7=\lambda_1-\lambda_3+f_7\lambda_4$ = −0.808"	θ_7+c_7 = 23°06'36.49"
	θ_8 = 71°55'49.0"		f_8 = 7			$0\lambda_2+42\lambda_2-50\lambda_3+6870\lambda_4=-437.984$		$c_8=\lambda_1-\lambda_3-f_8\lambda_4$ = +3.616"	θ_8+c_8 = 71°55'52.62"

$C_1=360°-\sum_{i=1}^{8}\theta_i$; $C_2=(\theta_1+\theta_6)-(\theta_2+\theta_5)$; $C_3=(\theta_3+\theta_8)-(\theta_4+\theta_7)$; $C_4=[\Sigma\log\{\sin(\text{Left angles})\}-\Sigma\log\{\sin(\text{Right angles})\}]\times 10^7$

$f_i=[\log\{\sin(\theta_i+1'')\}-\log\{\sin(\theta_i)\}]\times 10^7$, $i=1$ to 8; $F=\sum_{i=1}^{8} f_i$; $F_{12}=f_1-f_2$; $F_{34}=f_3-f_4$; $F_{56}=f_5-f_6$; $F_{78}=f_7-f_8$; $F^2=\sum_{i=1}^{8} f_i^2$

Checks:

(1) $\sum_{i=1}^{8} c_i$; $= -2.671''+0.963''-3.827''-0.667''+0.327''+5.067''-0.808''+3.616''=2''$

(2) $(c_1+c_2)-(c_5+c_6)=(-2.671''+0.963'')-(0.327''-5.067'')=-7.1''$

(3) $(c_3+c_4)-(c_7+c_8)=(-3.827''-0.667'')-(-0.808''+3.616'')=-7.3''$