Heat and Mass Transfer with Variable Temperature and Exponential Mass Diffusion

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Abstract

In this paper an analytical study is performed to study heat and mass transfer with variable temperature and exponential mass diffusion, the results were obtained for velocity, temperature and concentration, the dimensionless governing equations are tackled by the Laplace transform method, and computed for parameters namely thermal Grashof number Gr, mass Grashof number Gc, Schmidt number Sc, Prandtl number Pr, time t, and acceleration a . It is observed that the velocity increases with increasing values of Gr, Gc, a and t, It was also observed that velocity, temperature and concentration decreases with increasing Pr and Sc respectively.

Key Word: exponential, mass transfer, variable temperature, mass diffusion.

1 Introduction

Heat and mass transfer plays an important role in drying, filtration processes, saturation of porous materials by chemicals, solar energy collectors, nuclear reactors, in manufacturing industries for the design fins, steel, rolling, nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, space craft design, satellites, combustion and furnace design, material processing, energy utilization, temperature measurement, remote sensing for astronomy and space exploration, food processing and cryogenic Engineering, as well as numerous agricultural, health and military application. The study of convection flow with mass transfer along a vertical plate is receiving considerable attention to many researchers because of its vast application in the field of cosmically and geophysical science. England and Emery (1969) have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate, Gupta et al. (1979) have studied free convective effects flow past accelerated vertical plate in incompressible dissipative fluid, Mass transfer and free convection effects on the flow past an accelerated vertical plate with variable suction or injection, was studied by Kafousia and Raptis (1981), Jha et al. (1991)analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Raptis and Perdikis (1999) analyzed the Radiation and free convection flow past a moving plate, Chamkha and Soundalgekar (2001) have analyzed radiation effects on free convection flow Past a semi-infinite vertical plate with mass transfer, Chaudhary and Jain (2006) analyzed Influence of fluctuating surface temperature and velocity on medium with heat absorption, Toki (2006) studied unsteady free convective flow on a vertical oscillating porous plate with heat , Alam et al. (2008) have analyzed the effects of variable suction and thermophoresis on steady MHD combined free – forced convective heat and mass transfer flow over a semi-infinite permeable inclined plate in the presence of thermal radiation, Muthucumaraswamy et al. (2009) have studied the exact solution of flow past an accelerated infinite vertical plate with heat and mass flux. It is proposed to study heat and mass transfer with variable temperature and exponential mass diffusion. The dimensionless governing equations are solved using the Laplace transform technique.
The solutions are in terms of exponential and complementary error function.

2 Problem Formulation:

Governing equation for Heat and mass transfer with variable temperature and exponential mass diffusion. Then under usual Boussinesq's approximation the unsteady flow equations are presented as momentum equation, energy equation, and mass equation respectively.

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_\rho \frac{\partial T}{\partial t'} = K\frac{\partial^2 T}{\partial y^2} + q_o(T - T_\infty) \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - KC' \tag{3}$$

The initial and boundary conditions are:

$$\left.\begin{array}{lllll} & U=0 & T=T_\infty & C'=C'_\infty & for\ all\ \ y,t'\le 0 \\ t'>0: & U=u_0 t' & T=T_\infty+(T_w-T_\infty)At' & C'=C'_\infty+(C'_w-C'_\infty)e^{a't'} & at\quad y=0 \\ & U\to 0 & T\to T_\infty & C'\to C'_\infty & as\quad y\to\infty \end{array}\right\} \quad (4)$$

$$where \qquad A=\frac{u_0^{\,2}}{\nu}$$

Where u is the velocity of the fluid, T is the fluid temperature, C' is the concentration, g is gravitational constant, $\beta\, and\, \beta^*$ are the thermal expansion of fluid, t' is the time, ρ is the fluid density, C_ρ is the specific heat capacity, V is the velocity of the fluid, k is the thermal conductivity. The non-dimensional quantities are:

$$\left.\begin{array}{l} U=\dfrac{u}{u_0},\quad t=\dfrac{t' u_0^{\,2}}{\nu},\quad Y=\dfrac{y u_0}{\nu},\quad \theta=\dfrac{T-T_\infty}{T_w-T_\infty} \\ \Pr=\dfrac{\mu C_p}{k},\quad a=\dfrac{a'\nu}{u_0^{\,2}},\quad Sc=\dfrac{\nu}{D},\quad F=\dfrac{q_0 u_0^{\,2}}{k},\quad R=\dfrac{k\nu}{u_0^{\,2}}, \\ Gr=\dfrac{g\beta\nu(T_w-T_\infty)}{u_0^{\,3}},\quad C=\dfrac{C'-C'_\infty}{C'_w-C'_\infty},\quad Gc=\dfrac{\nu g\beta^*(C'_w-C'_\infty)}{u_0^{\,3}}, \end{array}\right\} \quad (5)$$

Substituting the non-dimensional quantities of (5) in to (1) to (4) leads to dimensionless equations as:

$$\frac{\partial u}{\partial t}=Gr\theta+GcC+\frac{\partial^2 u}{\partial y^2} \quad (6)$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}-\frac{F\theta}{\Pr} \quad (7)$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-RC \quad (8)$$

Where Sc is the Schmidt number, Pr is Prandtl number, and Gr, Gc are the Grashof numbers, F is the heat source, R is the Concentration parameter.

The initial and boundary conditionsare reduces to:

$$\left.\begin{array}{llllll} & U=0, & \theta=0, & C=0, & for\ all & y,t\le 0 \\ t>0: & U=t, & \theta=t, & C=e^{at}, & at & y=0 \\ & U\to 0, & \theta\to 0, & C\to 0, & as & y\to\infty \end{array}\right\} \quad (9)$$

3 Method Of Solution

The dimensionless governing equations (6) to (8) with initial boundary conditions are solved using Laplace transform techniques and the results for temperature, concentration and velocity in terms of exponential and complementary error function:

$$L(\theta)=\frac{e^{-y\sqrt{(S\Pr+F)}}}{s^2} \quad (10)$$

$$L(C)=\frac{e^{-y\sqrt{Sc(s+R)}}}{s-a} \quad (11)$$

$$L(U)=\frac{e^{-y\sqrt{s}}}{s^2}+\frac{Gr(e^{y\sqrt{s}}-e^{-y\sqrt{(S\Pr+F)}})}{d^2(1-\Pr)(s-d)}-\frac{Gr(e^{-y\sqrt{s}}-e^{-y\sqrt{(S\Pr+F)}})}{d(1-\Pr)s^2}$$
$$-\frac{Gc(e^{-y\sqrt{s}}-e^{-y\sqrt{Sc(s+R)}})}{(1-Sc)(a-b)(s-a)}-\frac{Gc(e^{-y\sqrt{s}}-e^{-y\sqrt{Sc(s+R)}})}{(1-Sc)(b-a)(s-b)} \quad (12)$$

The Laplace inversion gives,

$$\theta=\frac{t}{2}\Big[\big(2\eta\sqrt{Ft}\big)erfc\big(\eta\sqrt{\Pr}+\sqrt{ct}\big)+\exp\big(-2\eta\sqrt{Ft}\big)\,erfc\big(\eta\sqrt{\Pr}-\sqrt{ct}\big)\Big] \quad (13)$$
$$-\frac{\eta\Pr\sqrt{t}}{2\sqrt{Ft}}\Big[\exp\big(-2\eta\sqrt{Ft}\big)\,erfc\big(\eta\sqrt{\Pr}-\sqrt{ct}\big)+\exp\big(2\eta\sqrt{Ft}\big)\,erfc\big(\eta\sqrt{\Pr}+\sqrt{ct}\big)\Big]$$

$$C=\frac{\exp(at)}{2}\Big[\exp\Big(2\eta\sqrt{Sc(a+R)t}\Big)\,erfc\Big(\eta\sqrt{Sc}+\sqrt{(a+R)t}\Big)+\exp\Big(-2\eta\sqrt{Sc(a+R)t}\Big)\,erfc\Big(\eta\sqrt{Sc}-\sqrt{(a+R)t}\Big)\Big] \quad (14)$$

$$
\begin{aligned}
U = {} & t\left[(1+2\eta^2)\,ercf\,\eta-\frac{2\eta\exp\left(-\eta^2\right)}{\pi}\right] -\frac{Gr\exp\left(dt\right)}{2d^2(1-\Pr)}\left[\exp\left(2\eta\sqrt{dt}\right)erfc\left(\eta+\sqrt{dt}\right)+\exp\left(-2\eta\sqrt{dt}\right)erfc\left(\eta-\sqrt{dt}\right)\right] \\
& +\frac{Gr\exp\left(dt\right)}{2d^2(1-\Pr)}\left[\exp\left(2\eta\sqrt{\Pr(d+F)t}\right)erfc\left(\eta\sqrt{\Pr}+\sqrt{(d+F)t}\right)+\exp\left(-2\eta\sqrt{\Pr(d+F)t}\right)erfc\left(\eta\sqrt{\Pr}-\sqrt{(d+F)t}\right)\right] \\
& -\frac{Gr\ t}{d(1-\Pr)}\left[(1+2\eta^2)\,ercf\,\eta-\frac{2\eta\exp\left(-\eta^2\right)}{\pi}\right] \\
& +\frac{Gr\,t}{2d(1-\Pr)}\left\{\begin{array}{l}\left[\exp\left(2\eta\sqrt{Ft}\right)erfc\left(\eta\sqrt{\Pr}+\sqrt{ct}\right)+\exp\left(-2\eta\sqrt{Ft}\right)erfc\left(\eta\sqrt{\Pr}-\sqrt{ct}\right)\right] \\ -\frac{\eta\Pr\sqrt{t}}{2\sqrt{Ft}}\left[\exp\left(-2\eta\sqrt{Ft}\right)erfc\left(\eta\sqrt{\Pr}-\sqrt{ct}\right)+\exp\left(2\eta\sqrt{Ft}\right)erfc\left(\eta\sqrt{\Pr}+\sqrt{ct}\right)\right]\end{array}\right\} \\
& -\frac{Gc\exp\left(at\right)}{2(1-Sc)(a-b)}\left[\exp\left(2\eta\sqrt{at}\right)erfc\left(\eta+\sqrt{at}\right)+\exp\left(-2\eta\sqrt{at}\right)erfc\left(\eta-\sqrt{at}\right)\right] \\
& +\frac{Gc\exp\left(at\right)}{2(1-Sc)(a-b)}\left[\exp\left(2\eta\sqrt{Sc(a+R)t}\right)erfc\left(\eta\sqrt{Sc}+\sqrt{(a+R)t}\right)+\exp\left(-2\eta\sqrt{Sc(a+R)t}\right)erfc\left(\eta\sqrt{Sc}-\sqrt{(a+R)t}\right)\right] \\
& -\frac{Gc\exp\left(bt\right)}{2(1-Sc)(b-a)}\left[\exp\left(2\eta\sqrt{bt}\right)erfc\left(\eta+\sqrt{bt}\right)+\exp\left(-2\eta\sqrt{bt}\right)erfc\left(\eta-\sqrt{bt}\right)\right] \\
& +\frac{Gc\exp\left(bt\right)}{2(1-Sc)(b-a)}\left[\exp\left(2\eta\sqrt{Sc(b+R)t}\right)erfc\left(\eta\sqrt{Sc}+\sqrt{(b+R)t}\right)+\exp\left(-2\eta\sqrt{Sc(b+R)t}\right)erfc\left(\eta\sqrt{Sc}-\sqrt{(b+R)t}\right)\right]
\end{aligned}
\qquad (15)
$$

$$where \qquad d=\frac{F}{(1-\Pr)},\quad b=\frac{ScR}{(1-Sc)},\quad c=\frac{F}{\Pr},\quad \eta=\frac{y}{2\sqrt{t}}.$$

4. Results and Discussion

The problem of heat and mass transfer has been formulated, analyzed and solved analytically, for physical understanding to the problems numerical computations were carried out for different physical parameters such asthermal Grashof number Gr, mass Grashof number Gc, Schmidt number Sc, Prandtl number Pr, time t, and acceleration a, upon the nature of flow and transport, the value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor, also the value of Prandtl number Pr are chosen such that they represent air (Pr=0.71). It is observed that the velocity increases with increasing values of Gr, Gc, and a.To access the effects of the various parameters in the flow fields, graphs are presented as follows:

4.1 Velocity profiles

Figures 1 to 6 represent velocity profile for the flow

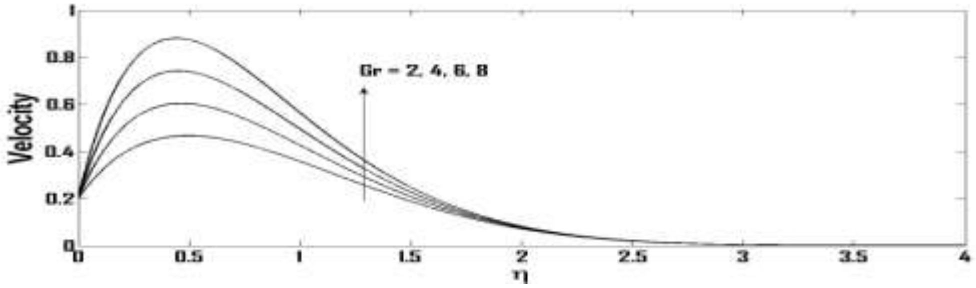

Figure 1: Velocity profiles for different Gr

The velocity profiles for different values of thermal Grashof number (Gr=2, 4, 6, 8) is presented in figure 1. It observed that velocity increases with increasing Gr.

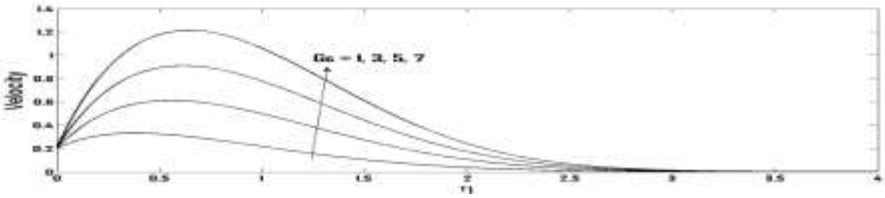

Figure 2 : Velocity profiles for different Gc

The velocity profiles for different values of mass Grashof number (Gc=1, 3, 5, 7) is presented in figure 2. It observed that velocity increases with increasing Gc.

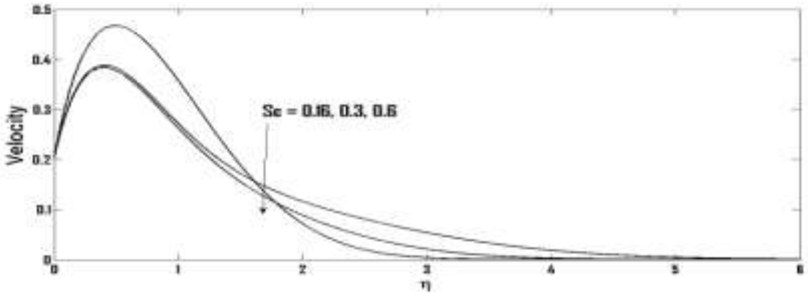

Figure 3 : Velocity profiles for different Sc

The velocity profiles for different values of Schmidt number (Sc= 0.16, 0.3, 0.6) is presented in figure 3. It observed that velocity decreases with increasing Sc.

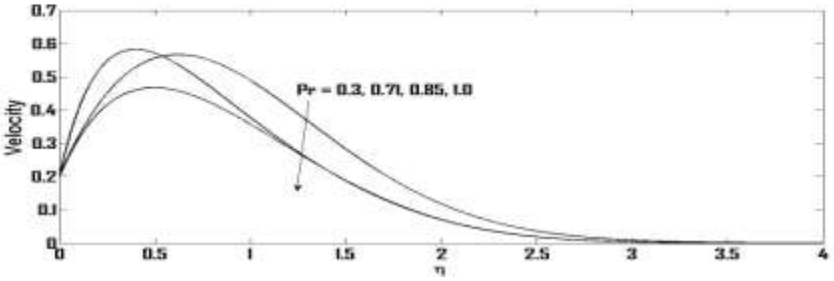

Figure 4 : Velocity profiles for different Pr

The velocity profiles for different values of Prandtl number (Pr= 0.3, 0.71, 0.85, 1.0) is presented in figure 4. It observed that velocity decreases with increasing Pr.

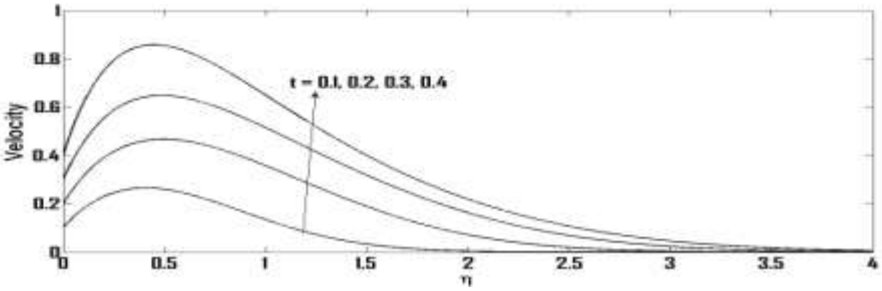

Figure 5 : Velocity profiles for different t

The velocity profiles for different values of time (t= 0.1, 0.2, 0.3, 0.4) is presented in figure 5. It observed that velocity increases with increasing t.

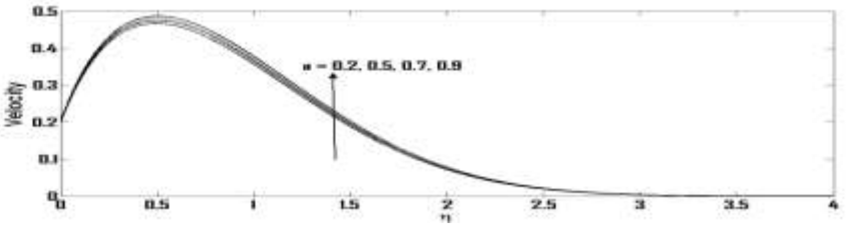

Figure 6 : Velocity profiles for different a .

The velocity profiles for different values of a (a= 0.2, 0.5, 0.7, 0.9) is presented in figure 6. It observed that velocity increases with increasing a.

4.2 Temperature profiles

Figures 7 and 8 represent temperature profiles for the flow

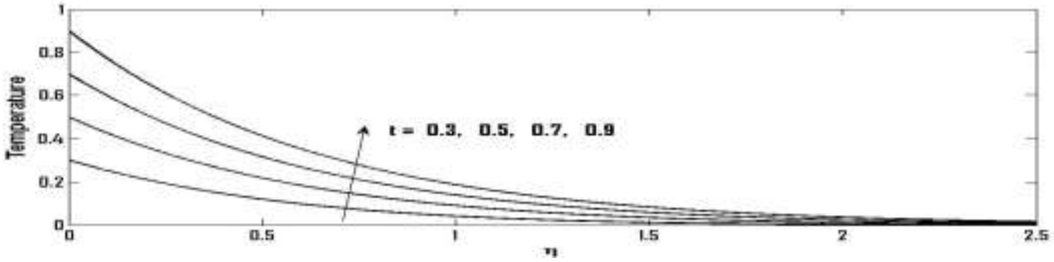

Figure 7: Temperature profiles for different t

The temperature profiles for different values of time (t=0.3, 0.5, 0.7, 0.9) is presented in figure 8. It is observed that temperature increases with increasing t.

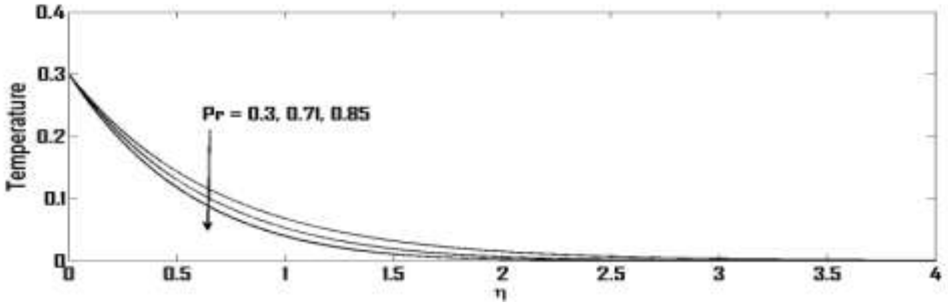

Figure 8 Temperature profiles for different Pr

The temperature profiles for different values of prandtl number (Pr=0.3, 0.71, 0.85) is presented in figure 8. It is observed that temperature decreases with increasing Pr.

4.3 Concentration profiles

Figures 9 and 10 represent concentration profiles for the flow

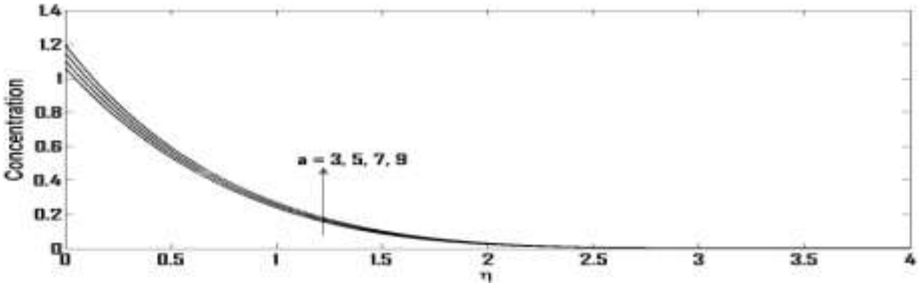

Figure 9: Concentration profiles for different a

The concentration profiles for different values of a (a=3, 5, 7, 9) is presented in figure 9. It is observed that concentration increases with increasing a.

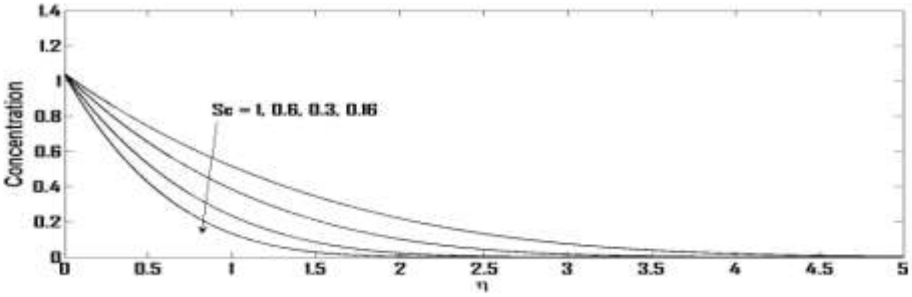

Figure 10: Concentration profiles for different Sc

The concentration profiles for different values of Schmidt number (Sc=1, 0.6, 0.3, 0.16) is presented in figure 10. It is observed that concentration decreases with increasing Sc.

Conclusion:

Analytical solutions of heat and mass transfer with variable temperature and exponential mass diffusion have been studied. The dimensional governing equations are solved by Laplace transform technique. The effect of different parameters such as Schmidt number, Prandtl number, mass Grashof number, thermal Grashof number, and time are presented graphically. It is observed that velocity profile increases with increasing parameter a, t, Gc, and Gr and also decreases with increasing Sc and Pr respectively, it is also observed that temperature and concentration profile increases with increasing t, and inversely, decreases as Sc and Pr increases respectively.

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69. 6 Abbreviations

70. C' Species concentration in the fluid $kg: m^{-3}$
71. C dimensionless concentration
72. C_p Specific heat at constant pressure $J: kg^{-1}: K$
73. D mass diffusion coefficient m^2, s^{-1}

74. Gc mass Grashof number
75. Gr thermal Grashof number
76. g acceleration due to gravity $m\,s^{-2}$
77. k thermal conductivity $W{:}\,m^{-1}s^{-1}$
78. Pr Prandtl number
79. Sc Schmidt number
80. T temperature of the fluid near the plate K
81. t' times
82. t dimensionless time
83. u veocity of the fluid in the x'-direction $m\,s^{-1}$
84. u_0 velocity of the plate $m\,s^{-1}$
85. u
dimensionless velocity
86. y coordinate axis normal to the plate m
87. Y dimensionless coordinate axis normal to the plate
88. α thermal diffusivity $m^{-2}s^{-1}$
89. β volumetric coefficient of thermal expansion k^{-1}
90. β^* volumetric coefficient of expansion with concentration k^{-1}
91. μ coefficient of viscosity $Ra.s$
91.1. kinematic viscosity $m^{-2}s^{-1}$
92. ρ density of the fluid $kg\,m^{-3}$
93. T dimensionless skin-friction $kg,\,m^{-1}s^{-2}$
94. θ dimensionless temperature
95. η similarity parameter
96. $ercf$ complementary error function