Oscillating Supersonic delta wing with Straight Leading Edges

[1]Asha Crasta, [2]S.A. Khan

1. Research Scholar, Department of Mathematics, Jain University, Bangalore,
2. Principal, Department of Mechanical Engineering, P.A College of Engineering, Mangalore

Abstract:
A Supersonic similitude has been used to obtain stability derivatives in pitch and roll of a delta wing with straight leading edge for the attached shock case. Ghosh's strip theory is been used in which strips at different span wise locations are independent of each other. This combines with the similitude to give a piston theory which gives the closed form of solution to stability derivatives in pitch and roll. Some of the results obtained have been compared with those of Hui et al ,Ghosh and Lui &Hui. Results have been obtained for supersonic flow of perfect gas over a wide range of Mach numbers, incidences and sweep angles.

Key Words: Aspect ratio, Attached shock wave, Delta wings, Damping derivative, Leading edge, Rolling derivative, Stiffness derivative, Supersonic Flow, unsteady flow

1. Introduction

Sychev's [1] large incidence hypersonic similitude is applicable to a wing provided it has an extremely small span in addition to small thickness. Cole and Brainered [2] have given a solution for a delta wing of very small span at large incidence. Messiter [3] has found a solution, in the realm of thin shock layer theory, for a steady delta wing with a detached shock; the attached shock case has been studied by Squire. Malmuth [5] obtained an analytical solution for the attached shock case at small incidence based on hypersonic small disturbance theory. Pike [4] and Hui [6]have analytically treated the steady delta wing in supersonic/hypersonic flow with an attached shock.

The role of dynamic stability at high incidence during re-entry or maneuver has been pointed out by Orlik-Ruckemann [7]. The shock-attached relatively high aspect ratio delta is often preferred (Townend) [8] for its high lift and drag ratio. Hui and Hemdan [9] have studied the unsteady shock detached case in the context of thin shock layer theory. Lui and Hui [10] have extended Hui's theory to an oscillating delta. Hui et. al. [11] has treated flat wings of arbitrary plan forms oscillating in pitch in supersonic/hypersonic flow. Ericsson [12] has used embedded Newtonian concept for unsteady flows. Ghosh [14] has developed a large incidence 2-D hypersonic similitude and piston theory; it includes lighthill's and Miles piston theories. This theory has been applied for oscillating plane ogives. Ghosh [15] has extended the large deflection similitude to non-slender cones, quasi cones and shock attached delta wings. This similitude in this paper has been extended to oscillating delta wings with straight leading edges past a supersonic flow.

2. Analysis:

A thin strip of the wing, parallel to the centerline, can be considered independent of the z dimension when the velocity component along the z direction is small. This has been discussed by Ghosh's [16]. The strip theory combined with Ghosh's large incidence similitude leads to the "piston analogy" and pressure P on the surface can be directly related to equivalent piston Mach number M_p. In this case both Mp and flow deflections are permitted to be large. Hence light hill piston theory [17] or miles strong shock piston theory cannot be used but Ghosh's piston theory will be applicable.

$$\frac{P}{P_\infty} = 1 + AM_P{}^2 + AM_P(B + M_P{}^2)^{\frac{1}{2}} \qquad \text{, Where } P_\infty \text{ is free stream pressure} \qquad (1)$$

Since strips at different span wise location are assumed independent of each other, the strip can be considered as a flat plate at an angle of attack. The angle of incidence is same as that of wing. Angle ϕ is the angle between the shock and the strip. A piston theory which has been used in equation (1) has been extended to supersonic flow. The expression is given below.

$$\frac{p}{p_\infty} = 1 + A\left(\frac{M_p}{\cos\phi}\right)^2 + A\left(\frac{M_p}{\cos\phi}\right)\left(B + \left(\frac{M_p}{\cos\phi}\right)^2\right)^{\frac{1}{2}} \qquad (2)$$

Where p_∞ is free stream pressure, $A = \frac{(\gamma+1)}{4}, B = (4/(\gamma+1)^2, \gamma$ is the specific heat ratio and M_p = the local piston Mach number normal to the wedge surface.

2.1 Pitching moment derivatives

Let the mean incidence be α_0 for the wing oscillating in pitch with small frequency and amplitude about an axis x_0. The piston velocity and hence pressure on the windward surface remains constant on a span wise strip of length 2z at x. The pressure on the lee surface is assumed Zero. Therefore the nose up moment is

$$m = -2\int_0^c p.z(x - x_0)dx \tag{3}$$

The stiffness derivative is non-dimensionalized by dividing with the product of dynamic pressure, wing area and chord length.

$$\therefore -C_{m_\alpha} = \frac{2}{\rho_\infty U_\infty{}^2 C^3 \cot\varepsilon}\left(-\frac{\partial m}{\partial \alpha}\right)_{\substack{\alpha=\alpha_0 \\ q=0}} \tag{4}$$

The damping derivative is non-dimensionalised by dividing with the product of dynamic pressure, wing area, chord length and characteristic time factor $\left(\frac{c}{U_\infty}\right)$

$$\therefore -{}^{C}m_q = \frac{2}{\rho_\infty U_\infty c^4(\cot\varepsilon)}\left(-\frac{\partial m}{\partial q}\right)_{\substack{\alpha=\alpha_o \\ q=0}} \tag{5}$$

The local piston Mach number normal to the wing surface is given by

$$M_p = M_\infty \sin\alpha + \frac{q}{a_\infty}(x - x_o) \tag{6}$$

Where ρ_∞, a_∞ are density and velocity of sound in the free stream? Combining (2) through (6), differentiation under the integral sign is performed.

Defining $x_0 = hL$, $S_1 = \frac{M_\infty \sin\alpha_o}{\cos\phi}$, the derivatives in pitch of a delta wing become equal to

$$-C_{m_\alpha} = \frac{\sin\alpha_o \cos\alpha_o f(S_1)}{\cos^2\phi}\left[\left(\frac{2}{3} - h\right)\right] \tag{7}$$

$$-C_{m_q} = \frac{\sin\alpha_o f(S_1)}{\cos^2\phi}\left[\left(h^2 - \frac{4}{3}h + \frac{1}{2}\right)\right] \tag{8}$$

$$\text{Where}\quad f(S_1) = \frac{(r+1}{2S_1}[2S_1 + (B + 2S_1{}^2/(B + 2S_1{}^2)^{\frac{1}{2}}] \tag{9}$$

2.2Rolling Damping Derivative:

Let the rate of roll be $\overline{p}$ and rolling moment be L, defined according to the right hand system of reference.

$$\therefore L = 2\int_0^c \left(\int_0^{Z=f(x)} p.zdz\right)dx \tag{10}$$

The piston Mach number is given by

$$M_p = M_\infty \sin\alpha - \frac{z}{a_\infty}\overline{p} \tag{11}$$

The roll damping derivative is non-dimensionalized by dividing with the product of dynamic pressure, wing area, and span and characteristic time factor $\frac{C}{U_\infty}$

$$\therefore -C_{l_p} = \frac{1}{\rho_\infty U_\infty C^3 b \cot\varepsilon}\left(\frac{-\partial L}{\partial p}\right)_{\substack{\alpha=\alpha_0 \\ \bar{p}=0}} \qquad (12)$$

Combining through (10) to (12)

$$\therefore -C_{l_p} = \frac{\sin\alpha_o f(S_1)}{(\cos^2\phi)}\left[\frac{\cot\ \varepsilon}{12}\right] \qquad (13)$$

Where $f(S_1) = \frac{(r+1}{2S_1}[2S_1 + (B+2S_1^2/(B+2S_1^2)^{\frac{1}{2}}]$

3. Results and discussions:

The variation of the stability derivatives with pivot position for various Mach numbers and angle of incidence is shown in Figs. 1 to 4. The stiffness and damping derivative have been compared with Hui et al. (Fig 5. to Fig. 8).The Stiffness derivative shows good agreement. The difference in the damping derivative is attributed to the present theory being a quasi-steady one whereas Liu and Hui [13] give an unsteady theory which predicts$C_{m\dot\theta}$. The present work invokes strip theory arguments. Hui et al [11] also use strip theory arguments whereby the flow at any span wise station is considered equivalent to an oscillating flat plate flow; this is calculated by perturbing the known steady flat plate flow (oblique shock solution) which serves as the 'basic flow' for the theory. For a pitching wing the mean incidence is the same for all 'strips' (irrespective of span wise location) and hence there is a single 'basic flow' which Hui et al have utilized to obtain closed form expression for stiffness and damping derivatives. They have not calculated the roll damping derivative. For a rolling wing the 'strips' are at different incidences and there is no single 'basic flow'; hence it is doubtful whether approach can be extended to yield a closed form expression for roll damping derivative. Their theory is valid for supersonic as well as hypersonic flows; whereas the present theory also gives closed form expressions for Stiffness & damping derivatives in pitch as well as roll damping derivative. Liu and Hui's [10] theory is more accurate than of Hui et al [11] as far hypersonic flow is concerned.

The present theory is in good agreement with Hui et al [11] for angle of incidence up to thirty degrees and then there is no matching with the results of Hui et al [11]. This may be due to the detachment of the shock wave and stalling of the flow (Fig. 8). The present theory is simpler than both Liu and Hui [9] and Hui et al [13] and brings out the explicit dependence of the derivatives on the similarity parameters S1. (Fig. 9). Fig.10 presents the variation of damping derivatives with pivot position. There is a disagreement with Liu & Hui [9] as well as Hui [13] and the reasons for the disagreement are the same as discussed earlier. Fig. 11, Fig.12, and Fig.13 present the variation of damping derivatives with mean flow incidence for h = 0, h= 1.0 and h = 0.6. They are in disagreement with Liu & Hui [9] as well as Hui [13] for the angle of incidence more than thirty degrees. This may be due to the detachment of the shock wave and stalling of the flow. Fig.14 and Fig. 15 show the dependence of Roll damping derivative with Mach number and with the aspect ratio. The roll damping derivative decreases with Mach number initially then confirms the Mach number independence principle for large Mach numbers. Further, the roll damping derivative increases with aspect ratio of the wing.

There was an error in the formulation in Ghosh [16] for roll damping derivatives and hence in the present work the same has been corrected and implemented

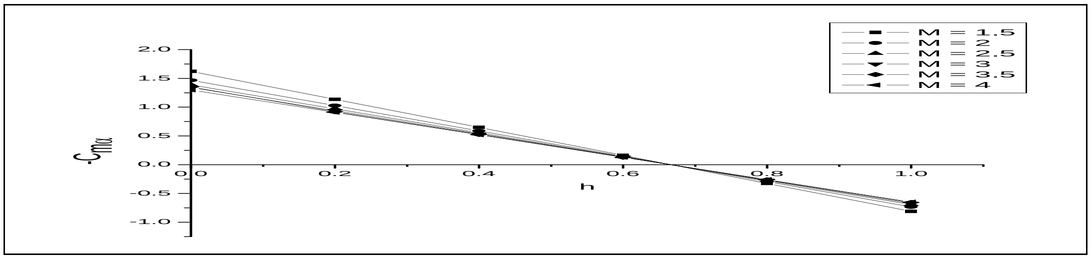

Fig.1: Variation of Stiffness derivative with pivot position α_0 =25

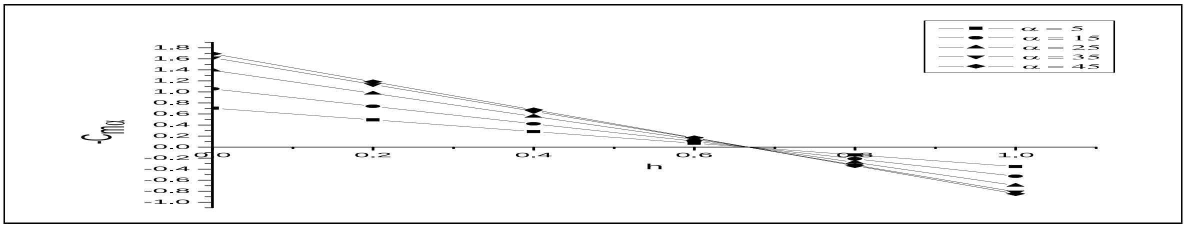

Fig. 2: Variation of stiffness derivative with the pivot position

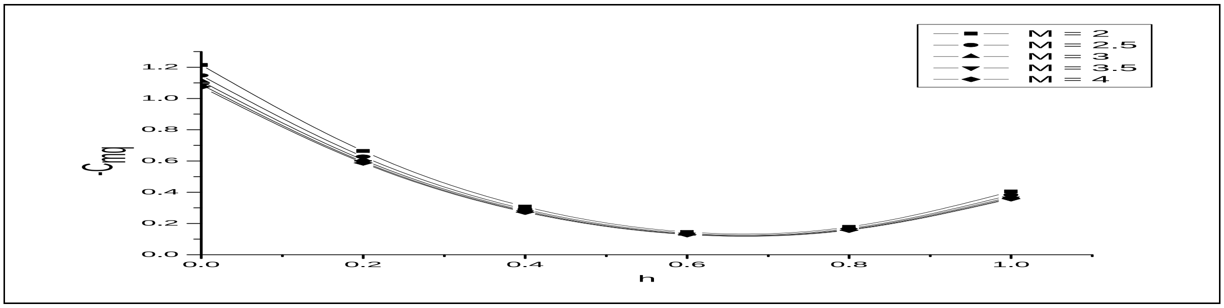

Fig. 3: Variation of Damping derivative with pivot position $\square_0$=25

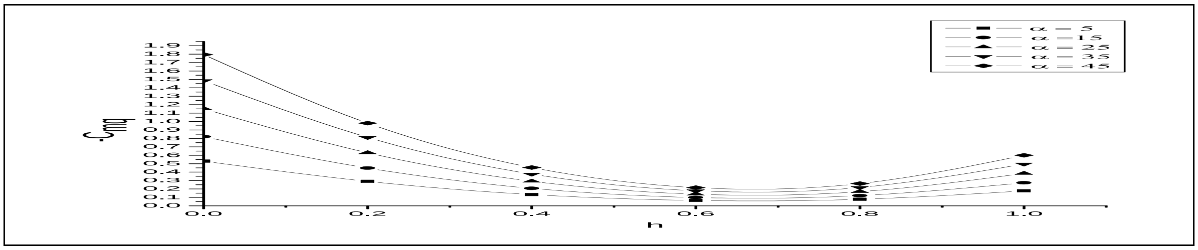

Fig. 4: Variation of damping derivative with pivot position

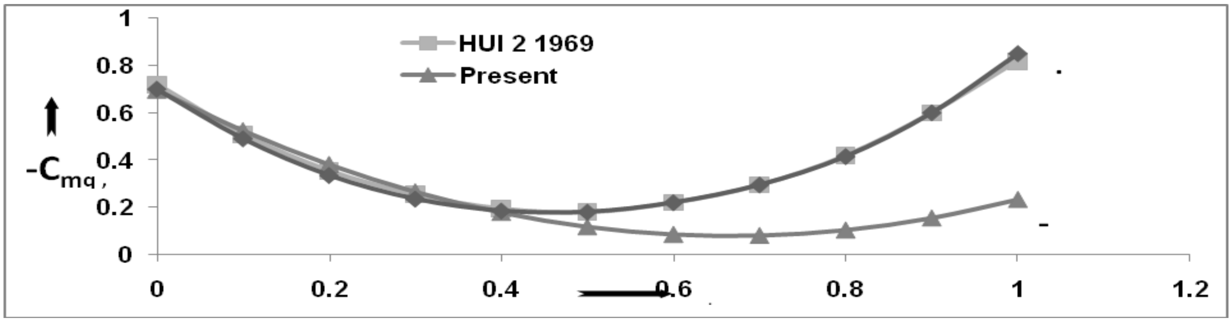

Fig. 5: Variation of Damping Derivative of a Wing with Pivot position with M=3, $\square_0$ =10 ,γ=1.4

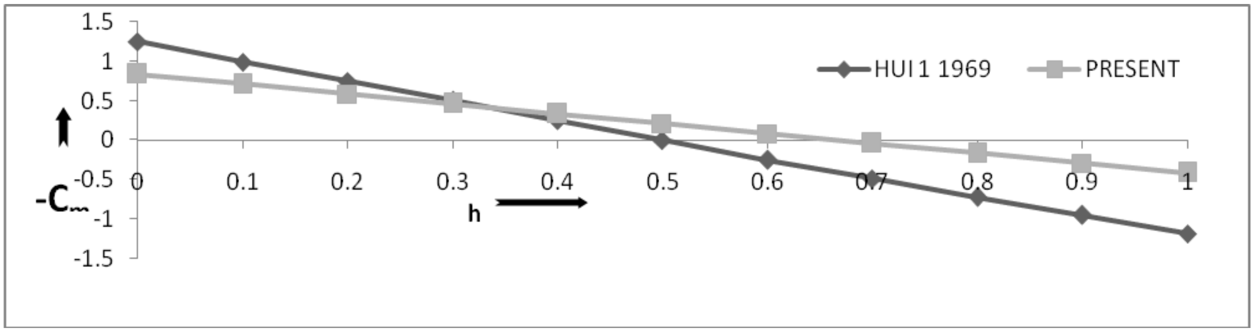

Fig. 6: Variation of Stiffness Derivative of a Wing with Pivot position with M=2.47, $\square_0$ =6°51', γ =1.4

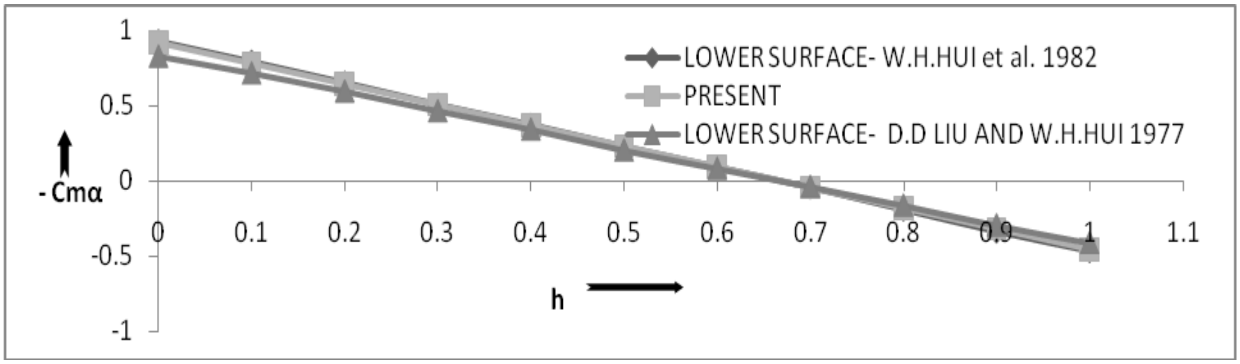

Fig. 7: Comparison of Stiffness Derivative with Theory of Liu and Hui for Triangular Wing with M=4, $\theta_0 = 15^0$, γ =1.4

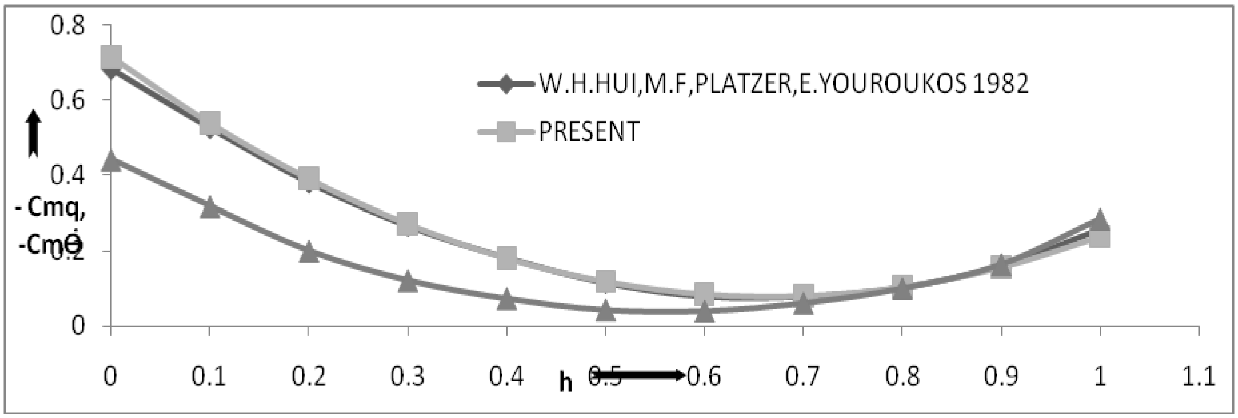

Fig. 8: Comparison of Damping Derivative of Triangular Wing with M=4, θ_0=15^0, γ=1.4

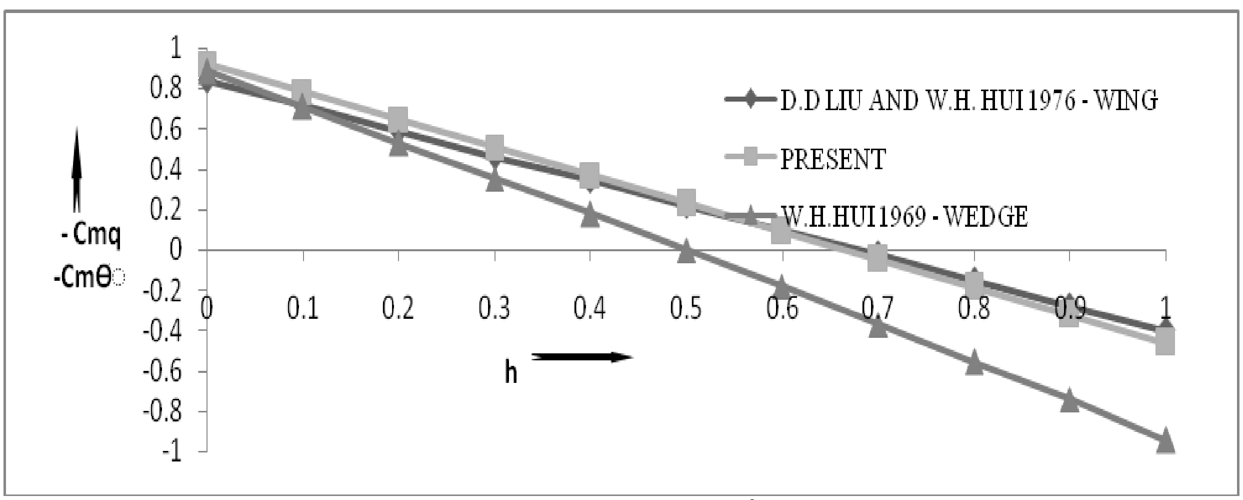

Fig.9: Stiffness Derivative Vs Pivot Position for M= 4, θ_0 = 15^0, γ = 1.4

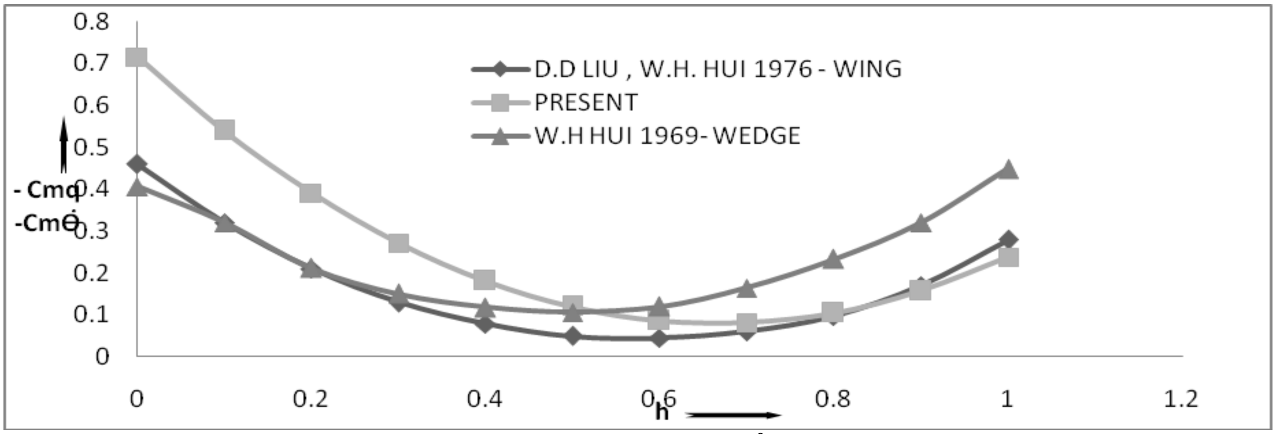

Fig. 10: Damping Derivative Vs Pivot Position for M=4, $\theta_0 = 15^0$, γ=1.4

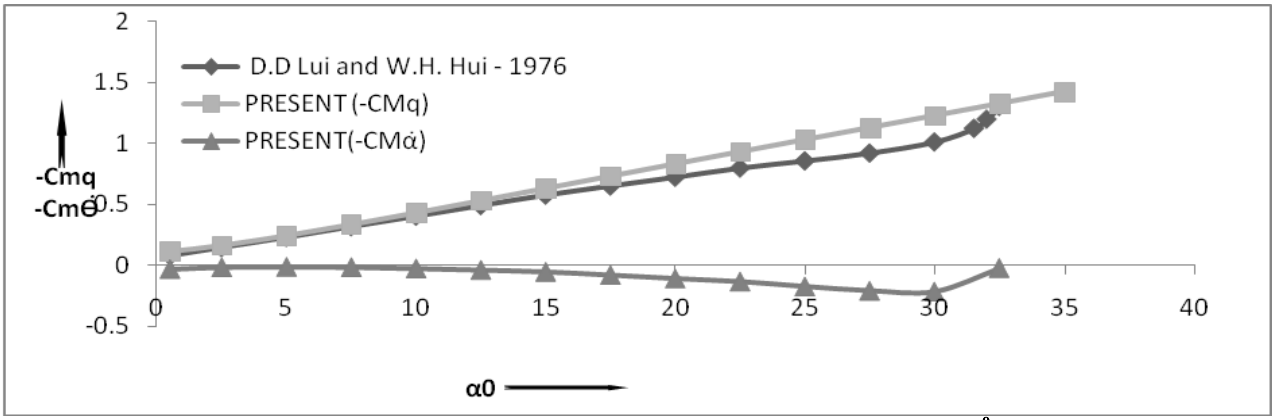

Fig. 11: Variation of Stability Derivatives with Mean Flow Incidence with M=4, $\theta_0=15^0$,Pivot position, h=0

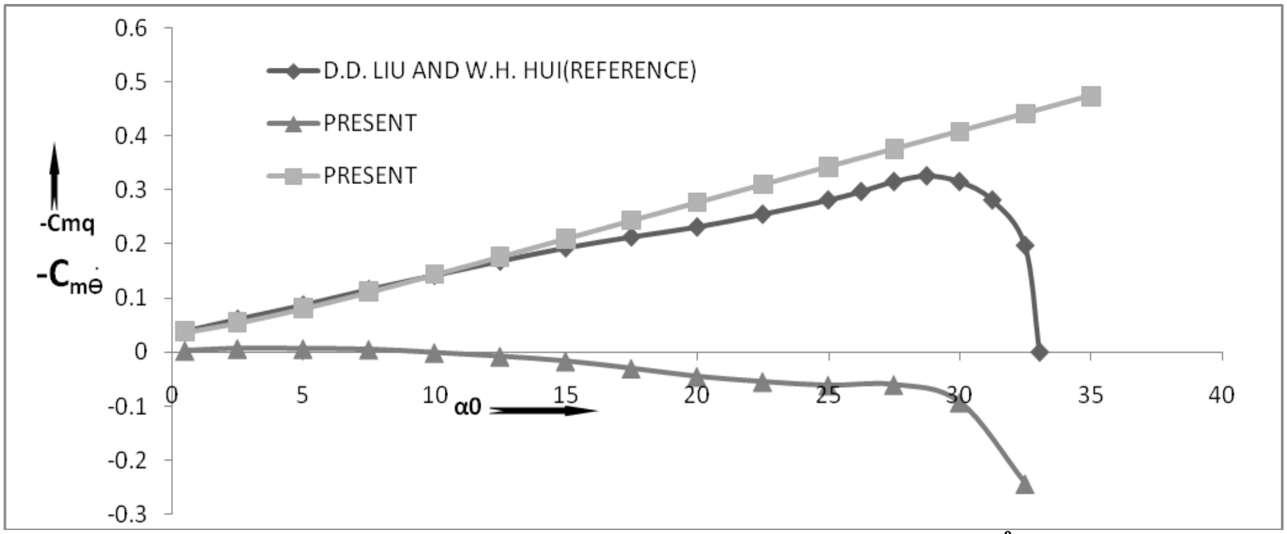

Fig. 12: Variation of Stability Derivative with Mean Flow Incidence with M=4, $\theta_0 = 15^0$ Pivot position, h = 1

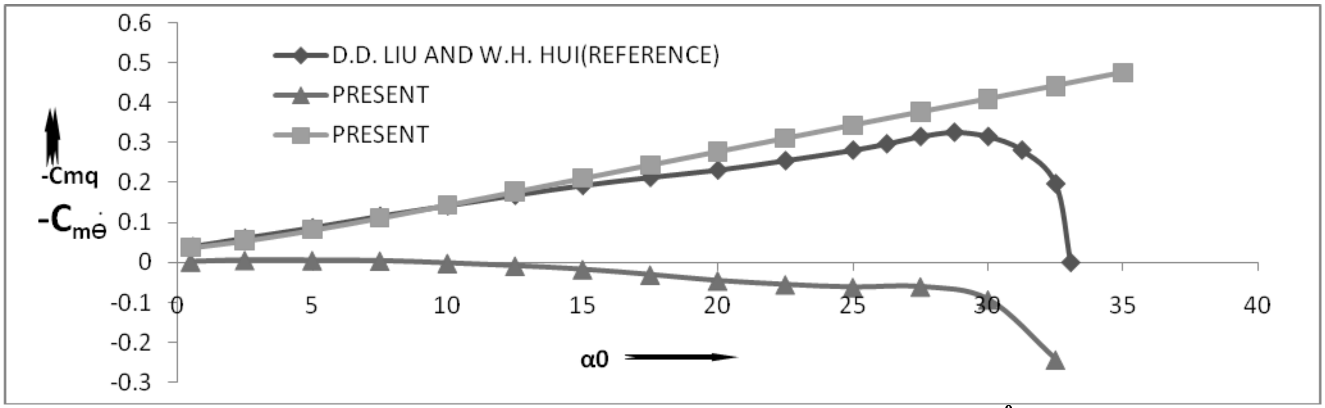

Fig.13: Variation of Stability Derivative Vs Mean Flow Incidence with M=4, $\theta_0 = 15^0$, Pivot position, h = 0.6

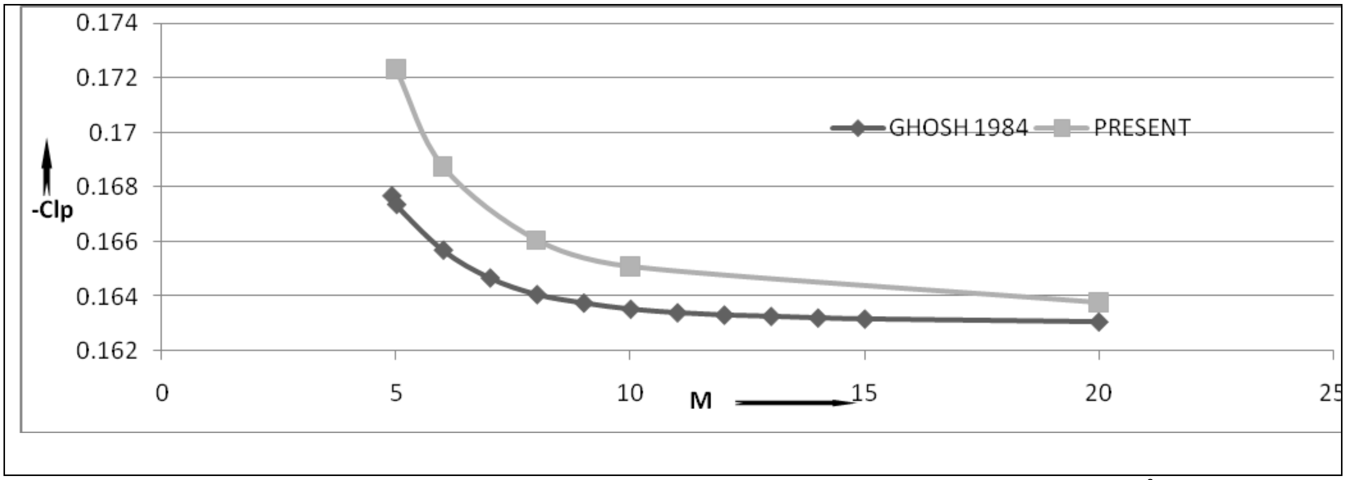

Fig.14: Rolling Moment Derivative Vs Mach number with Aspect Ratio, AR= 4.764, $\theta_0 = 25^0$, γ=1.4

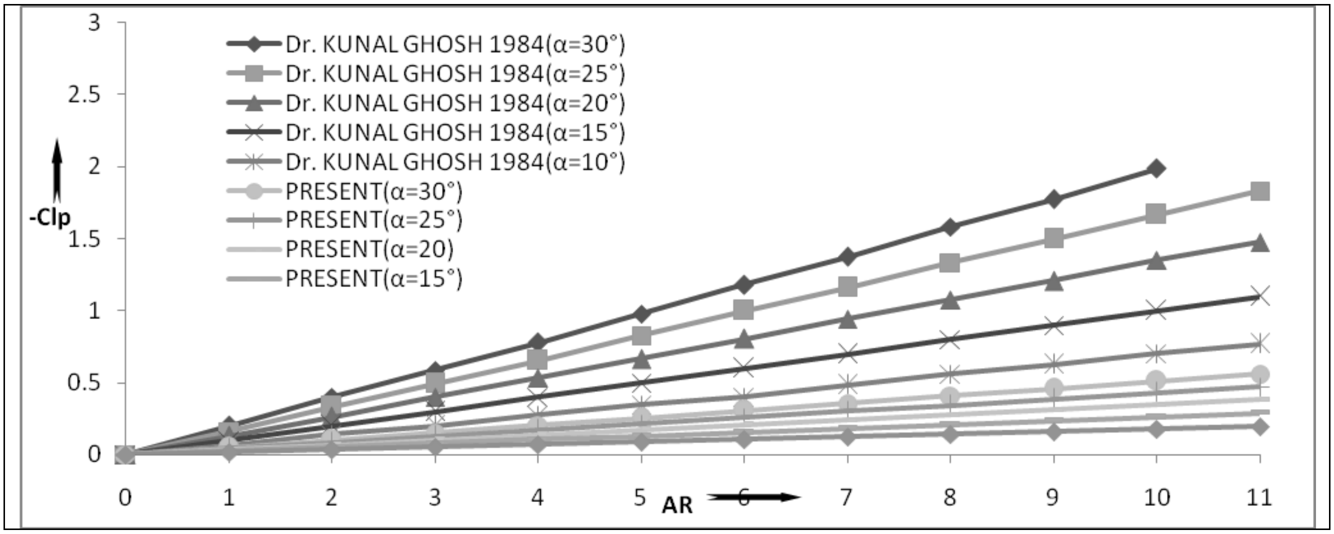

Fig.15: Roll Damping Derivative Vs Aspect Ratio of Delta Wings with M = 10

References

[1.] Sychev, V. V, “Three Dimensional Hypersonic Gas flow Past Slender Bodies at High Angles of Attack”, Journal of Applied Mathematics and Mechanics”, Vol. 24, Aug. 1960, pp. 296-306.

[2.] Cole, J.D. and Brainerd, J. J., “Slender wings at high angles of attack in hypersonic flow”, ARS Reprint 1980-61, 1961.

[3.] Messiter, A.F., “Lift of slender delta wings according to Newtonian theory”, AIAA Journal, 1963, 1, 794-802.

[4.] Pike. J.The pressure on flat and anhydral delta wings with attached shock waves, the Aeronautical Quarterly, November 1972, XXIII, Part 4, pp. 253-262.

[5.] Malmuth, N. D., “Hypersonic flow over a delta wing of moderate aspect ratio”, AIAA Journal, 1966, 4, pp. 555-556.

[6.] Hui, W. H., “Supersonic and hypersonic flow with attached shock waves over delta wings”, Proc. of Royal Society, London, 1971, A. 325, pp. 251-268.

[7.] Orlik-Ruckemann, K.J., “Dynamic stability testing of aircraft needs versus capabilities”, Progress in the Aerospace Sciences, Academic press, N.Y., 1975, 16, pp. 431- 447.

[8.] Towend, L. H., “Some design aspects of space shuttle Orbiters’”, RAE TR 70139, 1970.

[9.] Hui, W. H. and Hemdan, H. T., “Unsteady hypersonic flow over delta wings with detached shock waves”, AIAA Journal, April 1976 , 14, pp. 505-511.

[10.] Lui D. D. and Hui W. H., “Oscillating delta wings with attached shock waves”, AIAA Journal, June 1977, 15, 6, pp. 804-812.

[11.] Hui,W. H., Platzer, M. F. and Youroukos E., “Oscillating supersonic/hypersonic wings at high incidence”, AIAA Journal, March 1982, 20, pp. 299-304.

[12.] Ericsson L. E., “Viscous and Elastic Perturbation Effects on Hypersonic Unsteady Airfoil Aerodynamics”, AIAA Journal, Vol.15 Oct. 1977, pp. 1481-1490.

[13.] Hui, W. H., “Stability of Oscillating Wedges and Caret Wings in Hypersonic and Supersonic Flows”, AIAA Journal, Vol. 7, August 1969, pp. 1524-1530.

[14.] Ghosh, K. and Mistry, B. K., “Large incidence hypersonic similitude and oscillating non-planar wedges”, AIAA Journal, August 1980, 18, 8, 1004-1006.

[15.] Ghosh K., “Hypersonic large-deflection similitude for quasi wedges and quasi-cones”, The Aeronautical Journal, March 1984, 88, 873, pp. 70-76.

[16.] G hosh K., “Hypersonic large deflection similitude for oscillating delta wings”, The Aeronautical journal, Oct. 1984, pp. 357-361.

[17.] LightHill M. J., “Oscillating Aerofoil at High Mach Numbers, Journal of Aeronautical Sciences”, Vol. 20, June 1953, pp. 402-406.